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Fast Lagrangian analysis of the short-term bearing capacity of shallow foundations in clays on the basis of the cone penetration test

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Abstract: Based on a numerical modelling of the cone penetration test CPT as well as the vertical response of shallow foundation in saturated clays, derivation of the cone factor N_k led to propose a CPT-based method of computation of the bearing capacity. This study was carried out by launching a detailed parametric study using the software FLAC-2D (Fast Lagrangian Analyses of Continua) based on the finite difference method. The soil material was modeled as a homogeneous elastic-perfectly plastic medium obeying the Mohr-Coulomb criterion, and the effects of the soil rigidity index I_r, the cone-soil interface, and the initial depth of the penetration on the cone factor N_k were investigated leading to an analytical formulation of N_k. After a comparative study with those proposed in the literature, the cone factor N_k was used to derive the bearing capacity factor K_c of a shallow foundation. Compared to current CPT-based methods proposed in the literature, this method reasonably predicts the short-term bearing capacity of shallow foundations in saturated clayey soil.

1. Introduction

The CPT test is one of the most commonly used geotechnical tests because of many advantages compared to other tests, the main ones are the continuity of measurement of the soil resistance and the simplicity of the experimental procedure. Many researchers interpreted the soil response to the penetration of a conical tip in order to derive the geotechnical properties. Such an approach was initially undertaken within the theoretical framework, but the studies were subsequently diversified by using the physical modelling based on centrifuge models or in calibration chamber, or the numerical modelling usually based on the finite elements method (FEM) or the finite difference method (FDM), in order to correlate the soil strength and stiffness properties with the cone resistance q_c.

According to Lunne et al. (1997), Desai et al. (1984), Liyanapathirana (2009) and Kim et al. (2006) despite important efforts devoted to explore this field, it still does not have a rigorous solution, and the difficulty of this problem is mainly due to the large deformations involved during the penetration and the complexity of the soil stress-strain behavior.

The aim of this paper is to develop a practical method belonging to the category of theoretical methods of bearing capacity, by carrying out an extensive parametric study, based on the software FLAC, with focus on the key parameters affecting the cone factor N_k in a saturated fine soil, as well as on the load-settlement response of circular or strip footing resting on such a soil. The bearing capacity was derived by correlating the CPT-based bearing capacity factor K_c to N_k as well as to the soil rigidity index I_r.

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2. Brief review of the existing methods

Methods of computation of the bearing capacity of shallow foundations based on the CPT can be subdivided according to their basis into three main categories, namely empirical, semi-empirical or theoretical methods. The first category, generally derived from the analysis of full-scale loading tests of foundation, is based on a pragmatic approach by circumventing the difficulties of a theoretical model of the foundation/soil interaction, and the inherent uncertainties to such models (Bouafia 2021).

In fine soils exhibiting an undrained behavior, the CFEM (Canadian Foundation Manual) recommendations stipulate to calculate the bearing capacity q_l as follows:

$$q_l = K_c q_{ce} + q_0 \tag{1}$$

The CPT-bearing capacity factor K_c, depending on the foundation dimensions, its embedded length, the stress history, and the sensitivity of the clay, varies between 0.3 and 0.6.

q_{ce} is an average value of the cone penetration resistance q_c along a depth equal to B under the foundation, B being the width (or the diameter) of the foundation. q_0 is the total vertical overburden stress at the foundation base.

The French standard NF P94-261, accompanying the Eurocode 7, replaces two previous French standards, namely the DTU-13 applicable to the building sector and the CCTG-93 (Fascicule 62, Title 5) applicable to the public works sector. It is based on the equation (1) where q_{ce} is computed as a weighted average value as follows:

$$q_{ce} = \frac{1}{h_r}\int_D^{D+h_r} q_c(z).dz \tag{2}$$

h_r is the thickness of the mobilized zone of the soil and equal to 1.5B in case of a vertical and centered loading.

The factor K_c depends on the equivalent embedded length D_e and the foundation dimensions, and is obtained by linear interpolation of the factor K_c^1 of a square footing and K_c^0 of a strip footing:

$$K_c = K_c^1 \frac{B}{L} + K_c^0\left(1 - \frac{B}{L}\right) \tag{3}$$

$$K_c{}^1 = 0.27 + \left(0.1 + 0.007\frac{D_e}{B}\right)\left(1 - exp\left(-1.5\frac{D_e}{B}\right)\right) \tag{4}$$

$$K_c{}^0 = 0.27 + \left(0.07 + 0.007\frac{D_e}{B}\right)\left(1 - exp\left(-1.3\frac{D_e}{B}\right)\right) \tag{5}$$

Based on a detailed correlation study between the cone resistance q_c and the limit pressure P_l measured by the pressuremeter test (PMT) in low to high plastic clays (CL/CH) located in 23 sites in Algeria, Mir (2018) suggested to calibrate the PMT method prescribed by the French standard DTU-13.12, and derive an empirical CPT-based method to compute the bearing capacity of shallow foundations in saturated clayey soils. The bearing capacity is formulated as follows:

$$q_l = K_c q_{ce}^* + q_0 \tag{6}$$

q_{ce}* *is the "equivalent net cone resistance*" computed as an arithmetic average of the net values of q_c within a zone thick of 1.5B below the foundation base.

K_c is called the "CPT bearing capacity factor", depending on the slenderness ratio D/B and the ratio L/B (ratio Length to width). It is determined on the basis of the equation (3), the factors K_c^0, K_c^1 corresponding respectively to a strip footing and a circular foundation are given in table 4.

The second category, encompassing the semi-empirical methods, generally proposes to estimate the soil resistance properties, namely the angle of internal friction ϕ and the cohesion C, in correlation with the cone resistance q_c, and to calculate the bearing capacity via the classical methods, usually called the (C, ϕ) methods.

