

Free Vibration Analysis of Stiffened Lock Gate Coupled with Reservoir Fluid considering Linearised Free Surface Condition

Deepak Kumar Singh ^{1,*}, Priyaranjan Pal ²

¹ Civil Engineering Department, Research Scholar, Motilal Nehru National Institute Technology Allahabad, Prayagraj, 211 004, India

² Civil Engineering Department, Associate Professor, Motilal Nehru National Institute Technology Allahabad, Prayagraj, 211 004, India

Paper ID - 250184

Abstract

The effect of surrounding reservoir fluid on the stiffened lock gate structure is investigated using the finite element method. A single stiffener is used to stiffen the plate, which is placed edge to edge along the height on the plate's center nodal line. Mindlin's plate bending and Euler's beam theories are used to formulate plate and stiffener, respectively. The stiffened lock gate material is assumed to be isotropic, homogeneous, uniformly thick and elastic in nature. The fluid is assumed to be incompressible and inviscid, resulting in an irrotational flow field. The fluid domain's top free surface is assumed to be linear based on Airy's linear wave theory. The far boundary of the fluid domain is truncated numerically close to the lock gate to control the size of computation without influencing the results, very much. It is truncated by solving the Laplace equation using Fourier half range cosine series expansion in the finite element formulation. Pressure and displacement are considered as nodal variables for the fluid domain and the lock gate, respectively. The interaction between the fluid domain and the lock gate is established by finite element formulation and transformed into a computer code, written in FORTRAN. The natural frequencies of clamped and simply supported stiffened lock gates are evaluated by the varying extent of the fluid. Both stiffened and unstiffened gates are compared. The results are beneficial to the engineers/designers when the gate structure is subjected to cataclysmic events.

Keywords: Stiffened Lock gate, Stiffener, Finite element method, Linearised free surface, Frequencies.

1. Introduction

The analysis of structure influenced by the surrounding fluid is a popular subject of research. Many researchers have carried out to investigate the behaviour of the structure, fluid and their effects on each other by experimental and numerical techniques. When there is relative motion between the fluid and the adjacent structure, the type of phenomena is known as fluid-structure interaction. The structure has to be strong enough to the dynamic impact of the fluid and the changing behaviour of fluid due to waves' effect at the surface. The lock gate structure is one such structure under the influence of fluid as it controls the water flow in the dam structure system, and most of the time, it is in contact with water. Thus, the surrounding fluid affects the lock gates' behaviour. Also, the natural frequency of structure decreases when in contact of fluid than in the absence of fluid. Hence, the determination of frequencies of such structures become very crucial. Such fluid-structure interaction problems are addressed very frequently in literature, but the effect of the free surface wave is not given much importance each time.

Some important literature is discussed below to show the significance of these studies. Free vibration analysis of plate influenced by the surrounding fluid is investigated [1, 2]. A novel truncated infinite boundary of the fluid is developed for compressible fluid [3]. Further, author has developed the same for incompressible fluid [4]. The plate-water system is

solved using an analytical Ritz procedure and validated the outcome with AVMI (Added Virtual Mass Incremental) factor approach [5]. The problem of the gate is solved by the finite element method [6-7]. A liquid-filled container is investigated to extract the free vibration frequencies using a meshless local Petrov-Galerkin (MLPG) approach [8-9]. Unstiffened and stiffened lock gates are investigated to determine the free vibration frequencies [10-12].

The present study aims to explore the impact of fluid on the natural frequencies of unstiffened and stiffened lock gates, considering the linearised free surface condition at the top of the reservoir fluid.

2. Modelling

Figure 1 shows a stiffened lock gate structure's model considered for investigation. The infinite length is curtailed near the gate to minimise the computational time without affecting the outcomes.

3. Mathematical Formulation

The mathematical formulation for fluid and lock gate is done separately. The coupling between the two is established utilising the finite element formulation and finally converted into a FORTRAN computer program to determine the natural frequencies of the gate structure. The detailed formulation is found elsewhere [11, 12].

*Corresponding author. Tel: +91-8560930378; E-mail address: erdeepak.india@gmail.com

