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Nonlinear Seismic Analysis of RC Elevated Liquid Storage Tanks

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Abstract

Liquid storage tanks are strategically important due to their essential requirement of service in the post-earthquake situation. Numerical modeling of the liquid storage tank needs special attention and cannot be done in the same manner as that of the conventional buildings. In the present paper, a numerical simulation of the RC elevated liquid storage tank is presented. The staging of the tank is modeled as a multi-degree freedom system, and the container with contained liquid is modeled as a two-mass system. Free vibration analysis of the tank is carried out, and mode shapes are extracted. Further, to study the seismic response of the tank, nonlinear time history analysis is carried out. The tank is subjected to time histories of real earthquake ground motions. The varying level of the liquid in the container is another characteristic feature of tanks. The filled condition of the tank is taken into account by considering the aspect ratio (*S*), defined as the ratio of height of the liquid to the radius of the container. The response of the tanks with two different aspect ratios viz. 0.5 (broad) and 2.0 (slender) is studied. The linear modal analysis also carried out to understand the significance of nonlinear analysis, particularly in liquid storage tanks. Displacement, velocity, and acceleration response at the bracing levels, as well as at container levels, are obtained. Additionally, the base shear response is also obtained. The effect of aspect ratio on the free vibration analysis and the seismic response of the tanks are presented. Liquid storage tanks are special structures that have typically low fundamental natural frequencies. The nonlinear time history response of the tank showed that the higher displacement and velocity response occurs at the convective level. It is found that the linear modal analysis significantly underestimates the response of the liquid storage tank.

Keywords: Seismic analysis, RC elevated liquid storage tank, nonlinear analysis, linear modal analysis

1. Introduction

Earthquakes are the most commonly occurring natural disasters. Concerning the performance of structures during and after earthquakes, many research works are being carried out globally. Liquid storage tanks are lifeline structures as they have to remain functional even after the natural disasters. Numerical modelling of the liquid storage tank needs special attention and cannot be done in the same manner as that of conventional buildings. The varying level of the liquid in the container is another characteristic feature of tanks. Many researchers have carried out work in the field of seismic analysis and design of ground supported liquid storage tanks [1, 2, 3, 4, 5, 6, 7, 8]. International codes such as ACI 350.3 [9], EN 1998-4 [10], and IS 1893-2 [11], specify the guidelines for seismic analysis and design of RC liquid storage tanks. Housner [1,2]; and Haroun [3] carried out vibration studies on deformable cylindrical water tanks, and derived parameters of the mechanical model. Ibrahim et al. [4] carried out an extensive review of recent advances in fluid sloshing dynamics; further, mass-spring dashpot and pendulum models for modelling sloshing were discussed [5]. Jain and Jaiswal [12] and; Tung and Kiremidjian [13] Shenton III and Hampton [14], Shrimali and Jangid [15, 16], Panchal and Jangid [17, 18] worked on elevated liquid storage tanks. The primary assumption in all these studies is the staging of the tank is considered as a single degree of freedom system.

In the present study, a brief idea of the mathematical model for a multi-degree freedom system of the staging of the RC elevated tank, and two-mass system for container and the contained liquid is presented. Further, the governing equation of motion and its solution by the state-space method is also described. The results of the free vibration analysis, linear modal, and nonlinear time history analysis are obtained and presented in the subsequent sections.

2. Mathematical Modelling

Housner [1] proposed a two-mass model which is adopted by several researchers. The modified Housner's model presented in the ACI: 350.03 [6] is used in the present study. Figure 1 displays the schematic diagram of the tank model showing the total height of the liquid inside the tank (*H*) and the radius of the circular tank (*R*). In this model, the convective mass (m_c) of the liquid is considered to be connected to the solid tank wall with stiffness (k_c) at height H_c from the base of the container, whereas the rigid mass (m_r) is connected rigidly to the tank wall with stiffness

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(k_i) at a height H_r, while H_{st} denotes the height of the staging, and m is the mass of liquid.

$$m = \pi R^2 H \rho_l \tag{1}$$

where, ρ_l is the density of the contained liquid.

