ASPS Conference Proceedings 1: 1847-1852 (2022)

Alwaha Sci. Publ. Services

Proceedings of

12th Structural Engineering Convention-An International Event (SEC 2022)

Available at https://asps-journals.com/index.php/acp

# Static Stress Analysis of Functionally Graded Cylindrical Stiffened Shells

S. M. Shiyekar [1, *], Akshaya Awari [2]

[1] Department of Civil Engineering, Professor, D Y Patil College of Engineering, Akurdi, Pune, 411 044, India
[2] Department of Civil Engineering, PG Research Scholar, Sanjay Ghodawat Institutes, Atigre, 416 118, Kolhapur, India

**Paper ID - 010514**

**Abstract**

In this paper, a study on bending performance of functionally graded (FGM) cylindrical shells under transverse mechanical load is presented. Computational and analytical tools are used to study the behavior of FGM cylindrical shells under bending. Analytical modeling is based on first order shear deformation theory (FOST) and a finite element computational tool ABAQUS is used to model the FGM cylindrical shell. Material properties are estimated by power law index. Results from computational tools for isotropic and FGM cylindrical shells with various boundary and loading conditions are validated with literature and present FOST. Stiffened FG cylindrical shells with cutouts are analyzed. The FGM circular cylindrical shells subjected to an internal pressure with various arrangements of stiffeners are also analyzed and von – Mises stresses are also studied.

*Keywords*: Analytical solution, von – Mises stresses, FGM, cylindrical shells, ABAQUS

## 1. Introduction

The Functionally Graded Materials (FGMs) is a relatively new technology and are being studied for the use in components exposed to high temperature gradients. Laminated composite materials provide design flexibility to achieve desirable stiffness and strength through the choice of lamination scheme. The anisotropic constitution of laminated composite structures often results in stress concentrations near material and geometric discontinuities that can lead to damage in the form of delamination, matrix cracking, and adhesive bond separation. To overcome this issue, the material properties are functionally graded (FG). The concept of FG Materials (FGM) were proposed by Koizumi and Yamanouch in 1984. FGMs allow to bear these problems because they consist of continuous variation of material properties from one surface to the other. The continuous nature of the variation reduces the stress concentrations which become troublesome in a laminated composite material. Smooth transition through the various material properties reduces both thermal and residual stresses. In most cases, material progresses from a metal on one surface to a mostly ceramic on the opposite surface, with a smooth transition. The material transitions from a metal to a ceramic by increasing the percentage of ceramic material present in the metal until the appropriate percentage is reached or a pure ceramic is achieved. FGMs have high resistance to temperature gradients results in capability to withstand to high load and high durable properties, subsequently reduction in residual and thermal stresses. Many researchers are being working on FGMs. Stiffened shells are used in several engineering applications to improve efficiency of the structures in terms of strength/weight.

Thai and Kim [1] presented the review of various theories for the modelling and analysis of FGM plates and shells. The review is devoted to theoretical models which were developed to predict the global responses of functionally graded plates and shells under mechanical and thermal loadings. Asadi et al. [2] have studied the static and vibration analyses of thick deep laminated cylindrical shells using 3D and various shear deformation theories.

Bich et al. [3] presented an analytical approach to investigate the nonlinear static and dynamic buckling of imperfect eccentrically stiffened functionally graded thin circular cylindrical shells subjected to axial compression. Ardestani et al. [4] presented the analysis of functionally graded stiffened plates based on FSDT using Reproducing Kernel Particle Method (RKPM). Viola et al. [5] studied the static analysis of functionally graded conical shells and panels using the generalized unconstrained third order theory coupled with the stress recovery. Oktem et al. [6] presented an analytical solution to the static analysis of functionally graded plates and doubly-curved shells, modelled using a higher order

*Corresponding author. Tel: +919822377577; E-mail address: smshiyekar@dypcoeakurdi.ac.in





Published online: December 19, 2022
doi:10.38208/acp.v1.727

shear deformation theory (HSDT). Huang and Han [7] studied the non-linear buckling of torsion-loaded functionally graded cylindrical shells in thermal environment based on nonlinear large deflection theory of cylindrical shells. Shiyekar and Lavate [8] presented analytical model based on Reddy's TOT and modelling of FG material is done by using FEM computational tools ABAQUS [9].