The cone resistance q_c mobilized during the CPT test, at a depth z within a saturated fine soil, is described by the fundamental equation involving the cone factor N_k, the undrained shear resistance C_u, and the total initial vertical overburden stress, as follows (Bouafia 2021) :

$$q_c = \sigma_{v0} + C_u N_k \tag{7}$$

The cone factor is defined as the ratio :

$$N_k = \frac{q_c - \sigma_{v0}}{C_u} \tag{8}$$

Bouafia (2021) adapted the method of Vesic (1975) by introducing the cone factor N_k, which leads to compute the bearing capacity of a rectangular foundation on the basis of equation (1) provided that the K_c is determined as follows:

$$K_c = \frac{(\pi+2)}{N_k}\left(1 + 0.2\frac{B}{L}\right)(1 + 0.4k) \tag{9}$$

k is a dimensionless factor equal to D/B if D/B ≤1, and $tg^{-1}(D/B)$ else, D and B being the embedded length and the width (or the diameter) of the foundation. N_k is taken equal to 14 for normally consolidated clays and 22 for over-consolidated clays.

The last category, encompassing the theoretical methods, is based on the modelling on the one hand the cone test and on the other hand the behaviour of the shallow foundation, resulting in a correlation of the bearing capacity q_l with the cone resistance q_c via the cone factor N_k.

Schmertmann (1978) recommended for square foundations, with a slenderness D/B below 1.5 in cohesive soil, the following bearing capacity equation:

$$q_l = 500 + 0.34\, q_c \quad (10)$$

where all stresses are expressed in kPa. This equation can also be used for circular footings.

Tand et al (1986) proposed a procedure for estimating q_l of circular or square footings having B up to 2.5 m in diameter, based on equation (1). The K_c factor is given by a chart as function of the slenderness ratio D/B, varying within a margin of 0.30-0.42 for fissured clays, and 0.44-0.55 for intact clays.

Bouafia & Benali (2002) proposed a numerical method for computing the bearing capacity of circular foundation embedded in a saturated clayey soil, based on the CPT, and derived from the theory of slip lines. The soil material is assumed to be rigid-perfectly plastic obeying Mohr-Coulomb failure criterion and characterised by an undrained shear strength C_u and a unit weight γ. The indefinite equilibrium equations written in axi-symmetric conditions were combined to the Mohr-Coulomb failure criterion and to the Haar & Von-Karman plasticity hypothesis in axial symmetry, which led to a hyperbolic system of partial derivative equations of the second order. The equations were solved according to an iterative process based on the centered finite differences method, and the method was programmed through a Fortran source of a software called CAPCIR (**CAP**acity of **CIR**cular foundations).

Detailed parametric study by using CAPCIR led to conclude that the factor K_c linearly increases with OCR (Over-Consolidation Ratio) of the clayey soil, and slightly varies with D/B for usual circular foundation/soil systems. For practical purposes, it was suggested for K_c the values of 0.44 and 0.47 for normally consolidated and heavily over-consolidated clays (OCR >1.7) respectively (Bouafia 2021).

There are two main categories of approaches to determine N_k, one based on theoretical models and the other one on empirical correlations.

Among the theoretical methods, the classical theory of bearing capacity of piles was applied to the CPT test by modelling the cone penetrometer as a slender pile. Methods of De-Beer, Begemann, l'Herminier, Berezantsev, Biarez & Grésillon are reported by Bouafia (2009). Moreover, modern theories like the theory of spherical cavities expansion (Ladanyi, 1963; Vesic, 1972; Ladanyi and Longtin, 2005) and the theory of strains path (Baligh, 1985; Baligh and Levadoux, 1986) were used within this framework to interpret the CPT test. It was demonstrated that the cone resistance q_c clearly depends on the soil compressibility and may thus be seen as a mean of indirect evaluation of the soil stiffness. Consequently, the CPT test enables to characterize the soil resistance and the soil stiffness as well. Moreover, some experimental research works demonstrated the possibility of correlation of the soil stiffness to the cone resistance q_c (Lunne et al, 1997).

If considered as an elastic-plastic material, a fine saturated soil is usually characterized by the soil rigidity index I_r involving the soil shear modulus G and the undrained shear strength C_u, as follows:

$$I_r = \frac{G}{C_u} = \frac{E}{2(1+\upsilon)C_u} \quad (11)$$

It was theoretically shown that in the most theoretical approaches, N_k is a logarithmic function of I_r as follows :

$$N_k = a + b\, ln(c\, I_r) \quad (12)$$

a, b and c are specific constants shown in table 1 for each model. Table 1 summarizes some of the usual values of N_k according to these theoretical models.

Further feedbacks showed the theory of bearing capacity of a rigid-plastic soil leads to a simplistic and insufficient interpretation of the CPT test. In addition, the multitude of parameters affecting the tip resistance mechanism led many researchers to undertake more realistic approaches based on numerical modeling (finite element methods, finite differences, etc).

The finite element method is a powerful tool for numerical modeling in soil mechanics, leading to great success both in the field of applied research and that of geotechnical engineering. Several research works have been undertakn with the aim of understanding the mechanism of penetration of the cone into the ground, among others: Ladanyi and Johnson (1974), Teh and Houlsby (1991), Yu et al (1996), Lu et al. (2004), Liyanapathirana (2009) and Boufrina (2012). This latter carried out a Lagrangian- based FEM analysis of the cone penetration within a purely elastic-plastic clayey soil, obeying the Drucker-Prager failure criterion, and exhibiting large deformations. Interface elements were inserted between the cone and the surrounding soil, which enabled a realistic cone/soil interaction, taking into consideration a possible relative displacement of the cone. This study showed a linear logarithmic increase of N_k as function of I_r, as formulated in equation (12).

As it can be seen in table 1, Values of N_k strongly depend on the model used, and no clear accordance is noticed between them. N_k values derived from the bearing capacity lead to the margin 7-11 which is the lowest and independent on the soil rigidity quantified by I_r. Moreover, the strains path theory as well as the FEM method show the dependance on the soil rigidity and the in-situ stress quantified by Δ.

Category 2 of the methods of determination of N_k encompasses the empirical correlations developed on the basis of the physical modelling of the CPT test, namely by scale models in the centrifuge or by prototype cone in the calibration chamber. Moreover, a pragmatic approach to determine N_k is to correlate q_c obtained from the CPT test to the undrained cohesion measured from a direct shear test, a triaxial compression test, or a vane shear tests (VST). Table 2 sumarizes some of these correlations, most of them were reported by various laboratories and research organizations during the 1st international symposium on the CPT test, held in 1995 in Linkoping, Sweden, on the basis of analysis of local databases. In the column (Site), if the study covered a large number of sites, only the country is indicated. This table clearly shows that the cone factor depends on the degree of overconsolidation of the clayey soil. It is remakable that for soft or normally consolidated clays, the empirical values of N_k fluctuate around an average value of 14 with a coefficient of variation (standard deviation/mean) of 20%. For stiff or overconsolidated clay, N_k is rather around 22 with a coefficient of variation equal to 27%.