The stiffness of the spring attached to the convective mass (k_c) and the natural frequency of convective mass of liquid (ω_c) are given by,

$$k_c = \frac{1.84 m_c g}{R} \tanh(1.84S) \tag{2}$$

$$\omega_c = \sqrt{\frac{k_c}{m_c}} \tag{3}$$

where, g is the gravitational acceleration and $S = H/R$ is the aspect ratio of the liquid column.

$$c_c = 2\xi_c m_c \omega_c \tag{4}$$

where, c_c is the damping coefficient and ξ_c is the damping ratio of the convective mass. t_w is the thickness of the wall; the spring stiffness attached to the impulsive mass is given by, $k_r = m_i(\omega_i)^2$, and the natural frequency of impulsive mass of liquid (ω_i) is given by,

$$\omega_i = C_I \frac{1}{H} \sqrt{\frac{1000E}{\rho_c}} \tag{5}$$

$$C_I = C_w \sqrt{\frac{t_w}{10R}} \tag{6}$$

$$C_w = 0.09375 + 0.2039\left(\frac{H}{2R}\right) - 0.1034\left(\frac{H}{2R}\right)^2 - 0.1253\left(\frac{H}{2R}\right)^3 + 0.1267\left(\frac{H}{2R}\right)^4 - 0.03186\left(\frac{H}{2R}\right)^5 \tag{7}$$

The convective mass (m_c) and rigid mass (m_r) are expressed in terms of the total mass (m) of the liquid as,

$$\gamma_c = \frac{m_c}{m} = \frac{0.46}{S} \tanh(1.84\,S) \tag{8}$$

and

$$\gamma_r = \frac{m_r}{m} = \frac{S}{1.732} \tanh\left(\frac{1.732}{S}\right) \tag{9}$$

The modified convective mass (m_{rc}) and rigid mass (m_{rr}) are given by,

$$m_{rc} = \gamma_c (m + m_{tw}) \tag{10}$$

$$m_{rr} = \gamma_r (m + m_{tw}) \tag{11}$$

The mass matrix is an important part in mathematical modelling. The assumptions and coupling of masses are explicitly explained by Waghmare et al. [19]. The masses in the staging are assumed to be concentrated at the bracing levels as m_1, m_2, m_3, m_4, m_5, and m_6. Figure 2 shows the mathematical model of the RC elevated liquid storage tank with the lumped masses.

Lumped mass (m_{fb}), which is concentrated at the top-most panel, is the summation of the mass contributed from Panel 7 and the tank base (m_{tb}). k_r and c_{rr} are the stiffness and damping associated with the rigid mass.

