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Transformation of the Equivalent Linear Method for solving soil-structure interaction problems in the time domain

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Abstract: In the context of soil-structure interaction, often underestimated in design codes, accurate anticipation of dynamic amplification and foundation displacements has become increasingly crucial. Recent earthquakes like Mexico City (1985), Loma Prieta (1989), Port-au-Prince (2010), and Kumamoto (2016) highlighted the need for robust methods to analyze these complex nonlinearities. While both nonlinear and equivalent linear approaches offer valuable insights, limitations in computational efficiency and implementation can hinder their practical application. This paper addresses these challenges by presenting the formal development of a novel quasi-nonlinear method for real-time seismic response analysis, building upon the principles introduced by the author. This innovative approach, unlike traditional equivalent linear methods, dynamically calculates mean shear moduli and local secant stiffnesses, integrating them into a Newmark-based solution process. The paper further validates the method against the Santa Cruz earthquake (Loma Prieta 1989) on a sandy soil profile, demonstrating remarkable agreement with reference solutions obtained using established methods like Shake91 and Quad4m. This comprehensive formulation paves the way for accurate and efficient real-time reconstruction of seismic responses within a practical quasi-nonlinear framework.

1. Introduction

Soil-structure interaction (SSI), often underestimated in regulations, plays a crucial role in the seismic design of buildings, as evidenced by the lessons learned from the Mexico (1985), Loma Prieta (1989) and Kumamoto (2016) earthquakes. These major seismic events revealed unexpected seismic amplification phenomena at the foundations of structures, thus highlighting the need for a better understanding and consideration of this phenomenon in the seismic design of buildings.

Modelling SSI relies on two main approaches: purely non-linear (NL) and equivalent linear (EQL) methods. Choosing between them is often debated, as research reveals strengths and limitations in each. While NL methods offer greater accuracy (Fatahi and Tabatabaiefar 2014), their complexity and computational demands can be significant. Studies have demonstrated the effectiveness of EQL in approximating experimental results (Nguyen et al. 2017, Li et al. 2017), but comparisons by Fatahi and Tabatabaiefar (2014) highlight the underestimation of lateral deformations with EQL, confirmed by Luo et al. (2016) in 3D finite element analyses.

Despite limitations, EQL's simplicity and applicability make it attractive. However, a fundamental hurdle lies in the constant stiffness and damping moduli used throughout the analysis. This presents significant drawbacks when

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following the time history response within systems or interfaces (Bakhtaoui et al., 2024).

This paper addresses these challenges by proposing a novel improvement to the equivalent linear method, tackling its limitations while preserving its advantages. This innovative approach, unlike traditional EQL methods, dynamically calculates mean shear moduli and local secant stiffnesses, integrating them into a Newmark-based solution process, building upon the following methodology (Bakhtaoui 2024):

1. Determining the initial static stiffness and damping of the soil-structure system.
2. Calculating the mean shear modulus of the soil at each time step.
3. Calculating the local secant stiffnesses of the soil at each time step.
4. Solving the Newmark equations to obtain the dynamic response of the system.

It's important to note that this procedure isn't meant to re-demonstrate the Newmark formulation or the equivalent linear method. Rather, it's about demonstrating the technique that enables us to transition from a classical equivalent linear method, implemented in codes such as Shake91 (Idriss and Sun 1992) or Quad4m (Hudson et al. 1994), to a method that can be classified as pseudo-nonlinear, or an equivalent linear method that's adjusted to a temporal approach.

The effectiveness of the new procedure is validated against a classic case of earthquake propagation. This case study, represented in this paper, considers the response of a geotechnical profile of sandy soil subjected to the seismic acceleration of Santa Cruz (Loma Prieta 1989). The results are compared to the reference solutions provided by the established benchmark codes Shake91 (Idriss and Sun 1992) and Quad4m (Hudson et al. 1994). It is worthy to note that the work presented and the applications used have focused primarily on the restitution of soil behaviour at each time step. However, the method is intended to be implemented in a model including structures for the study of soil-structure interaction effects.

2. Methodology

From the virtual work theorem, the equations of motion of a given body subjected to an external load {Q(t)}, can generally be summarised by the differential equation system:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{Q(t)\} \tag{1}$$

[M], [C], and [K] are respectively the nodal matrices of mass, damping and stiffness of the idealised body, whose motion is characterised by the accelerations $\{\ddot{u}\}$, velocities $\{\dot{u}\}$, and displacements $\{u\}$ at the nodes of the finite elements composing the object of study.

The resolution of this system can be carried out by several resolution processes, the most popular of which are summarized by the finite differences, the Newmark approach or the Wilson-Θ approach (Bathe 1996, Chopra 2012). The EQL method applied to finite elements, which generally uses an implicit type Newmark resolution process, which means that the equations are solved simultaneously for all time steps. The method also uses an iterative loop to ensure convergence, which means that the solution is repeated until it converges to a stable value.