Table 1. Summary of parameter of some theoretical values of the cone factor N_k

Author	Category	Theory	a	b	c	Reference
Terzaghi (1943)	1	Bearing capacity	7.41	0		3
Caquot & Kérisel (1956)	1	Bearing capacity	7	0		3
Meyerhof (1951)	1	Bearing capacity	9.34 (smooth cone) 9.74 (rough cone)	0		3
De-Beer (1974)	1	Bearing capacity	9.94	0		3
L'Herminier (1967)	1	Bearing capacity	11	0		1
Meyerhof (1951) Skempton (1951)	1	SCE	2.33	1.33	1	3
Gibson (1950)	1	SCE	3.06	1.33	1	3
Vesic (1972)	1	SCE	1.33	1.33	1	3
Baligh and Scott (1976)	1	CCE	12	1	1	3
Wittle (1992)	1	Strain path (smooth cone)	1.51	2	1	5
Yu (1993)	1	CCE (smooth cone)	4.18	1.15	0.87	6
		CCE (rough cone)	9.40	1.15	0.87	
Ladanyi and Johnson (1974)	1	SCE (smooth cone)	1.33	1.33	1	20
		SCE (rough cone)	3.06	1.33	1	
Teh and Houlsby (1991)	1	Strain path (smooth cone)	1.25-2Δ	1.84	1	9, 11
		Strain path (rough cone)	3.25-2Δ	1.84	1	
Boufrina (2012)	1	FEM	3.79	4.13	1	25
Lu et al (2004)	1	FEM (smooth cone)	3.4-1.9Δ	1.60	1	11
		FEM (rough cone)	4.7-1.9Δ	1.60	1	
Liyanapathirana (2009)	1	FEM (smooth cone)	1.0-1.7Δ	1.83	1	5
		FEM (rough cone)	1.5-1.7Δ	2.15	1	

SCE: Spherical Cavity Expansion, CCE: Cylindrical Cavity Expansion, FEM: Finite Elements Method
Δ: In-situ stress ratio=$(\sigma_{v0}-\sigma_{h0})/(2C_u)$

Table 2. Summary of some empirical values of the cone factor N_k

Author	Site(s)	Margin of N_k	Recommended value of N_k	Soil type	Reference
De Ruiter (1982)		10-15 15-20		NCC OCC	Cited in 27
Barakat et al (1983)	Egypt		10	NCC	Cited in 27
Mtynarek et al (1983)		20-24		OCC	Cited in 27
Sanglerat (1965)	Annecy (France)	10-20	15	Soft clay	26
Amar and Jézéquel (1972)	France		12 30	C_u < 50 kPa C_u > 600 kPa	28
Thom et al (1995)	Australia	12-15			29
Nuyens et al (1995)			15		29
Rocha et al (1995)	Bresil	10.5-15.5	14	Soft clay	29
Denver (1995)	Danemark		10		29
Tanaka (1995)	Japan	8-16			29
Formonavicius (1995)			15-20	Soft clay	29
Georges et al (1995)	Nigeria	12-20		NCC	29
Lunne et al (1995)			15		29
Ming-Fang (1995)	Malaisia		10	Marine clay	29
Ajdic et al (1995)			16	NCC	29
Manas et al (1995)			15		29
Moller et al (1995)	Sweden		16.3		29
Nguen-Truong (1995)	Vietnam	14-30	20	Soft silty clay	29
Olsen (1995)	USA	9-13 14-17		NCC OCC	29
Bouafia (2002)		17.9		OCC	1

NCC : Normally consolidated clays
OCC : Overconsolidated clays

3. Description of the numerical models

FLAC 2D (**F**ast **L**agrangian **A**nalyses of **C**ontinua) is an explicit finite difference software in two dimensions developed by Itasca (2008). This software includes several constitutive laws describing the stress-strain soil behaviour, in addition to the ability to integrate groundwater flow, heat, and dynamic soil analysis elements, and to link them together. FLAC 2D uses the Lagrangian elements whose geometry is updated at each time step, and is able to launch a static as well as dynamic analysis, in small as well as in large displacements.

The cone penetration test CPT was modelled by an axi-symmetrical model, considering half the cross-sectional plane of the massif soil. The penetration process was simulated by a prescribed vertical velocity of 20 mm/s imposed to the cone, as per the international standard ISO 22476-3 of the CPT test. Dynamic analysis option was chosen to take into consideration the dynamic loading, a usual value of the material damping of 5% was input, and absorbent boundaries option was activated within the software to prevent any process of wave reflections. Moreover, the soil model was designed in such a way as its dimensions are enough large to mobilize spatial (or radial) damping of waves.

The soil material is assumed to be elastic-perfectly plastic obeying the Mohr-Coulomb failure criterion, and characterized by the mechanical properties summarized in table 3. The penetrometer device is composed of a steel conical tip having an apex angle of 60°, a base diameter of 35.7 mm, connected to a series of cylindrical steel rods. The steel material is assumed to be a linearly elastic material characterized by a Young's modulus of 2.1×10^5 MPa, a Poisson's ratio of 0.15, and a unit wight of 78.5 kN/m^3.

Table 3. Summary of the mechanical properties of the clayey soil

	E (MPa)	C_u (kPa)	I_r	Unit weight γ (kN/m^3)	Poisson's ratio υ	Shear angle φ (°)	At-rest lateral pressure coefficient K_0
Model 1	2	100	6.66				
Model 2	10	200	16.66				
Model 3	40	300	44.44	20	0.499	0°	1.0
Model 4	60	200	100				
Model 5	300	200	500				

The soil model was designed in such a way as its boundaries correspond to a free field. Consequently, the dimensions adopted for the model are as follows: total height equal to 4z, z being the initial depth of the cone, and a width of 20ϕ, ϕ being the cone diameter. As illustrated in figure 1, the automatic mesh made by the software is refined in the vicinity of the cone in order to get reliable results in terms of stresses and displacements.