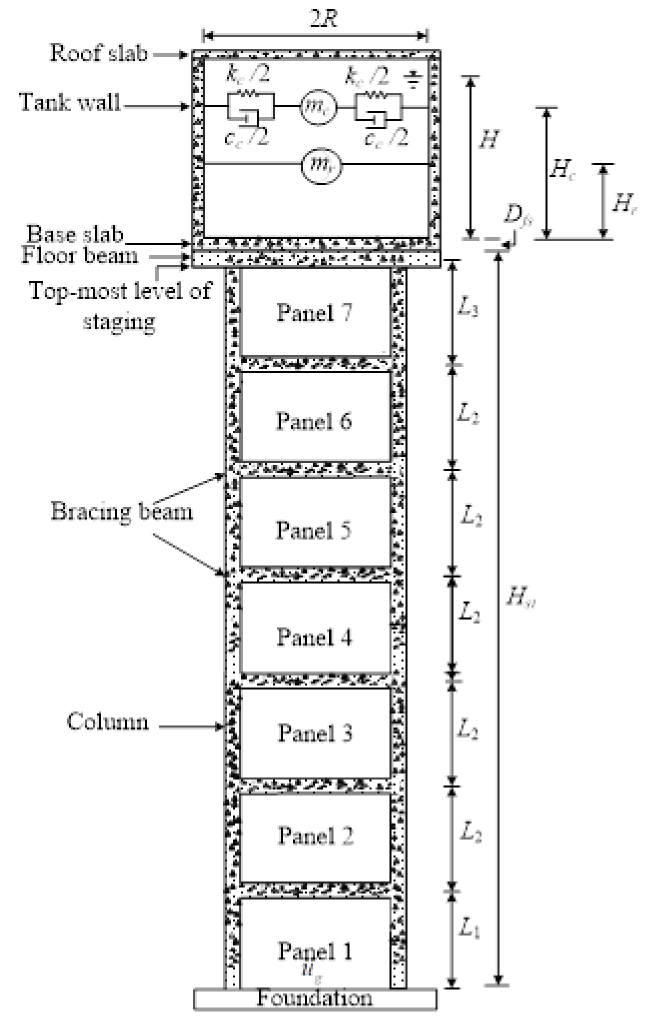

Fig 1 Schematic diagram of RC elevated liquid storage tank

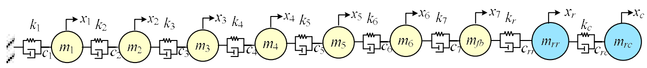

Fig. 2 Mathematical model of RC elevated liquid storage tank

Height (H_c) of the convective mass and impulsive mass height (H_r) are given by,

$$\frac{H_c}{H} = 1 - \frac{\cosh(1.84S) - 2.01}{(1.84S)\sinh(1.84S)} \tag{12}$$

$$\frac{H_r}{H} = \frac{0.866}{S} \coth\left(\frac{1.732}{S}\right) - 0.125 \tag{13}$$

The panel stiffnesses are obtained as discussed by Sameer and Jain [20].

$$k_p = \frac{12\,N_c E I_c}{L_p^{\,3}} \left(\frac{(EI_b)/L_b}{(EI_b/L_b) + (EI_c/L_p)} \right) \tag{14}$$

In Eq. (14) E is the modulus of elasticity of concrete; the moment of inertia of the beams and columns are I_b and I_c, respectively; L_b is span of the beam and; N_c is the number of columns in the staging. The height of the panel in the staging is denoted by L_p, and p is the panel number. The damping associated with column members denoted by c_p , and is given by,

$$c_p = 2\xi_t \sqrt{k_p m_p} \tag{15}$$

The damping ratio of the staging in the elevated tank is ξ_t.

The governing differential equation of motion under earthquake excitation is given by,

$$M\left\{\ddot{X}(t)\right\}+C\left\{\dot{X}(t)\right\}+K\left\{X(t)\right\}=F_e(t) \tag{16}$$

where, $M_{n\times n}$ is mass matrix; $K_{n\times n}$ is stiffness matrix; $C_{n\times n}$ is damping matrix, and $F_e(t)=\{F_1(t) \quad F_2(t) \quad ... \quad F_n(t)\}^{\mathrm{T}}$ is a vector such that $F_e(t)=-M\{r\}\ddot{u}_g(t)$ where $\{r\}=\{1 \quad 1 \quad ... \quad 1\}^T$ is the influence coefficient vector. Moreover, $\{X\ (t)\}$ is the vector of displacement of the masses relative to the ground, expressed as:

$$\{X(t)\}=\{x_1(t) \quad x_2(t) \quad \cdots \quad x_r(t) \quad x_c(t)\}^T \tag{17}$$

where, $x_1=(d_1-u_g)$,

$x_2=(d_2-u_g),\cdots,x_r=(d_r-u_g),x_c=(d_c-u_g)$; u_g is the ground displacement; and d_c, d_r, …, d_2, d_1 are the absolute displacements at the container level and bracing level masses.