This method can be expressed at time t, as follows (Hudson et al 1994):

$$\mathbf{M}\ddot{\mathbf{u}}_{\mathbf{i+1}} + \mathbf{C}_i\,\dot{\mathbf{u}}_{\mathbf{i+1}} + \mathbf{K}_i\,\mathbf{u}_{\mathbf{i+1}} = \boldsymbol{Q}_{i+1} \tag{2}$$

In general, the direct resolution of the motion by Newmark's method is based on an approximation of the velocities and displacements at time i+1. This leads to :

$$\left(\mathbf{K} + \frac{\gamma}{\beta\,\Delta t}\,\mathbf{C} + \frac{\mathbf{M}}{\beta\,\Delta t^2}\right)\mathbf{u}_{\mathbf{i+1}} = \mathbf{a_i}\,\mathbf{M} + \mathbf{b_i}\,\mathbf{C} + \mathbf{Q}_{\mathbf{i+1}} \tag{3}$$

With:

$$\mathbf{a_i} = \frac{\mathbf{u_i}}{\beta\,\Delta t^2} + \frac{\dot{\mathbf{u}}_\mathbf{i}}{\beta\,\Delta t} + \ddot{\mathbf{u}}_\mathbf{i}\left(\frac{1}{2\beta} - 1\right) \tag{4}$$

$$\mathbf{b_i} = \frac{\gamma\,\mathbf{u_i}}{\beta\,\Delta t} + \dot{\mathbf{u}}_\mathbf{i}\left(\frac{\gamma}{\beta} - 1\right) + \ddot{\mathbf{u}}_\mathbf{i}\left(\frac{\gamma}{2\beta} - 1\right)\Delta t \tag{5}$$

In these equations, the coefficients γ and β are the Newmark coefficients, ensuring the stability condition of the method (Bathe 1996). In the simulations used, the values used to have an unconditionally stable method are: γ=0.5 and β=0.25

Typically, the solution of the system (equations 2), at time step i+1 (for the seismic event j), gives a displacement vector $\boldsymbol{u}_{i+1}^{j}$, which allows us to calculate the shear strain γ_{i+1}^{e} for each element (e). The classic EQL method recommends calculating the maximum shear deformations obtained for each element over the entire seismic event during the j^{th} iteration, which represents the moments going from i=1 to N.

Assuming that γ_1^e is known at time step i=1, and the maximum elementary deformations can be expressed vectorially at the end of seismic event j by:

$$(\boldsymbol{\gamma_N})^{\boldsymbol{j}} = [\max \underbrace{\gamma_{i+1}^e}_{1<i\leq N}]^j \quad (6)$$

Where:

γ_{i+1}^e: represents the elementary shear deformation (e) at the instant i+1

$(\boldsymbol{\gamma_N})^{\boldsymbol{j}}$: represents the vector of maximum shear deformations obtained in the elements at the end of the event; that is, the instant i=N, in iteration j.

In addition, the equivalent linear method generally assumes a uniform distribution coefficient $\alpha_{uniforme}$ of shear deformations that can be calculated from the magnitude "M" of the earthquake, by

$$\alpha_{uniforme} = (M-1)/10 \quad (7)$$

This makes it possible to evaluate the uniform shear strain vector at the end of the seismic event j by:

$$(\boldsymbol{\gamma_N})^{\boldsymbol{j}}_{\boldsymbol{max\,uniforme}} = \alpha_{uniforme} \times (\boldsymbol{\gamma_N})^{\boldsymbol{j}} \quad (8)$$

This relation, which is the basis of EQL, uses the maximum deformation of each finite element composing the soil model subjected to seismic action, which is only obtained at the end of a given seismic event (itération j), by the maximum uniform deformation $(\boldsymbol{\gamma_N})^{\boldsymbol{j}}_{\boldsymbol{max\,uniforme}}$. This implies that the respective stiffness and damping parameters, d^j and G_N^j/G_{max}, are only determined in each element at the end of the iteration, using the appropriate constitutive curves.

This method has a drawback in that the solution is deemed reliable only after a number of events "j," which depends on the predetermined convergence criterion "kc." This can be illustrated by a convergence ratio vector $\boldsymbol{kr}$ for the elements.

$$\boldsymbol{Ratio}(\boldsymbol{kr^j}) = (\boldsymbol{G}_N^j - \boldsymbol{G}_N^{j-1}) / \left[(\boldsymbol{\gamma}_N)^{\boldsymbol{j}}_{\boldsymbol{max\,uniforme}} - (\boldsymbol{\gamma}_N)^{\boldsymbol{j-1}}_{\boldsymbol{max\,uniforme}}\right] \quad (9)$$

implying that;

> $If : kc \geq \boldsymbol{Ratio}(\boldsymbol{kr^j})$, *for all elements* ➔ There is Convergence ➔ the analysis is successful.