To allow a possible separation of the cone with respect to the surrounding soil, which has an non negligible influence on the stresses and displacements of the cone, the interface elements, having a zero thickness, were included between the soil and the cone as well as the rods. These elements may be smooth, mid-rough, or completely rough, which allows analyzing this effect on the response of the cone. The foundation model consists of a reinforced concrete rigid foundation embedded in a homogeneous isotropic soil mass. The models of foundation were built in axi-symmetry condition for a circular foundation and in plane strains condition for a strip footing. Due to the symmetry of the problem, a half-plan can be analyzed. The boundary conditions are taken into account by blocking the horizontal and vertical displacements for the base of the mesh, and only the horizontal displacements for the lateral limit (see Figure 2).

The dimensions of the mesh are 20Bx20B, B being the width or the diameter of the footing, taken equal to 1 m for all the models. The mesh has an average size of 4000 elements (zones), 50 in the horizontal direction, including 10 elements under the foundation and 80 in the vertical direction. The embedded length D has taken the values 0, 0.5, 1 and 2. The foundation was loaded by prescribing a set of vertical displacements at its base with a rate of 10^{-7} m/step, this choice was adopted as a compromise between the precision of results and the time required for calculation.

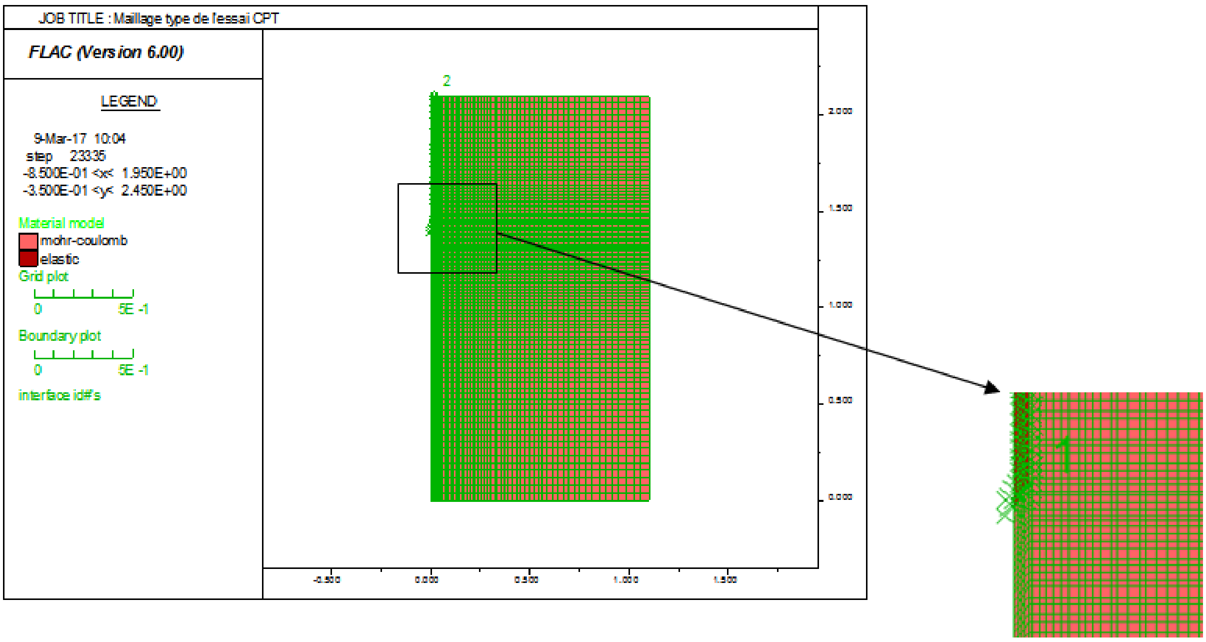


Figure 1: Typical mesh of the CPT model

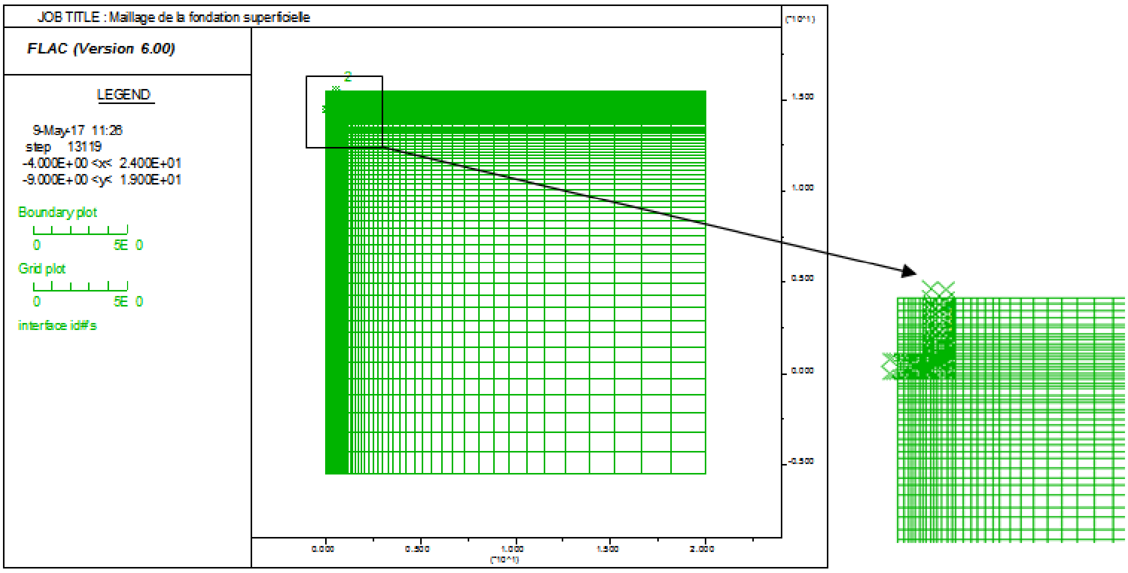


Figure 2. Mesh and boundary conditions for the foundation model

4. Dimensional analysis

4.1. The cone penetration test

The cone resistance q_c may be formulated as follows :

$$f(q_c, z, r_0, C_u, \gamma, G) = 0 \quad (13)$$

Where z, r_0 are respectively the initial depth of the cone, prior the penetration into the soil, and the cone radius.