In this paper, analytical model based on First Order Shear Deformation Theory (FOST) [10, 11] and finite element model based on ABAQUS [9] of FG cylindrical shell is presented for static stress analysis. The variation of volume of ceramic is done by using power law index. The results of FG shells are validated with the literature results. The FG shells with cut-outs and with different arrangements of stiffeners are analysed. An isotropic cylindrical shell with different aspect ratios is also validated with literature results. The FG cylindrical shells with cut-outs and with different arrangements of stiffeners are also studied for von-Misses stresses under internal pressure.

## 2. Formulation for FOST

The First Order Shear Deformation Theory (FOST) developed by Mindlin [10] accounts for the shear deformation effect by the way of a linear variation of the in-plane displacements through the thickness. It is noted that the theory developed by Reissner [11] also accounts for the shear deformation effect. The displacement field is stated as per the following form.

$$u = u_0 + z\theta_x$$
$$v = v_0 + z\theta_y \qquad (1)$$
$$w = w_0$$

In the above relations, the terms $u$, $v$ and $w$ are the displacements of a general point $(x, y, z)$ in the cartesian coordinate system. The parameters $u_0$, $v_0$ are the in plane displacements and $w_0$ is the transverse displacement of a point (x, y) on the middle plane. The terms $\theta_x$ and $\theta_y$ are the rotations of the normal to the element middle plane about y- and x-axis respectively. With the definition of strains from the linear theory of elasticity, assuming, $h/R_x$ and $h/R_y <= 1$, the general strain-displacement relations in the curvilinear co-ordinate system are given as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x}$$
$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y}$$
$$\varepsilon_z = \frac{\partial w}{\partial z}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad (2)$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u}{R_x}$$
$$\gamma_{yz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} - \frac{v}{R_y}$$

Assuming the principal material axes (1,2,3) and the shell axes (x,y,z) in the curvilinear coordinate system, the three dimensional stress-strain relations for an cylindrical shell with reference to the principal material axes for the theory to be developed based on the displacement are defined as follows:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{33} & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & C_{55}\end{bmatrix} \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix}$$

Where,

$$C_{11} = \frac{E_1}{(1-\nu_{12}\nu_{21})}, \quad C_{12} = \frac{\nu_{21}E_1}{(1-\nu_{12}\nu_{21})}, \qquad (3)$$

$$C_{22} = \frac{E_2}{(1-\nu_{12}\nu_{21})}, \quad \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2},$$

$$C_{33} = G_{12}, \quad C_{44} = G_{13}, \quad C_{55} = G_{23}$$

The principal material axes of shell may not coincide with the reference axes of the shell (x,y,z). It is therefore necessary to transform the constitutive relations from the shell's material axes (1,2,3) to reference axes, (x,y,z). This is conveniently accomplished through the transformations. The final relations are as follows:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0\\ Q_{12} & Q_{22} & Q_{23} & 0 & 0\\ Q_{13} & Q_{23} & Q_{33} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & Q_{45}\\ 0 & 0 & 0 & Q_{45} & Q_{55}\end{bmatrix} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \qquad (4)$$

The stress resultants are defined as

$$\begin{bmatrix} N_x & M_x\\ N_y & M_y\\ N_{xy} & M_{xy}\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}(1, z^2, z, z^3)\,dz$$

$$= \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14}\\ Q_{12} & Q_{22} & Q_{23} & Q_{24}\\ Q_{13} & Q_{23} & Q_{33} & Q_{34}\\ Q_{14} & Q_{24} & Q_{34} & Q_{44}\end{bmatrix} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}(1, z^2, z, z^3)\,dz$$

$$\begin{bmatrix} Q_x & S_x\\ Q_y & S_y\end{bmatrix} = \sum_{L=1}^{NL}\int_{Z_L}^{Z_{L+1}} \begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}(1, z^2, z, z^3)\,dz$$

Following equilibrium equations are obtained using principle of minimum potential energy.

$$\frac{\partial N_x}{\partial x} + \frac{\partial(N_{xy} + c_0 M_{xy})}{\partial y} + \frac{Q_x}{R_x} = 0$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial(N_{xy} - c_0 M_{xy})}{\partial x} + \frac{Q_y}{R_y} = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} + q = 0$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x + \frac{S_x}{R_x} = 0$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y + \frac{S_y}{R_y} = 0 \qquad (5)$$