$If : kc < Ratio(\boldsymbol{kr}^j)$ *For all or some elements* ➔ It appears that the calculation has not converged, and therefore it is required to redo the calculation in a new iteration, denoted as j+1.

This entails adjusting the stiffness for a new iteration (event) j+1, while using the same seismic action. However, correction coefficients must be applied to the subsequent elementary stiffnesses:

$$\boldsymbol{\alpha}_{corr}^j = \boldsymbol{G}_N^j / \boldsymbol{G}_N^{j-1} = f(\boldsymbol{\gamma}^{\mathbf{j}}, \mathbf{Po}, \mathbf{PI}, \mathbf{Gmax}, \mathbf{etc} \ldots.) \quad (10)$$

In this writing, $\boldsymbol{G}_N^j$ and $\boldsymbol{G}_N^{j-1}$ represent the vectors of maximum shear moduli for the elements during events j and j-1. The function $f(\boldsymbol{\gamma}^{\mathbf{j}}, \mathbf{Po}, \mathbf{PI}, \mathbf{etc} \ldots.)$ depends on several parameters defined by the constitutive model adopted. In this expression, $\boldsymbol{\gamma}^{\mathbf{j}}$, $\mathbf{Po}$, $\mathbf{PI}$, **G**max are respectively, the shear deformation, the average effective confining stress, the index plasticity, and the shear modulus. These parameters could express in vector form, depending on the form given to the formulation of the correction coefficient.

The correction of the local stiffness matrices can therefore be written as:

$$\boldsymbol{k^j} = \boldsymbol{\alpha}_{corr}^j \ \boldsymbol{k^{j-1}} \quad (11)$$

This enables updating equation (2) with corrected local stiffness and damping, previously formed using Hudson et al's procedure (1994). Which gives after assembling in the global system:

$$\mathbf{M\ddot{u}_{i+1}} + \mathbf{C}^{\boldsymbol{j}} \, \mathbf{\dot{u}_{i+1}} + \mathbf{K}^{\boldsymbol{j}} \, \mathbf{u_{i+1}} = \boldsymbol{Q_{i+1}} \quad (12)$$

In this system of equations, the initial conditions are generally fixed such that $\boldsymbol{k^1} = \boldsymbol{k_0}$, for the local stiffness matrices, $\boldsymbol{K^1} = \boldsymbol{K_0}$, for the global stiffness matrices, $\mathbf{C^1} = \mathbf{C_0}$, for damping ratios, and $\boldsymbol{\alpha}_{corr}^1 = \{1\}$, for the correction coefficient.

The method of updating the system of equations at the end of an event is a useful technique in soil dynamics. It allows for moving on to the next calculation in the next iteration j+1, until convergence is ensured on all elements. However, this method has its limitations in the case of complex free-field analyses, contact problems or non-linear structural behaviour, where it is necessary to have a solution at each instant i+1.

Proposed modifications

To overcome this obstacle, a modification can be proposed by adjusting the stiffness and damping of the system of equations at each instant. This allows for a solution at each instant, according to an iterative procedure that ensures convergence. Compared to the previous method, this modified method does not depend on the number of events, but on the necessary iterations k at each instant i.

Thus, equation (8) is rewritten, providing a more accurate and detailed solution.

$$(\boldsymbol{\gamma_{i+1}})^{\boldsymbol{k+1}}_{\boldsymbol{uniforme}} = \alpha_{uniforme} \times (\boldsymbol{\gamma_{i+1}})^{k+1} \quad (13)$$

As we see, it is no longer a question of calculating the maximum shear deformation $(\boldsymbol{\gamma_N})^{\boldsymbol{j}}$ in the elements, at the end of the j event (when i=N), but of managing the deformation at each moment i+1. This involves using a convergence criterion similar to that of equation 9, but iteratively (with k iteration) for each element. That gives:

$$Ratio(\boldsymbol{kr}^{k+1}) = (\boldsymbol{G}^{k+1}_{i+1} - \boldsymbol{G}^{k}_{i+1}) / [(\boldsymbol{\gamma_{i+1}})^{\boldsymbol{k+1}}_{\boldsymbol{uniforme}} - (\boldsymbol{\gamma_{i+1}})^{\boldsymbol{k}}_{\boldsymbol{uniforme}}] \quad (14)$$

implying that;

If $\boldsymbol{kc} \geq Ratio(\boldsymbol{kr}^{k+1})$ ➔ *for all elements* ➔ There is Convergence, ➔ the analysis is successful