Dimensional analysis by the Vashy-Buckingham theorem was used to define the problem by dimensionless parameters, called terms in π, which leads to reduce the number of variables used in a parametric study. Equation (13) is then reduced to :

$$g(\pi_1, \pi_2, \pi_3) = 0 \quad (14)$$

$$\pi_1 = \frac{q_c - \gamma z,}{C_u} \quad (15)$$

$$\pi_2 = \frac{G}{C_u} \quad (16)$$

$$\pi_3 = \frac{z}{r_0} \quad (17)$$

These dimensionless terms are respectively cone factor N_k, the rigidity index I_r and the normalized depth. Equation (13) leads to identify N_k as function of I_r and z/r_0 as follows:

$$N_k = h\left(I_r, \frac{z}{r_0}\right) \quad (18)$$

The aim of the parametric study carried out hereafter is to identify the function **h** by varying I_r, z/r_0 and the roughness of the interface elements, in order to propose a practical formulation of the cone factor.

4.2. The bearing capacity of foundations

The bearing capacity q_l of a shallow foundation having a width (or diameter) B, an embedded length D, and resting on a semi-infinite purely cohesive soil mass characterized by an undrained shear strength C_u and a saturated unit weight γ may be formulated as follows:

$$j(q_l, D, B, C_u, \gamma) = 0 \quad (19)$$

This equation, involving physical and mechanical parameters, may be transformed based on the Vashy-Bukingham theorem of dimensional analysis, into the following equation involving only dimensionless parameters:

$$i(\pi_1, \pi_2) = 0 \quad (20)$$

$$\pi_1 = \frac{q_l - \gamma D}{C_u} = N_c \quad (21)$$

$$\pi_2 = \frac{D}{B} \quad (22)$$

The terms π_1 and π_2 are respectively called the bearing capacity factor due to cohesion and the foundation slenderness ratio. The parametric study may lead to formulate N_c as function of D/B:

$$N_c = k\left(\frac{D}{B}\right) \quad (23)$$

By using the equations (21), and (1), the coefficient K_c may be written as follows:

$$K_c = \frac{q_l - \gamma D}{q_{ce}} = \frac{N_c . C_u}{q_{ce}} \quad (24)$$

The cone factor N_k may be introduced based on equation (8), which leads, when assuming the initial vertical overburden pressure σ_{v0} negligible with respect to the cone resistance q_c, to:

$$K_c = \frac{N_c . C_u}{q_{ce}} = \frac{N_c}{N_k} = \frac{\boldsymbol{k}(\frac{D}{B})}{\boldsymbol{h}(I_r)} \quad (25)$$

The function **h** and **k** may be derived by fitting respectively the results of the parametric studies of the CPT test and the foundation loading test.

5. Analysis of the cone factor N_k

5.1. Determination of the cone resistance

As depicted by figure 3, the cone resistance q_c is derived by integrating the nodal forces F_i acting on a half-plan, which leads to:

$$q_c(z) = \frac{Q}{\pi {r_0}^2} \quad (26)$$

r_0 is the cone radius (r_0=17.85 mm) and Q is the total vertical force acting on the cone such as:

$$Q = \sum_{i=1}^{i=N} F_i . (2\pi r_i) \quad (27)$$

The cone factor is then computed based on equation (8).

5.2. Parametric study of N_k

5.2.1. Effect of the initial depth of penetration

As shown in figure 4, the cone factor, computed for two different initial depths of penetration, is slightly affected by the initial depth of penetration, which implies that a homogeneous clayey deposit may be characterized by a unique value of N_k independently of the depth.

5.2.2. Effect of the roughness of the cone/soil interface

According to figure 5, the cone factor decreases with the roughness of the cone/soil interface. N_k of a smooth interface is lesser than that of a rough interface by a default of 12-14% depending on I_r. However, this effect disappears when comparing the N_k of mid-rough and rough interfaces. Such a statement implies the roughness of the interface has generally a limited effect on the cone factor.

5.2.3. Effect of the soil rigidity

As illustrated by figures 4 and 5, the cone factor exhibits a logarithmic increase with the soil rigidity index I_r, which is in conformity with the usual formulation of N_k, as described by equation (12).

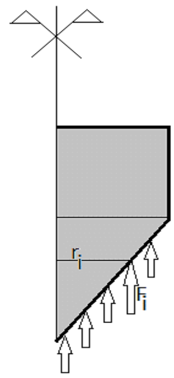
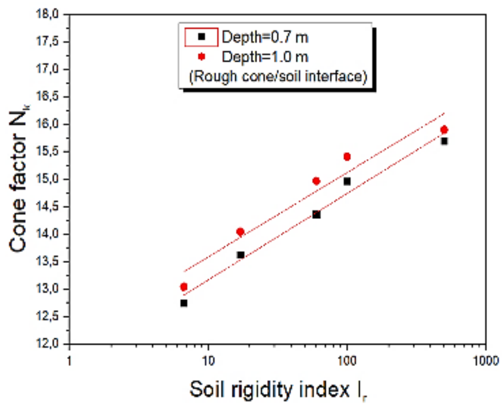


Fig. 3. Scheme of computation of q_c



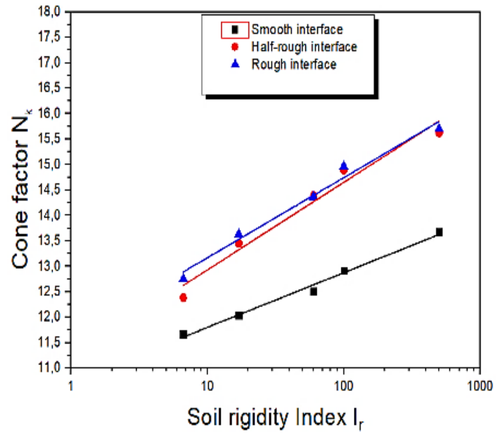


Fig. 4. Effect of the initial depth on N_k

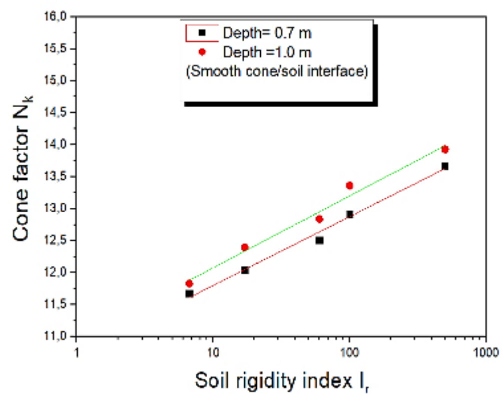


Fig. 5. Effect of roughness of the cone/soil interface on N_k

5.3. Formulation of the cone factor

Function **h**, as formulated in equation (18), may be formulated by disregarding the effect of the initial depth of penetration. Although the roughness of the cone/soil interface has a quite limited effect, it is possible to fit the values of N_k based on the technique of least-squares, which gives the following formulas:

- for a smooth interface

$$N_k = 10.73 + 0.467\ ln(I_r) \quad (28)$$

- for a mid-rough interface

$$N_k = 11.20 + 0.747\ ln(I_r) \quad (29)$$

- for a rough interface

$$N_k = 11.60 + 0.683\ ln(I_r) \quad (30)$$

The coefficient R of regression is equal to 99.5% and 98.5% respectively for smooth and rough interfaces, showing an excellent quality of fitting.

5.4. Comparative study of the cone factor

According to figures 6 and 7 illustrating the variation of N_k for smooth and rough interfaces, the proposed values of N_k reasonably lie within the other studies. Values of N_k proposed by Boufrina (2012) are the greatest, while those of Gibson (1950), Meyerhof (1951) and Skempton (1951) are the lowest ones. For a smooth interface, the propose values are in excellent accordance with those of Baligh and Scott (1976) and Yu (1993) based on the cavity expansion theory.

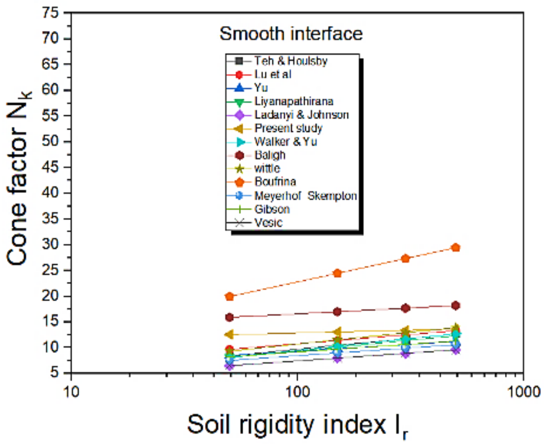


Fig. 6. Comparison of N_k values for a smooth interface

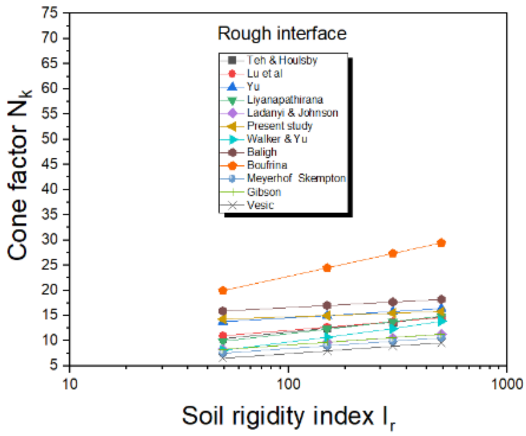


Fig. 7. Comparison of N_k values for a rough interface

6. Analysis of the bearing capacity factor K_c

6.1. Determination of the factor K_c

For a prescribed settlement s, the soil mobilizes nodal forces F_i acting on nodes i at distance r_i from the vertical axis of the foundation ($0 \leq r_i \leq r_0$), r_0 being the half-width or the radius of the foundation. For a circular foundation, the vertical pressure q is therefore computed according to equation (27), by dividing Q by the foundation area, whereas for a strip footing, q is the sum of the nodal forces divided by the width B, such as:

$$q = 2\sum_{i=1}^{i=N} \frac{F_i}{B} \quad (31)$$

All the load-settlement curves, as depicted in figure 8, were hyperbolic shaped, and may be fitted by the following hyperbolic function, where q and s respectively correspond to the applied vertical pressure and the foundation settlement assumed to be uniform:

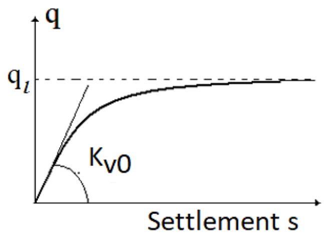


Fig. 8. Typical load-settlement curve

$$q = \frac{s}{a+bs} \quad (32)$$

It can easily be shown that **a** is the inverse of the initial slope of the curve, usually called the initial vertical stiffness of the foundation K_{v0} ($a=1/K_{v0}$), and **b** is the inverse of the bearing capacity q_l ($b=1/q_l$).

The load-settlement curves obtained were fitted by the hyperbolic function on the basis the least-squares technique, which led to obtain the bearing capacity q_l. The CPT bearing capacity factor K_c was therefore determined by using the following equation, N_k being already computed according to the equations (28) through (30):

$$K_c = \frac{(q_l - \gamma D)}{C_u}\frac{1}{N_k} \quad (33)$$

According to equations (28) through (30), N_k varies as a logarithmic function of the rigidity index I_r, this latter being difficult to be practically determined since it requires the knowledge of the soil deformation modulus. It was however noticed that N_k varies slightly with I_r. In fact, for a smooth interface, the values of N_k may statistically be characterized by a mean value of 13.17 and a coefficient of variation COV of 3.3%, whereas those of a rough interface, the mean value is 15.55 and COV=4.2%. At last, for a mid-rough interface, the mean value is 15.10 and COV=5.0%.

For practical purposes, the mean values of a mid-rough interface were input in equation (33), which enables computing K_c as function of D/B according to equation (25). Hereafter, K_c^1 corresponds to a circular foundation, whereas K_c^0 correspond to a strip footing.

6.2. Parametric study of factor K_c

6.2.1. Effect of the slenderness ratio

As can be seen from figures 9 and 10, the factor K_c regularly increases with the slenderness ratio, which implies that in a homogeneous clayey soil, the short-term bearing capacity increases with D/B. Moreover, it is remarkable that whatever D/B and the roughness of the cone/soil, the factor K_c^0 is less than K_c^1. The difference is negligible for a slenderness D/B= 0 but significantly increases with D/B.