Navier's solution has been adopted

$$u_0 = \sum_{m,n}^{\infty} u_{0mn} \cos \alpha x \sin \beta y$$

$$v_0 = \sum_{m,n}^{\infty} v_{0mn} \sin \alpha x \cos \beta y$$

$$w_0 = \sum_{m,n}^{\infty} w_{0mn} \sin \alpha x \sin \beta y \quad (6)$$

$$\theta_x = \sum_{m,n}^{\infty} \theta_{xmn} \cos \alpha x \sin \beta y$$

$$\theta_y = \sum_{m,n}^{\infty} \theta_{ymn} \sin \alpha x \cos \beta y$$

The sinusoidal load is considered as

$$q = \sum_{m,n}^{\infty} q_{mn} \sin \alpha x \sin \beta y \quad (7)$$

$\alpha = \pi/a$ and $\beta = \pi/b$

## 3. Functionally Graded Material (Power Law)

Consider a shell of total thickness $h$ and made of an isotropic but inhomogeneous material through the thickness of the shell. Suppose that a typical material property P is varied through the shell thickness according to the power law expressions as per following.

$$P(z) = (P_t - P_b)V + P_b$$

$$V = \left(\frac{z}{h} + \frac{1}{2}\right)^n$$

Where $P_t$ and $P_b$ denote the property of the top and bottom faces of shell, respectively, and $n$ is a parameter that indicates the material variation profile through the thickness. Here we assume that Young's modulus E, Modulus of rigidity G, Density varies through thickness and Poisson's ratio is assumed to be constant.

We take $P_t = P_c$ and $P_b = P_m$ as the properties of the ceramic and metal, respectively. The metal content in the shell increases as the value of $n$ increases. The value of $n$=0 represents a fully ceramic shell. The above power law assumption reflects a simple rule of mixtures used to obtain the effective properties of the ceramic-metal shell.

## 4. Numerical Investigations

Numerical investigations are presented in this section. Firstly, the results for an isotropic cylindrical shell subjected to uniformly distributed load are validated with the literature results for various aspect ratios. The results obtained for FG cylindrical shell are validated with literature results, and new results are generated for different boundary conditions. The numerical investigations are presented for an FG cylindrical shell with different arrangements of stiffeners for various boundary conditions. Functionally graded circular cylindrical shell subjected to internal fluid pressure with different arrangements of stiffeners are studied. The FG cylindrical shell subjected to sinusoidal loading shows good agreement with the analytical results. The comparison of numerical results of (ABAQUS), analytical (FSDT) and literature results are shown in succeeding sections.

### *3.1 Numerical investigations of FG cylindrical shell subjected to transverse sinusoidal load.*

In this example, an FG cylindrical shell is simply supported on its edges and is subjected to sinusoidal loading *q (x, y) = q0 Sin (πx/a) Sin (πy/b),* where *q0* is the amplitude of Sin load. Central angle of the shell is assumed $180^0$. This case is studied for the central deflection and the results are compared with the analytically obtained results [FSDT]. The results are compared in Fig. 1.

### *3.2 Numerical investigations of an isotropic cylindrical shell*

In this example, an isotropic cylindrical shell subjected to uniformly distributed load is studied for different aspect ratios. The non-dimensional central deflections are obtained for different aspect ratios presented in Table-1.

Present ABAQUS results are in good agreement with other literature results [2].

### *3.3 Numerical investigations of FG cylindrical shell*

In this example, functionally graded cylindrical shell subjected to uniformly distributed load for various boundary conditions are studied. The results are validated with the literature [5] results and shown in Fig. 2.

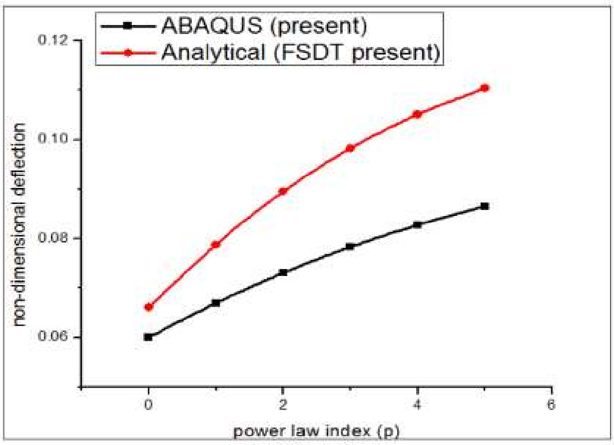



Fig. 1. Non-dimensional transverse deflection vs. power law index for FG cylindrical shell subjected to sinusoidal load.