The base shear, V_b and the overturning moment, M_o in the staging at the foundation level are expressed as:

$$V_b=m_{rc}a_c+m_{rr}a_r+\cdots+m_3a_3+m_2a_2+m_1a_1 \tag{18}$$

$$\begin{aligned}M_o&=m_{rc}a_c\,(H_c+D_{fs}+H_{st}+\frac{D_{fb}-D_{fs}}{2})+\\&m_{rr}a_r\,(H_r+D_{fs}+H_{st}+\frac{D_{fb}-D_{fs}}{2})+\\&\cdots m_3a_3(2L_2+L_1)+m_2a_2(L_2+L_1)+m_1a_1L_1\end{aligned} \tag{19}$$

Here, the height of the staging $H_{st}=\left[L_1+(p-2)L_2+L_3\right]$, D_{fs} is the depth of the floor slab, and D_{fb} is the depth of the floor beam. The absolute accelerations at the different levels of masses are denoted by a_c, a_r, …, a_2, a_1.

3. Solution to governing equation

The state-space model represents the mathematical model of the physical system. In the present study state space approach is used for modelling and solving the governing equation of motion. In the state-space method, the independent variables viz. displacement and velocity, are called states [21]. Vector, $z(t)$ is defined to represent states of the system viz. displacement and velocity.

$$z(t)=\begin{Bmatrix}X(t)\\ \dot{X}(t)\end{Bmatrix}. \tag{20}$$

If the degree of freedom of a structure is n, then there will be $2n$ states, first n for the displacement and remaining n for the velocity. It follows that Eq. (16) can be written in the following equivalent state-space form,

$$\dot{z}(t)=Az(t)+F(t) \tag{21}$$

where, **0** and I are respectively, null matrix with all zero elements and identity matrix.

where,

$$A=\begin{bmatrix}\mathbf{0} & I\\ -M^{-1}K & -M^{-1}C\end{bmatrix} \tag{22}$$

$$F(t)=\begin{Bmatrix}\mathbf{0}\\ M^{-1}F_e(t)\end{Bmatrix}. \tag{23}$$

The continuous state-space equation of motion is solved over the entire range of the time history of the earthquake ground motion.

4. Numerical study

The RC elevated liquid storage tank of 20 m staging height is considered. Here, the degrees of freedom (n) are nine, and the number of panels in the staging is seven. Damping coefficients are ξ_c = 0.5% and $\xi_i=\xi_t$ = 5%. The number of columns is four, and $L_1=L_3$ = 2.5m; L_2 = 3m. Height (H) of the container is 5 m and radius (R) is 2.53 m. Aspect ratio S, (H/R) is considered as 0.5 and 2.0. The dynamic properties and mode shape are shown in Figure 3. A code has been written in MATLAB [22] for the time history analysis of the system. The results obtained are compared with the results of nonlinear time history analysis, and linear modal time history analysis is performed using SAP 2000 [23]. The tank is subjected to Tabas, 1978 earthquake ground motion with peak ground acceleration as 0.93 g.

4.1 Free vibration analysis

Free vibration analysis of the tanks is carried out, and the mode shapes are extracted. Figure 3 presents the different natural frequencies and the respective modal response at the different levels of the tank obtained for the nine modes, for S = 0.5 and S =2.0. It is noted from Figure 3 that fundamental natural frequency (Mode1) of the liquid storage tank is very low as 0.2 Hz for S =0.5 and 0.27 Hz for S = 2.0. The long natural period makes the liquid storage tank more special, as it will take more time to come to rest.

Form Figure 3 it is observed that the liquid storage tanks have low fundamental natural frequency (Mode 1). A long natural period indicates that the tank takes a long time to come to rest. This is attributed to the low stiffness of liquid in the container. Additionally, it is clear from Figure 3 that the first three modes (Mode1, Mode2, and Mode 3) are the significant modes of vibration. In case of elevated liquid storage tanks, a huge mass of the liquid is concentrated at the container level, while the mass of the staging is very less. It is observed that in the first three modes, the entire elevated tank along with container level masses (m_{rc} and m_{rr}), and bracing level masses, participate in the vibration. However, it is exciting to note that in case of higher modes (Mode 4 to Mode 9), the container level masses do not participate in the modal response, thereby decreasing the modal mass participation.