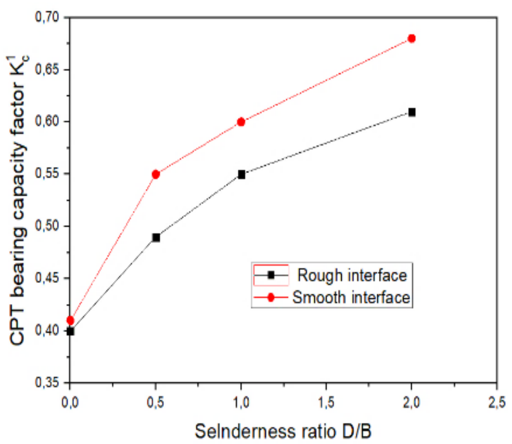


Fig. 9. Chart of K_c^1 for a circular foundation

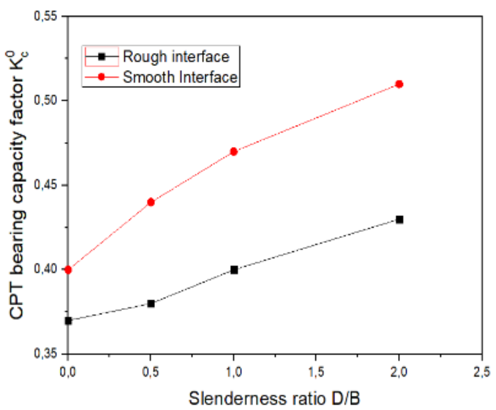


Fig. 10. Chart of K_c^0 for a strip footing

6.3. Formulation of the bearing capacity q_l

The values of the bearing capacity factor K_c were derived on the basis of equation (33) by assuming a mid-rough cone/soil interface, the cone factor corresponding to such an interface is equal to 15.1. To take account the size effect on the bearing capacity, the factor K_c of a rectangular foundation is computed by linear interpolation of the factor K_c^1 of a square footing, and K_c^0 of a strip footing, as formulated in equation (3). The values of K_c^0 and K_c^1 are given in table 4.

The bearing capacity may be calculated by the equation (6), where the equivalent net cone resistance q_{ce}* is computed by equation (2) as a weighted average value of the net cone resistances q_c* within a zone thick of 1.5B below the foundation base. As a beforehand interpretation of the cone resistance profile $q_c(z)$ based on safety considerations, it is recommended to filter this profile by eliminating the peaks of resistance which leads to relatively regular profile. Moreover, values of q_c* within the zone 1.5B below the foundation base, greater than 1.3q_{cm}*, q_{cm}* being the average value computed by equation (2), should be bounded to 1.3q_{cm}* and q_{ce}* computed by equation (2).

6.4. Comparative study of the factor K_c

As it can be seen in table 4, the suggested values of K_c fall within the margins prescribed by the CFEM, are greater than those of the French standard, and have the same order of those proposed by Schmertmann (1978) and Tand et al. (1986). A good agreement is noticed between the proposed values and those recommended by Bouafia (2021).

7. Validation of the proposed method

The predictive capability of the proposed method was assessed through a well-documented case history of a full-scale loading test. Eslami and Gholami (2006) presented the results of the vertical loading test of a circular foundation, 0.6 m in diameter and 1.5 m deep with respect to the surface, at the NGES (National Geotechnical Experimental Site) experimental site of the University of Houston (Texas). The soil is a layer of stiff and over-consolidated clay (by desiccation), 14 m thick, and overlying a sandy-silty horizon.

Table 4. Comparison of the values of the CPT-bearing capacity factor K_c

Author		Margins de K_c					Remarks	Reference
Present study	D/B	0.0	0.5	1.0	1.5	2.0		
	K_c^0	0.36	0.38	0.39	0.41	0.43		
	D/B	0.0	0.5	1.0	1.5	2.0		
	K_c^1	0.37	0.49	0.54	0.58	0.61		
Canadian standard CFEM		0.3-0.6						
French standard NF P94-261		0.27-0.35					Strip footing	
		0.27-0.38					Square	
Mir (2018)	D/B	0.0	0.5	1.0	1.5	2.0	Strip footing	34
	K_c^0	0.22	0.25	0.26	0.27	0.27		
	D/B	0.0	0.5	1.0	1.5	2.0	Square	
	K_c^1	0.22	0.26	0.29	0.30	0.30		
Bouafia (2022)	D/B	0.0	0.5	1.0	1.5	2.0	Strip footing	1
	K_c^0	0.37	0.44	0.51	0.51	0.53		
	D/B	0.0	0.5	1.0	1.5	2.0	Square	
	K_c^1	0.44	0.53	0.62	0.61	0.63		
Schmertman		0.34					Square and D/B<1.5	
Bouafia & Benali (2002)		0.44					Circular foundation NC clays	
		0.47					Circular foundation OC clays	
Tand et al. (1986)		0.30-0.42					Intact clays B< 2.5 m	
		0.44-0.55					Fissured clays B< 2.5 m	

NC Normally Consolidated

OC Over-Consolidated

The CPT test, conducted with a standard cone, provided the cone resistance profile illustrated in Figure 11. The groundwater was detected at a depth of 2.1 m as from the surface, and for calculation purposes the unit weights above and below the water table were respectively estimated at 18 kN/m^3 and 20 kN/m^3. As shown in figure 11, the cone resistance q_c varies, up to a depth of 2.0 m, from 0.83 to 2.50 MPa, which classifies it as a soft clay according to the French standard CCTG-93. However, by adopting the value 22 for the N_k factor of an over-consolidated clay, equation (8) gives an estimate of the cohesion C_u within the margin of 37-114 kPa, which also allows describing the consistency of this clay, according to the Canadian standard CFEM, as firm to very stiff.

The loading test was conducted by applying 10 increments of vertical pressure up to a maximum value of 600 kPa.

Usual bearing capacity criteria were applied to the experimental load-settlement curve in order to estimate the bearing capacity. Fitting by the hyperbolic function (see equation 32) on the basis of the least-squares method gave: q_l=785.9 kPa, K_{v0}=30.5 kPa/mm, and a coefficient of regression R equal to 99.5% indicating an excellent fitting procedure.