Table-1 non-Dimensional transverse deflection for simply supported isotropic cylindrical shell under uniformly distributed loading for a/h = 10.

| a/R | 3D (literature) | FSDTQ (literature) | FSDT (literature) | ABAQUS (present) |
|---|---|---|---|---|
| 0.5 | 40.875 | 40.956 | 40.699 | 41.1 [0.1%] |
| 1 | 28.415 | 28.333 | 28.009 | 29 [3.53%] |
| 2 | 12.242 | 12.108 | 11.972 | 11.52 [3.77%] |

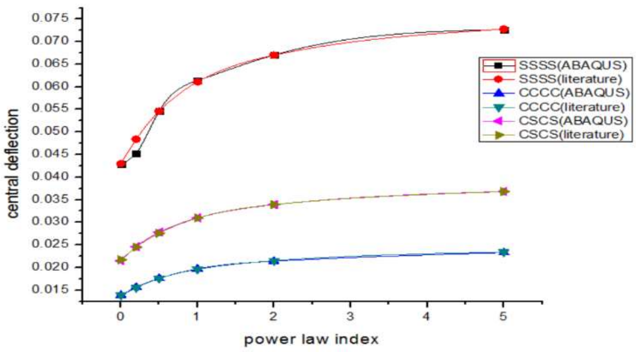



Fig 2. Non- dimensional transverse displacement for FGM cylindrical shell vs. literature results.

*3.4 Numerical investigations of FG cylindrical shell with cutouts and stiffeners.*

FGM cylindrical shell with the cutouts and stiffeners has been studied for central transverse deflection. The results are compared for different arrangements of stiffeners and for different power law index.

Fig. 3 to Fig. 6 show the variation of transverse displacement for CSCS and CCCC, von-Mises stresses and $\sigma_x$ stresses along power law index (p) respectively.

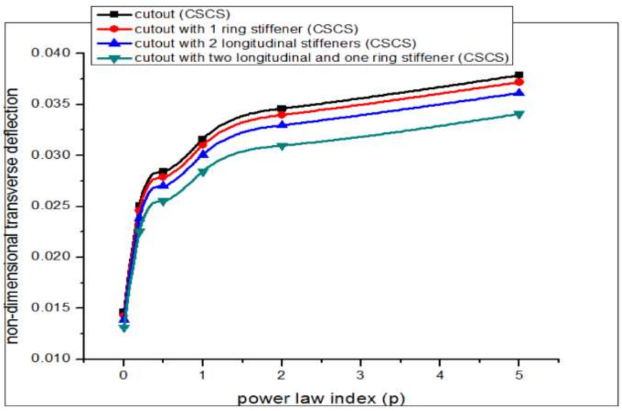



Fig. 3. non-dimensional transverse deflection vs. power law index (p) for FG cylindrical shell (CSCS)

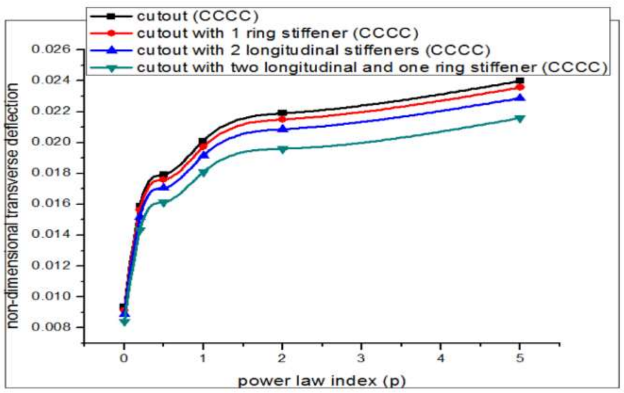



Fig. 4. non-dimensional transverse deflection vs. power law index (p) for FG cylindrical shell (CCCC)

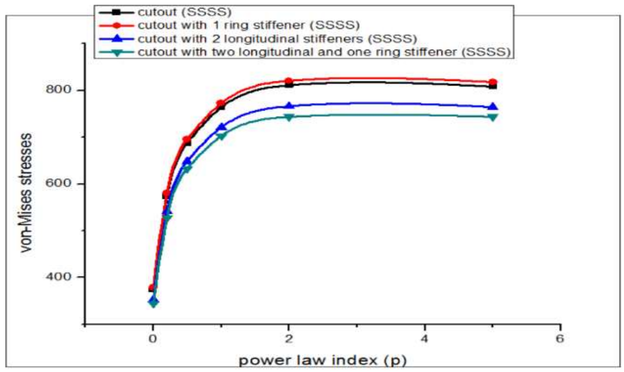



Fig. 5. von-Mises stresses vs. power law index for FG cylindrical shell (SSSS)