Further, in the case of slender tank (S = 2.0), in Mode 2, the response of the convective level mass is more than that of broad tank. The performance of the broad tank and slender tank in Mode 1 and Mode 2 is entirely different. In Mode 1, both tanks behave in the same way (as a cantilever fixed at the base), with slender tank having a slightly higher response than the broad tank. In the case of Mode 2, the response of the convective mass is maximum; at the same time, broad tank showed a negligible modal response at the convective level. This may be attributed to the change in natural frequencies; the natural frequency of the slender tank is almost half of that of the broad tank.

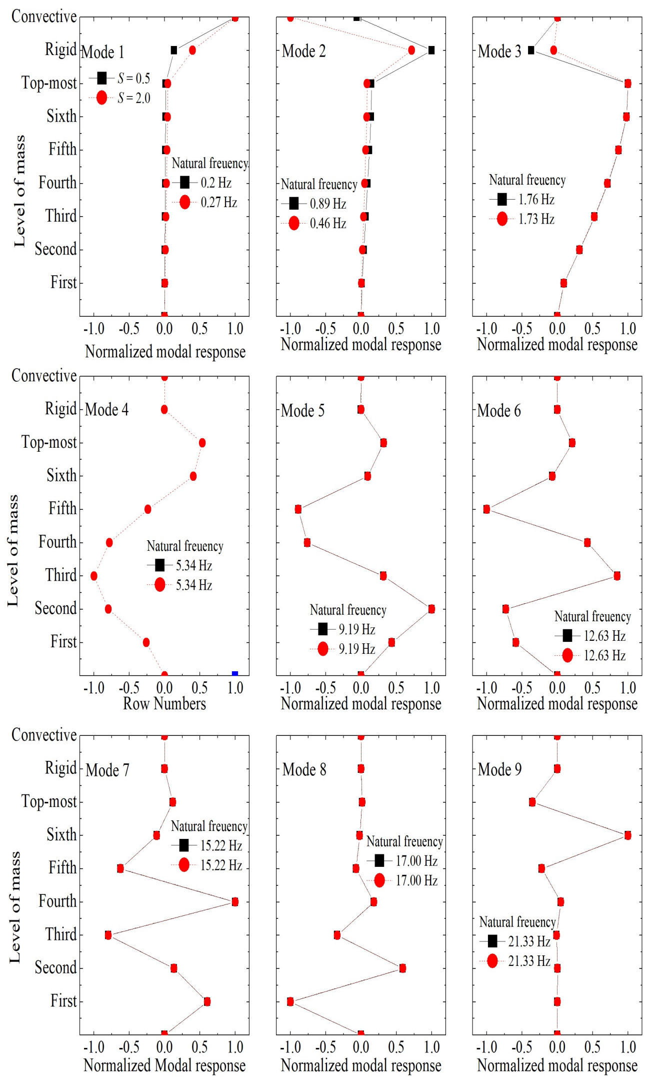

Fig. 3 Mode shapes of RC elevated liquid storage tank

4.2 *Time history analysis*

Nonlinear time history analysis has been performed using the MATLAB code for solving the Eq. 21.

The multi-degree freedom system, as discussed in section 2, is modelled in SAP 2000. Linear (modal) and Nonlinear (direct integration) time history analyses are carried out, and the results are compared with the results of nonlinear time history analysis performed using the state-space model.

Figure 4 shows the acceleration, velocity, and displacement response obtained by three different approaches viz: State Space (present study), nonlinear (direct integration), and linear modal (constant damping) analysis. Figure 5 presents the base shear response of the slender tank (S = 2.0) under the Tabas, 1978 earthquake. From Figure 4 and Figure 5, it can be seen that the acceleration, velocity, displacement, and the base shear response obtained from the present study, and the nonlinear time history analysis (SAP 2000), are in complete agreement. However, Linear modal analysis underestimates all responses of the tanks.