The graphical procedure of Asaoka (1978) (cited by Bouafia, 2021) gave: q_l=667 kPa, whereas the graphical method called the "*null secant stiffness*" proposed by Decourt (1999) (cited by Bouafia, 2021) gave: q_l=841 kPa. At last, for a settlement equal to B/10=60 mm, the load-settlement curve directly gave a pressure of 550 kPa, conventionally defined as being the bearing capacity: q_l=550 kPa. In overall, the criteria of the bearing capacity are in margin of 550-841, characterized by a mean value of

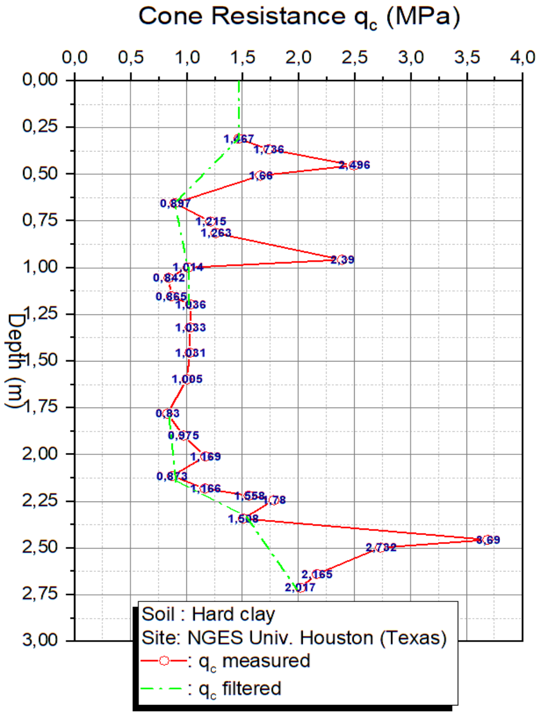


Fig. 11. Profile of the cone resistance

Table 5. Summary of the predicted bearing capacity values

Category	Method	q_l (kPa)	λ_{hyp}	$\lambda_{10\%}$
Empirical methods	Loading test (criterion of asymptote)	Hyperbole : q_l=785.9	1.00	1.43
		Chin : q_l=787.4	1.00	1.43
		Asaoka : q_l=667.0	0.85	1.21
		Raideur nulle : q_l=841	1.07	1.53
	Loading test (criterion 10% de B)	q_l=550	0.70	1.0
	Canadian Standard CFEM	318.1-609.2	0.40-0.77	0.57-1.10
	French Standard NF P94-261	---	---	---
	Mir (2018)	370.8	0.47	0.67
Semi-Empirical methods	Bouafia (2022)	501.4	0.64	0.91
Theoertical methods	Bouafia & Benali (2002)	523.6	0.66	0.95
	Proposed method	**719.0**	**0.91**	**1.30**
	Schmertmann (1978)	---	---	---
	Tand et al (1986)	531.2-657.3	0.67-0.84	0.96-1.20

711 kPa, and a coefficient of variation of 18.2%, which characterizes a quite homogeneous statistical population.

The different methods for calculating the bearing capacity in clay, as presented in the paragraph 2, were applied, and the results are summarized in table 5. The assessment of the quality of prediction of these methods is quantified by the parameter λ adopted "*bearing capacity ratio*" and defined as follows:

$$\lambda = q_l(pred)/q_l(crit) \tag{34}$$

where q_l(pred) is the predicted bearing capacity, and q_l(crit) is the experimental estimated bearing capacity according to a given criterion. The ratio λ is computed on the basis of the

hyperbolic criterion and denoted λ_{hyp}, as well as of the 10%B criterion and denoted $\lambda_{10\%}$.

It is stated that the most optimistic prediction is that of Bowles (1997) (q_l=889.6 kPa), while the most pessimistic is that of the CFEM (q_l=318.1 kPa). Moreover, the method of Bouafia & Benali (2002) is very close to the 10% B criterion, but far from the hyperbolic criterion.

In overall, the predicted values exhibit a relatively high dispersion characterized by a margin of λ of 0.4-1.07 according to the hyperbolic criterion, and of 0.57-1.53 according to the 10%B criterion.

Most of the methods gave values smaller than those given by both the hyperbolic criterion and 10%B criterion, consequently providing an underestimation of the bearing capacity, which is a pessimistic prediction but on the safety side.

It is remarkable that the proposed method gave the best prediction (λ=0.91) according to the hyperbolic criterion, but it was the most optimistic by providing the biggest value of bearing capacity (λ=1.30) according to the 10%B criterion. Such a statement leads to conclude that predicted

bearing capacity provided by the numerical modelling, presented in this study, falls into the margin of values obtained by of the most commonly used methods described in literature. However, such a finding must be confirmed by testing the quality of prediction of the proposed method on a larger sample of case histories.

8- Conclusions

This study describes a theoretical contribution to the calculation of the bearing capacity of shallow foundations in clayey soils on the basis of the CPT test. After a detailed review of the literature, the numerical model of the CPT test as well the loading test of a shallow foundation were presented. Detailed parametric study based on a fast Lagrangian analysis, using the software FLAC-2D, was undertaken and the interpretation of the results in terms of stresses and strains led to determine the cone resistance q_c, the applied pressure q on the foundation, the cone factor N_k, and the CPT-bearing capacity factor K_c.

The parametric study led to conclude that the initial depth of penetration as well as the soil rigidity have a limited effect on the factor K_c which led to formulate the bearing capacity.

Comparative study of the CPT-based methods of bearing capacity in clayey soils was undertaken within the scope of interpretation of a full-scale loading test of a square foundation embedded in a hard over-consolidated clay. In overall, the values of the bearing capacity predicted by the proposed method falls within the margin of the those given by the most commonly used methods in literature, and gave the best prediction according to the hyperbolic criterion, but the most optimistic prediction based on the conventional 10%B criterion. Such a statement should be subsequently confirmed by the assessment within the scope of a larger samples of case histories of loading test of foundations in clay.

Nomenclature

I_r	Rigidity index
N_k	Cone factor
K_c	Bearing capacity factor
B	Width of the footing, m
D	Embedded length, m
C_u	Undrained shear strength, KN/m^2
q_c	Cone resistance
q_l	bearing capacity

Greek symbols

γ	Saturated unit weight
ϕ	Cone diameter

Disclosures

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