If $\boldsymbol{kc} < Ratio(\boldsymbol{kr}^{k+1})$ ➔ There is divergence on part or all of the elements (e), which implies moving to a correction of stiffness and damping according to a correction vector of the elements by:

$$\boldsymbol{\alpha}^{k+1}_{corr} = \boldsymbol{G}^{k+1}_{i+1} / \boldsymbol{G}^{k}_{i+1} = f(\boldsymbol{\gamma}^{\mathbf{k+1}}, \mathbf{Po}, \mathbf{PI}, \mathbf{Gmax}, \mathbf{etc} \ldots.) \quad (15)$$

$$\boldsymbol{k}^{k+1}_{i+1} = \boldsymbol{\alpha}^{k+1}_{corr} \times \boldsymbol{k}^{k}_{i+1} \quad (16)$$

Where the terms of $f(\boldsymbol{\gamma}^{\mathbf{k+1}}, \mathbf{Po}, \mathbf{PI}, \mathbf{Gmax}, \mathbf{etc} \ldots.)$ were previously defined.

That leads for the time i+1, the updating of the system of equation (2) at iteration k, after assembling the updated stiffnesses and local damping, previously formed using the procedure described by Hudson et al (1994), can be written as:

$$\mathbf{M}\ddot{\mathbf{u}}^{\boldsymbol{k+1}}_{\boldsymbol{i+1}} + \mathbf{C}^{\boldsymbol{k+1}}_{\boldsymbol{i+1}}\,\dot{\mathbf{u}}^{\boldsymbol{k+1}}_{\boldsymbol{i+1}} + \boldsymbol{K}^{\boldsymbol{k+1}}_{\boldsymbol{i+1}}\,\mathbf{u}^{\boldsymbol{k+1}}_{\boldsymbol{i+1}} = \boldsymbol{Q}_{\boldsymbol{i+1}} \quad (17)$$

We can clearly see in this equation 17, compared to equation 12 that for a given instant i+1, the updating of the stiffness and damping matrices will occur iteratively several times depending on the updating of the movement until the total convergence of the system at k+1 iterations before moving to the next instant.

This approach allows step-by-step calculations during a seismic event, eliminating the need to wait for the entire event to finish. This provides a continuous understanding of how soil parameters, representing the system's state, change as the loading increases. Importantly, integrating this method with the direct secant approach, which solves nonlinear equations in the EQL method, is straightforward. In simpler terms, this method allows real-time adjustments to soil parameters at each time step, effectively capturing the dynamic nature of soil behaviour.

Constitutive model and reduction curves

The method uses the same constitutive model as the equivalent linear model (EQL), which represents the nonlinear stress-strain behaviour of soil by the secant stiffness Gsec and the equivalent damping ratio d^{k+1} compatible with the variation of soil deformation induced by soil vibrations.

In other words, the soil behaviour is represented by mean reduction curves of type d^{k+1} *and* G^{k+1}_{i+1}/G_{max}, as a function of compatible distortion $\boldsymbol{\gamma}^{\mathbf{k+1}}$. These curves, based on cyclic relationships, take into account the hysteretic properties of the soil, using the Hardin and Drnevich model (1972) with its necessary modifications (Darendeli 2001). These hyperbolic relationships actually depend on various parameters such as cyclic shear strain amplitude $\boldsymbol{\gamma}$, soil type, plasticity index (PI), average effective confining stress Po, loading frequency (f), number of loading cycles (N) and soil structure (Seed and Idriss 1970, Seed et al. 1986, Stokoe et al. 1999, Sun et al. 1988, Vucetic and Dobry 1991, Bedr et al. 2018). It is therefore easier to use the appropriate curves in generic or data form, which have already been the subject of numerous applications in geodynamics (Lysmer et al. 1975, Kramer 1996, Idriss and Sun 1992, Bardet et al. 2000).

In this paper, the reduction curves used refer to these formulations, which have been adapted in the form of tables in the original applications (Shake91 and Quad4M user manuals).

3. Applications

This example, drawn from the Quad4M user manual, simulates the propagation of seismic waves through a geological profile consisting of a 100-foot layer of sand overlying a rigid rock base. The soil profile, represented by 20 distinct layers (Figure 1), is assumed to be horizontally stratified. The rock base is subjected to the horizontal component of the Loma Prieta earthquake recorded at Santa Cruz, California (Figure 2). Nonlinear soil behaviour during the earthquake is represented by specific reduction curves for shear modulus and damping, while soil properties are specified as per the Quad4M user manual (see appendix 1 & 2 and figure 3). It is important to note that in this example, the amplitude of the horizontal component of the Loma Prieta earthquake applied to the sand profile is reduced by a factor coefficient of 0.6.