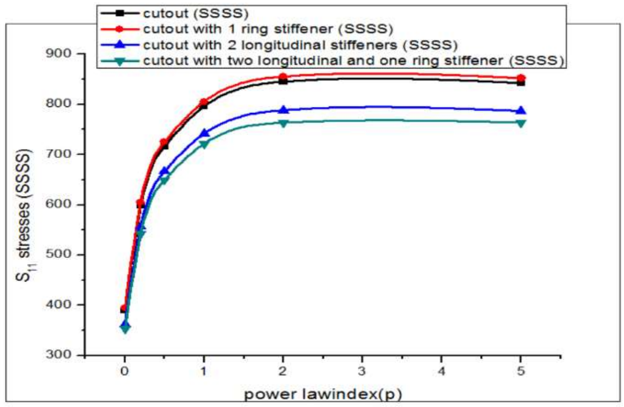


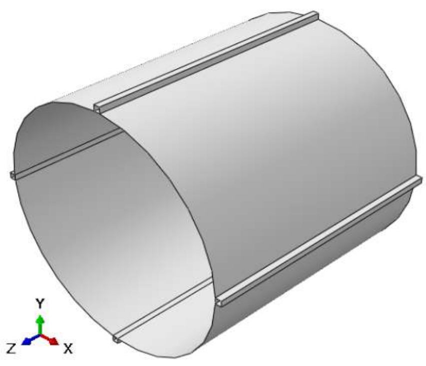

Fig. 6. $\sigma_x$ stresses vs. power law index for FG cylindrical shell (SSSS)

*3.5 Numerical investigations of FG circular cylindrical shell subjected to internal pressure.*

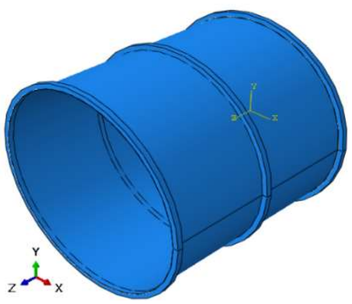
In this example, functionally graded circular cylindrical shell is studied when subjected to internal pressure of 20 KN/m$^2$ for different arrangements of stiffeners. Various arrangements of stiffeners:

A. Longitudinal (external) stiffeners.
B. Ring (external) stiffeners.
C. Ring (internal) stiffeners.

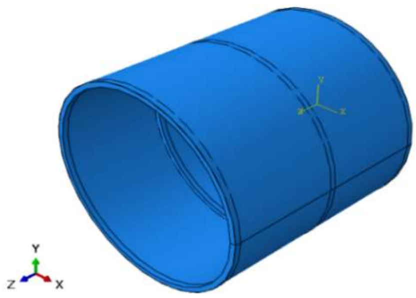
As shown in following Fig 7 and their deformed shapes in Fig. 8. von-Mises stresses are presented in Fig. 9 for various *p*.




**(a)**



**(b)**



**(c)**

Fig. 7. FG cylinder with external (a) longitudinal (b) ring and (c) internal ring stiffeners

The deformed shapes are presented in following figures.

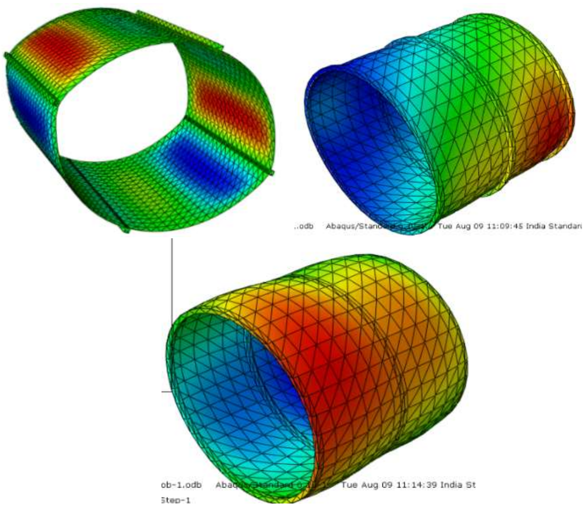


Fig. 8. Deformations in FG cylinders with external (a) longitudinal (b) ring and (c) internal ring stiffeners