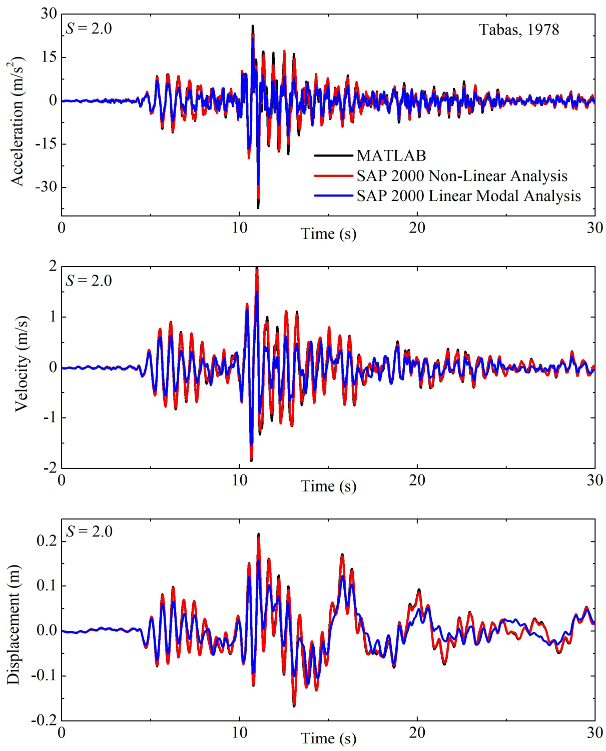

Fig. 4 Acceleration, velocity and displacement response at the top-most level (m_{fb}) of the staging

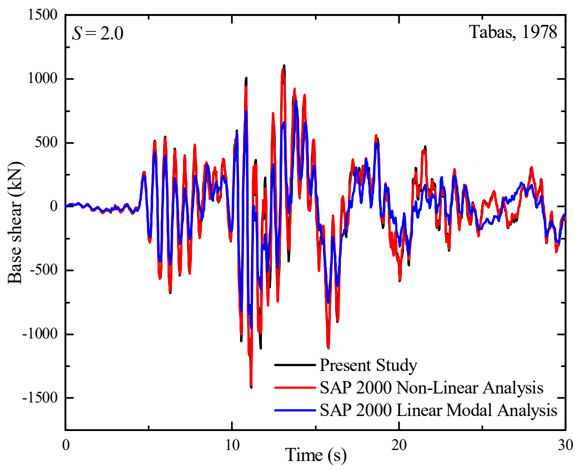

Fig. 5 Base shear response of RC elevated liquid storage Tank, subjected to Tabas, 1978, earthquake.

Table 1 summarizes the results of peak displacement, peak velocity, and peak acceleration at all levels of the masses, and the peak base shear. Further, the % difference between the results obtained by linear and nonlinear analysis in SAP 2000 and the present study is also presented.
It is observed from Table 1 that the displacement response of the tank increases from bottom to top such that the maximum response is at the convective mass level. However, it is very interesting to note that the velocity response increases with the height of the tank and attains its maxima at the convective level with a slight drop at rigid mass level.

The acceleration response is observed to be maximum in the staging of the tank with lower accelerations at the first level, second-level, and the container level masses.

Table 1 Uncontrolled response of the RC elevated liquid storage tank (S = 2.0).