The simulation utilizes three different numerical codes. The first, SHAKE91, employs an analytical approach to solve the Shear-wave propagation problem. The second,

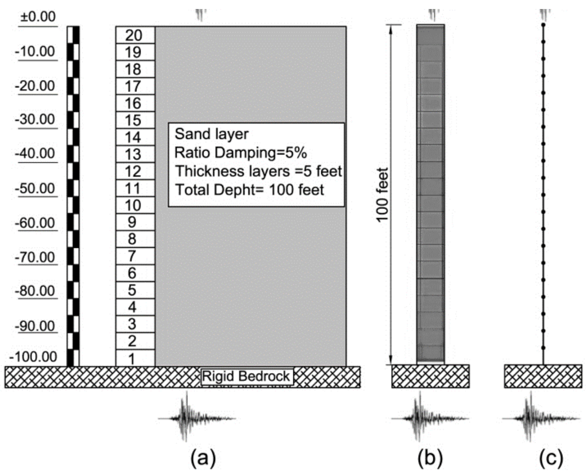


Figure 1: Representation of the sandy layer by: a) the geological model, b) a two-dimensional column, c) a one-dimensional column

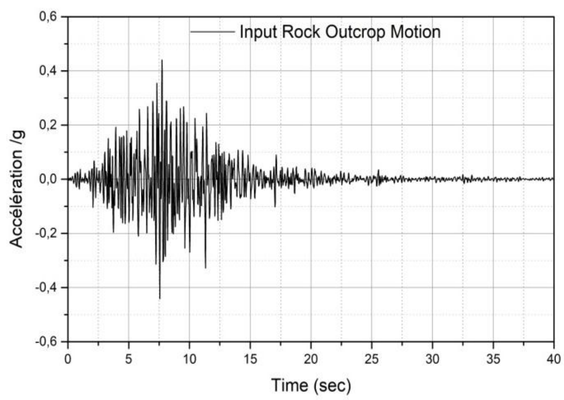


Figure 2: Values of accelerations at the bedrock (Santa Cruz)

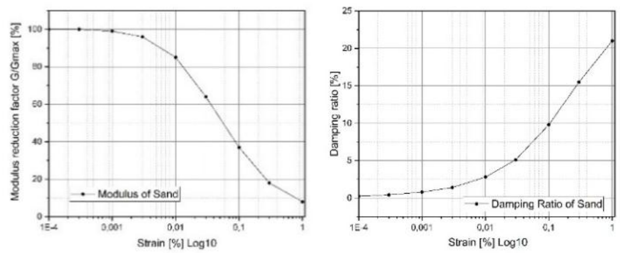


Figure 3: reduction Curves of sand used

CQuad4M, utilizes the finite element method (FEM) as described by Hudson et al. (1994), while the third is a FORTRAN code, developed previously for free-field problems with 3 nodes. This code, called Biss2D (Bakhtaoui 2024), also integrates the formulation developed previously for Flac2/3D and Abaqus for 2-node column elements. It was not planned to develop this part in this article, but the reader can refer to the references extracted from Flac2d/3d or Abaqus (NIELSEN 2006).

Regarding the calculations with 4-node bidimensional elements performed with the CQUAD4M code, it should be mentioned that the code used is essentially based on the Quad4M code, with a few minor differences such as the ability to calculate the approximate natural period of a geotechnical profile, to control the Newmark or Rayleigh parameters, or to be able to use beam elements. This procedure (CQUAD4M) has also already been used for validation (Bakhtaoui 2024).

All codes utilize finite element column-type models to represent the problem. While CQuad4M employs four-node quadrilateral elements, the proposed model utilizes two-and three-node linear elements.

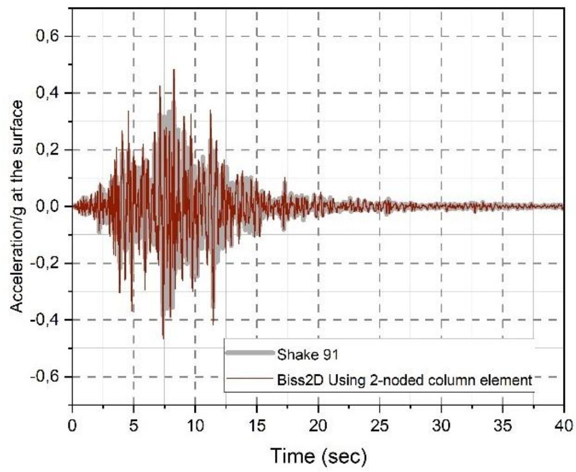
This comparative study aims to demonstrate the accuracy of the proposed model using the new code (BISS2D). Therefore, by comparing the results obtained from the three codes, we can evaluate the effectiveness of the modified approach to capture the complex nonlinear behaviour of the ground in a classic case of seismic wave propagation, used in several research works for validation (Shake91 manual, Quad4M, Flac3D7.0 etc.).