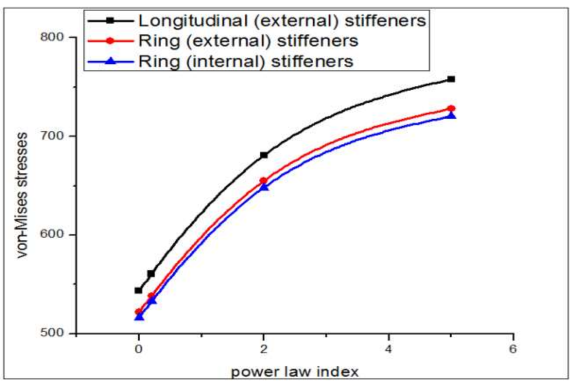



Fig. 9. von-Mises stresses of FG circular cylindrical shell vs. power law index.

## 5. Conclusion

The static stress analysis is presented for FGM cylindrical shells for various boundary conditions and loadings. New results are generated with cutouts and stiffeners. The results are compared with the available literature results and with analytical results based on present FOST.

Below are the conclusions of numerical investigations.

1. The non-dimensional central transverse deflection for an isotropic cylindrical shell increases as the ratio of length to thickness and length to radius increases. The present ABAQUS results shows good agreement with results presented in [2].

2. The non-dimensional central deflection for functionally graded cylindrical shell goes on increasing for the boundary conditions CCCC, CSCS, and SSSS. The present ABAQUS results shows good agreement with [5] FSDT results.

3. As the power law increases, the non-dimensional deflection also increases for functionally graded cylindrical shells.

4. For the purely ceramic shell ($p$=0), the cylindrical shell with cutouts shows lower deflection than the shell without cutouts. For all power law indices, the shell with cutouts shows higher deflection than the shell without cutouts.

5. In FGM cylindrical shell with cutouts, the longitudinally arranged stiffener shows lesser deflection than perpendicularly arranged stiffeners. Longitudinal and perpendicular stiffeners if arranged together, it may give further lesser deflection.

6. In case of FG circular cylindrical shell when subjected to internal pressure, the ring (internal) stiffeners arrangement shows lowest von-Mises stresses while the longitudinal (external) stiffeners show lowest shear stresses.

## Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

## References

1. Huu-Tai Thai, Seung-Eock Kim. A review of theories for the modeling and analysis of functionally graded plates and shells. Composite Structures, 2015; 128: 70-86.

2. Ebrahim Asadi, Wencho Wang, Qatu MS. Static and vibration analyses of thick deep laminated cylindrical shells using 3D and various shear deformation theories. Composite Structures, 2012; 94: 494-500.

3. Dao Huy Bich, Dao Van Dung, Vu Hoai Nam. Nonlinear dynamical analysis of eccentrically stiffened functionally graded cylindrical panels. Composite Structure, 2012; 94: 2465-2473.

4. Memar MA, Soltani B, Shams S. Analysis of functionally graded stiffened plates based on FSDT utilizing reproducing kernel particle method Composite Structures, 2014; 112: 231-240.

5. Viola E, Rossetti L, Fantuzzi N, Tornabene F. Static analysis of functionally graded conical shells and panels using the generalized unconstrained third order theory coupled with the stress recovery. Composite Structures, 2014; 112: 44-65.

6. Oktem AS, Mantari JL, Soares CG. Static response of functionally graded plates and doubly-curved shells based on a higher order shear deformation theory. European Journal of Mechanics A/Solids, 2012; 112: 163-172.7.

7. Huang H, Han Q, Wei D. Buckling of FGM cylindrical shells subjected to pure bending load. Composite Structures, 2011; 93: 2945-2952.

8. Shiyekar SM, Lavate P. Flexure of power law governed functionally graded plates using ABAQUS UMAT. Aerospace Science and Technology, 2015; 46: 51-59.

9. ABAQUS User‟s Manual (2003). ABAQUS Version.6.10

10. Mindlin RD. Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates. J Appl Mech, 1951;18.

11. Reissner E. The Effect of Transverse Shear Deformation on the Bending of Elastic Plates. J Appl Mech, 1945;12.