Response Quantity	Present Study	SAP 2000 (Non-Linear Analysis)	Difference (%)	Present Study	SAP 2000 (Linear Modal Analysis)	Difference (%)
x_1 (m)	0.018	0.018	0	0.018	0.012	32.44
x_2 (m)	0.060	0.058	3.33	0.06	0.039	35.13
x_3 (m)	0.097	0.092	5.15	0.097	0.069	28.53
x_4 (m)	0.138	0.135	2.17	0.138	0.100	27.27
x_5 (m)	0.177	0.175	1.13	0.177	0.129	27.09
x_6 (m)	0.211	0.209	0.95	0.211	0.153	27.43
x_7 (m)	0.217	0.216	0.46	0.217	0.158	27.34
x_8 (m)	1.016	1.011	0.49	1.016	0.843	17.05
x_9 (m)	**2.064**	**2.044**	**0.97**	**2.064**	**1.628**	**21.13**
$\dot{x}_1$ (m/s)	0.251	0.269	-7.24	0.251	0.201	20.09
$\dot{x}_2$ (m/s)	0.805	0.862	-7.05	0.805	0.610	24.24
$\dot{x}_3$ (m/s)	1.255	1.350	-7.61	1.255	0.945	24.67
$\dot{x}_4$ (m/s)	1.550	1.612	-3.97	1.550	1.164	24.88
$\dot{x}_5$ (m/s)	1.669	1.742	-4.33	1.669	1.353	18.99
$\dot{x}_6$ (m/s)	1.933	1.934	-0.05	1.933	1.516	21.57
$\dot{x}_7$ (m/s)	2.030	2.009	1.04	2.030	1.542	24.05
$\dot{x}_8$ (m/s)	1.730	1.707	1.33	1.730	1.505	13.00
$\dot{x}_9$ (m/s)	**3.634**	**3.614**	**0.55**	**3.634**	**2.816**	**22.49**
a_1 (m/s^2)	9.826	7.414	24.55	9.826	5.084	48.26
a_2 (m/s^2)	19.652	17.511	10.89	19.652	13.845	29.55
a_3 (m/s^2)	24.109	23.263	3.51	24.109	16.017	33.56
a_4 (m/s^2)	22.685	23.787	-4.86	22.685	16.429	27.58
a_5 (m/s^2)	23.932	23.632	1.25	23.932	20.692	13.54
a_6 (m/s^2)	34.961	33.894	3.05	34.961	27.545	21.21
a_7 (m/s^2)	**37.234**	**35.685**	**4.16**	**37.234**	**28.838**	**22.55**
a_8 (m/s^2)	9.303	9.330	-0.29	9.303	9.575	-2.92
a_9 (m/s^2)	10.687	10.665	0.21	10.687	10.009	6.34
Base Shear (kN)	1420.363	1398.493	1.51	1420.363	950.884	33.04

This is particularly due to a considerable mass of liquid concentrated at the container level, which has lower stiffness than the staging. The higher rigidity of staging attracts higher accelerations. From Table 1, it is noted that in the case of nonlinear analysis, results from the present study, and SAP 2000 for displacement, velocity, and base shear are in total agreement with each other, with maximum difference up to 5%, 7%, and 2%, respectively.

However, the difference is higher (up to 24%), in the case of acceleration. Further, in the case of Nonlinear and Linear Modal analysis, the maximum difference between the response quantities is of the order of 35% for displacement, 24% for velocity, 33% for base shear, and 48% for acceleration. The lower response in the linear modal analysis is attributed to the fact that it considers constant damping ratio (ξ = 5%), which is quite higher than that to be considered for the liquid storage tank. In the case of the liquid storage tank, the damping due to impulsive and convective masses is quite different (5% and 0.5%), and has to be considered in analysis appropriately. The lower results of the linear modal analysis method are due to the ignorance of varying damping ratios, and non-proportional damping.

5. Conclusions

The nonlinear time history analysis is carried out to evaluate the seismic response of the RC elevated liquid storage tank. A code has been developed using MATLAB® computing software. The seismic performance in terms of the response of displacement, velocity, acceleration, and base shear is obtained. It is observed that liquid storage tanks are long-period structures. Free vibration analysis is carried out to study the effect of aspect ratio (broad tank and slender tank) on the dynamic properties of the elevated RC liquid storage tank. The seismic performance is studied by plotting time variation of the response, and values of peak response are tabulated. It is concluded that the dynamic response of the elevated tank depends on the aspect ratio. This is due to the engagement of masses with different properties. In Mode 1, slender tank and broad tank behave in the same way (as a cantilever fixed at the base), with slender tank having a slightly higher response than the broad tank. In the case of Mode 2, the modal response of the convective mass is maximum for slender tank; at the same time, broad tank showed a negligible modal response at the convective level. The nonlinear time history response of the tank showed that the higher displacement and velocity response occurs at the convective level. However, due to the higher mass and lower stiffness of the container, the maximum acceleration response is observed in the staging of the tank. Additionally, the linear modal analysis underestimated the response, while the nonlinear analysis by the other two approaches is in good agreement.

Disclosures

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