Results

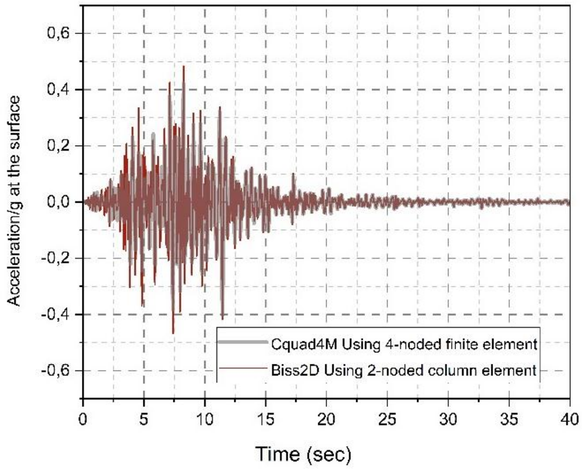
The results, presented in Figures 4 and 5, demonstrate a good overall agreement between the responses of the model's upper section, representing the soil profile, compared to those obtained with established codes like Shake91 and Quad4M (here CQuad4M).

However, minor discrepancies in response time can be observed. These discrepancies are likely due to the specific Rayleigh coefficients chosen for calculating the evolution of local damping matrices within the different software packages (Poul and Zerva 2018). Rayleigh coefficients essentially control how damping is incorporated into the soil behaviour, and their values can vary slightly depending on the implemented approach.

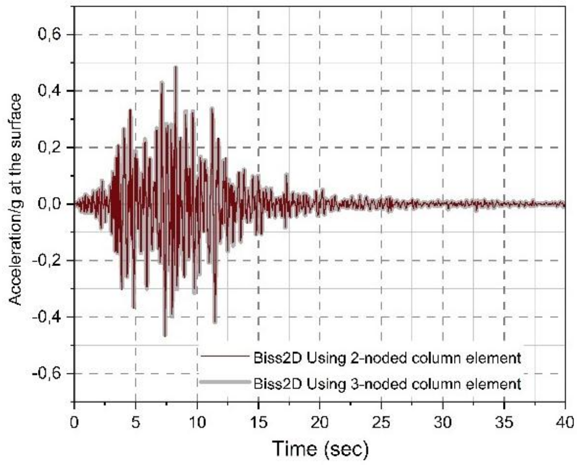
Achieving even closer alignment with precise analytical or experimental solutions, if desired, may require further investigations to meticulously examine the combined influences of damping and the proposed solution method on the overall outcome. This could involve exploring a wider range of Rayleigh coefficients (Kwok et al., 2007; Amorosi et al., 2010) and frequency terms within the Rayleigh formulation itself (Park and Hachach, 2004). Additionally, the possibility of employing alternative solution methods, inspired by those proposed by Bathe (1996), could be explored for achieving potentially improved accuracy and potentially different insights into the problem.



(a)



(b)



(c)

Figure 4: comparison of free field surface accelerations: a) with Shake 91, b) with Quad4m, c) with BISS2D using 2 and 3-noded column elements

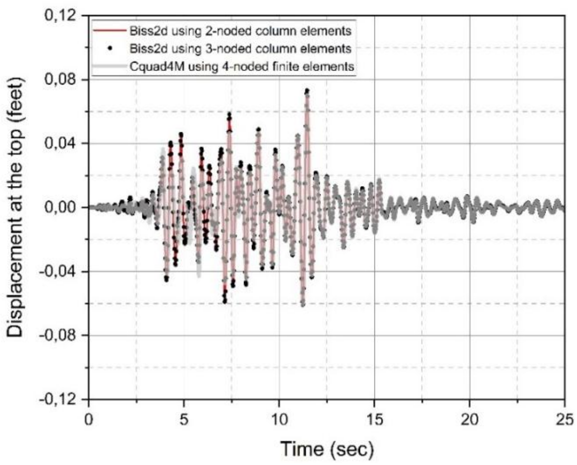


Figure 5: Comparison of surface displacements with CQuad4m

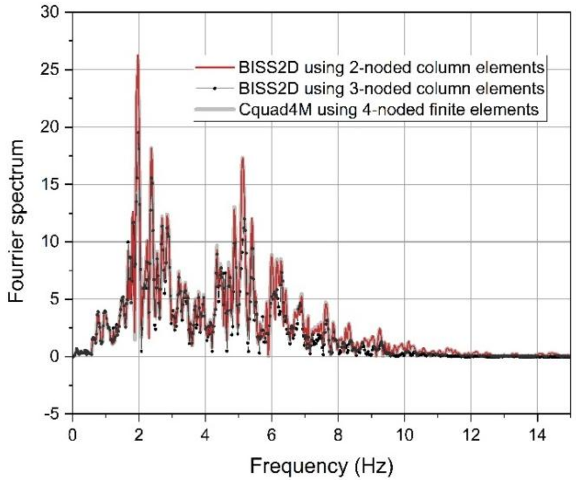


Figure 6: Comparison of free field surface Fourier spectra

The verification also consisted of comparing the surface Fourier spectrum of the accelerations provided by BISS2D and CQuad4M codes (Figures 6) to verify the restitution of the frequency content by the new procedure. In terms of total energy (97%) over the bandwidth (of approximately ± 8.35 Hz), the acceleration responses calculated by the 03 approaches show remarkable proximity to the results. The total energy represented on the surface by each of the three acceleration responses or by the power spectrum, according to the Parseval relation, is practically identical.

4. Conclusion

This article proposes an original modification of the equivalent linear method to simulate problems related to soil dynamics, by restoring the nonlinear behaviour of the medium using an update of the soil properties at each instant. Unlike the EQL method which updates these parameters only at the beginning of loading but uses the same seismic event several times, the procedure makes it possible to follow the soil response on a single seismic event, updating the parameters as they go along. Using a finite element program specially dedicated to this purpose, the procedure was validated on a classic case of seismic event by comparing the results found with those of the Shake 91 and Quad4M reference examples. This practical method for representing the evolution of geotechnical profile responses during a seismic event can be presented as a good alternative to comparable 2D models and can be integrated into an ISS calculation code, The proposed method offers several advantages over existing methods, including:

- Real-time capability: The proposed method can be implemented in real time, making it a practical tool for seismic design and engineering applications.
- Efficient implementation: The proposed method is more efficient than nonlinear methods, making it a more feasible option for practical applications.

In addition to the advantages mentioned above, the proposed method also has the potential benefits to improve the design of buildings to withstand earthquakes.

Disclosures

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References

Amorosi A, Boldini D, Elia G. Parametric study on seismic ground response by finite element modelling. Comput Geotech 37(4):515–528 (2010)

Bakhtaoui Y (2024). New procedure for soil-structure interaction problems incorporating free-field effects by 2D quadratic elements. Computers and Geotechnics, 165, 1-21

Bardet JP, Ichii K, and Lin CH, 2000. EERA: a computer program for equivalent-linear earthquake site response analyses of layered soil deposits. University of Southern California, Los Angeles.

Bathe KJ. Finite Element Procedures. Prentice Hall, 1996.

Bedr S, Mezouar N, Verrucci L, Lanzo G, 2018. Investigation on shear modulus and damping ratio of Algiers marls under cyclic and dynamic loading conditions.

Chopra Anil K. Dynamics of Structures. Theory and applications to Earthquake Engineering. Fourth Edition. Prentice-Hall International, 2012.

Darendeli MB. Development of a new family of normalized modulus reduction and material damping curves. Dissertation, The University of Texas at Austin (2001).

Fatahi B, and Tabatabaiefar HR, 2014. Fully Nonlinear Versus Equivalent Linear Computation Method for Seismic Analysis of MidRise Buildings on Soft Soils. International Journal of Geomechanics 14 (4): 04014016

FLAC3D Software (2019). Itasca Consulting Group, Inc. « Fast Lagrangian Analysis of Continua User's Manual, Version 7.0», "Dynamic analysis", Minneapolis, USA

Flac3D7.0 (2022). « Comparison of FLAC3D to SHAKE for a Layered, Nonlinear-Elastic Soil Deposit » Itasca Software 9.0 documentation: Options/Dynamic Analysis/ Verification problems. Itasca Consulting Group Inc.

Hardin BO and Drnevich VP. Shear Modulus and Damping in Soils: Design Equations and Curves. Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM6, June, pp. 603-624 (1972).

Hudson M, Idriss IM and Beikae M. User's Manual for QUAD4M: A Computer Program to Evaluate the Seismic Response of Soil Structures Using Finite Element Procedures and Incorporating a Compliant Base. Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California Davis, California (1994).

Idriss, I.M. and J.I. Sun. "User's Manual For Shake91: A Computer Program for Conducting Equivalent Linear Seismic Response Analyses of Horizontally Layered Soil Deposits." Center for Geotechnical Modeling, Department of Civil and Environmental Engineering. Davis, California: University of California. 1992.

Kramer SL, 1996. Geotechnical earthquake engineering. Prentice Hall, New Jersey.

Kwok A.O.L., Stewart J.P., Hashash Y.M.A., Matasovic N., Pyke R., Wang Z., and Yang Z., 2007. Use of Exact Solutions of Wave Propagation Problems to Guide Implementation of Nonlinear Seismic Ground Response Analysis Procedures. Journal of Geotechnical and Geoenvironmental Engineering Volume 133, Issue 11

Li Peizhen, Liu Shutong, Lu Zheng, Yang Jinping. 2017. Numerical Analysis of a Shaking Table Test on Dynamic Structure Soil-Structure Interaction Under Earthquake Excitations. Struct Design Tall Spec Build. (2017).

Luo et Chuan, Yang Xun, Zhan, Changbao, Jin Xianlong, Ding Zhenkun. 2016. Nonlinear 3D element analysis of soil-pile-structure interaction system subjected to horizontal earthquake excitation. Soil Dynamics and Earthquake Engineering 84, 145-156 (2016).

Lysmer J, Udaka T, Tsai C, Seed HB, 1975. FLUSH - A Computer Program For Approximate 3-D Analysis Of Soil-Structure Interaction Problems. Technical Report, EERC-75-30.

Nguyen, QV, Fatahi B, and Hokmabadi AS. 2017. Influence of size and load-bearing mechanism of piles on seismic performance of buildings considering soil–pile–structure interaction. Int. J. Geomech.,10.1061/(ASCE)GM.1943-5622.0000869, 04017007 (2017).

Nielsen Andreas H, 2006. Absorbing Boundary Conditions for Seismic Analysis in ABAQUS. ABAQUS Users' Conference.

Park D, Hashash YMA. Soil damping formulation in nonlinear time domain site response analysis. J Earthq Eng 8(2):249–274 (2004).

Poul MK, Zerva A. Efficient Time-Domain Deconvolution Of Seismic Ground Motions Using The Equivalent-Linear Method For Soil-Structure Interaction Analyses. Soil Dynamics and Earthquake Engineering 112 (2018) 138–151 (2018).

Schnabel PB, Lysmer J, and Seed HB. SHAKE - A computer program for earthquake response analysis of horizontally layered soils. Report No. EERC-72/12, University of California, Berkeley (1972).

Seed BH, Idriss IM, 1970. Soil Moduli and Damping Factors for Dynamic Response Analyses. UCB/EERC-70-10, Earthquake Engineering Resource Center, University of California, Berkley, California.

Seed BH, Wong RT, Idriss IM, and Tokimatsu K. 1986. Moduli and Damping Factors for Dynamic Analyses of Cohesionless Soils. Journal of Geotechnical Engineering Volume 112, Issue 11(1986).

Stokoe KH, Darendeli MB, Andrus RD, Brown LT. Dynamic soil properties: Laboratory, field and correlation studies. International Conference on earthquake geotechnical engineering (Lisboa 1999-06-21) A.A. Balkema, Brookfield (1999)

Sun J, Golesorkhi R, Seed HB. Dynamic moduli and damping ratio for cohesive soils. University of California Berkeley-Earthquake Engineering Research Center (1988).

Tabatabaiefar HR. Fully Nonlinear Versus Equivalent Linear Computation Method for Seismic Analysis of MidRise Buildings on Soft Soils. International Journal of Geomechanics 14 (4): 04014016 (2014).

Vucetic M and Dobry R, 1991. Effect of Soil Plasticity on Cyclic Response. Journal of Geotechnical Engineering, 117, 89-107 (1991).

Appendix 1: Sand Characteristics

	Layer- Characteristics				
	Gmax (ksf)	Thickness(ft)	Unit Weight (kcf)	Ratio damping (%)	Soil type
1	4549,74	5,0	0,125	5,0	1
2	4451,85	5,0	0,125	5,0	1
3	4351,76	5,0	0,125	5,0	1
4	4249,31	5,0	0,125	5,0	1
5	4144,32	5,0	0,125	5,0	1
6	4036,61	5,0	0,125	5,0	1
7	3925,94	5,0	0,125	5,0	1
8	3812,06	5,0	0,125	5,0	1
9	3694,67	5,0	0,125	5,0	1
10	3573,43	5,0	0,125	5,0	1
11	3447,93	5,0	0,125	5,0	1
12	3317,69	5,0	0,125	5,0	1
13	3182,11	5,0	0,125	5,0	1
14	3040,50	5,0	0,125	5,0	1
15	2891,96	5,0	0,125	5,0	1
16	2735,37	5,0	0,125	5,0	1
17	2482,23	5,0	0,125	5,0	1
18	2097,87	5,0	0,125	5,0	1
19	1625,00	5,0	0,125	5,0	1
20	938,19	5,0	0,125	5,0	1

Appendix 2: Reduction Curves

Dynamic properties of sand		
Shear Strain	G/Gmax	Ratio
0.0001	100	0.24
0.0003	100	0.42
0.001	99	0.80
0.003	96	1.40
0.01	85	2.80
0.03	64	5.10
0.1	37	9.80
0.3	18	15.50
1	8	21.00