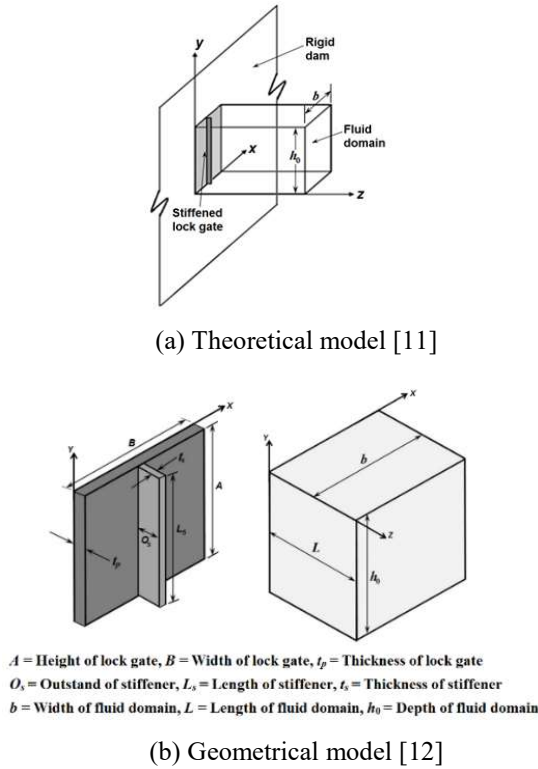


Fig. 1 Stiffened lock gate system with fluid domain

3.1 Truncation of Far Boundary

The fluid pressure might be composed by using Laplace equation for invisible, incompressible fluid and irrotational fluid flow neglecting the static pressure as

$$\nabla^2 p = \nabla^2 p(x, y, z) = 0 \quad (1)$$

where, ∇^2 = Laplacian operator and p = fluid pressure.

The fluid domain's infinite length is shortened closer to the gate numerically after incorporating the different boundary conditions [12]. The boundary at the infinite length can be established after solving Eqn. (1) and represented as

$$p = \frac{4\rho_f a}{h_0} \sum_{m=0}^{\infty} \frac{\sin(\lambda_m)}{(\lambda_m)^2} (e^{-\lambda_m \frac{z}{h_0}}) \cos(\lambda_m \frac{y}{h_0}) \quad (2)$$

$$\text{where, } \lambda_m = \frac{(2m-1)\pi}{2}$$

$$\frac{\partial p}{\partial n} = \left(\frac{\partial p}{\partial n} \right)_{z=L} = \frac{-p}{h_0} \mathbf{x}_i \quad (3)$$

where, L = length of fluid and

$$\mathbf{x}_i = \frac{\sum_{m=0}^{\infty} \frac{\sin(\lambda_m)}{(\lambda_m)^2} (e^{-\lambda_m \frac{z}{h_0}}) \cos(\lambda_m \frac{y}{h_0})}{\sum_{m=0}^{\infty} \frac{\sin(\lambda_m)}{(\lambda_m)^3} (e^{-\lambda_m \frac{z}{h_0}}) \cos(\lambda_m \frac{y}{h_0})}$$

here, x , y and z denote the axis in the direction of fluid's width, depth and length, respectively.

3.2 Finite Element Formulation

The obtained coupled equation is solved using the Jacobi iterative method to evaluate the gate's frequencies. 9 noded isoparametric plate bending elements for the gate, 3 noded

isoparametric beam elements for the stiffener, and 27 noded isoparametric hexahedron elements for the fluid domain have been considered to discretise the domains.

3.2.1 Governing Equation for the Gate

The undamped dynamic equilibrium equation of the stiffened lock gate may be defined as

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \quad (4)$$

where, $[M]$ and $[K]$ are overall mass and stiffness matrices of the gate. $\{\ddot{X}\}$ and $\{X\}$ are displacement, and acceleration vectors. Stiffness and mass matrices corresponding to each element are the summation of stiffness and mass matrices of the plate and the stiffener's contributions to that element.

3.2.2 Governing Equation for the Fluid Domain

The finite element formulation is presented somewhere else [12], considering the fluid's free surface as linear. Using the Galerkin's weighted residual method, the weighted average integral of Eqn. (1) may also be represented as

$$[G]\{\bar{p}\} = \{B\} \quad (5)$$

where, $[G] = \sum \int_{\Omega_e} (\nabla N^T \cdot \nabla N) d\Omega_e$ and

$$\{B\} = - \sum_{S_f} \left[\int_{\Gamma_e} (N^T N) d\Gamma_e \right] \left\{ \frac{\ddot{p}}{g} - \frac{1}{h_0} [R_{tis}] \{\bar{p}\} \right\}$$

where, $[R_{tis}] = \sum_{S_t} \int_{\Gamma_e} (N^T \mathbf{x}_i N) d\Gamma_e$

The boundary term $\{B\}$ are well defined in the literature [12] and after incorporating the boundary conditions, Eqn. (5) yields

$$[E_i]\{\ddot{p}\} + [G_{is}]\{\bar{p}\} = -\rho_f [R_{fs}]\{\bar{a}\} \quad (6)$$

where, $[E_i] = \frac{1}{g} \sum_{S_f} \left[\int_{\Gamma_e} (N^T N) d\Gamma_e \right]$ and

$$[G_{is}] = [G] + \frac{1}{h_0} [R_{tis}]$$

3.3.3 Coupling of the Lock Gate and the Fluid

The formulation of the domains is coupled to inquire about the effect of fluid on the lock gate structure. The movement of the interface between fluid and structure is prescribed by the stiffened gate. Substituting $\{\bar{a}\}$ with $\{-\ddot{X}\}$ in Eqn. (6), we get

$$[E_i]\{\ddot{p}\} + [G_{is}]\{\bar{p}\} = \rho_f [R_{fs}]\{\ddot{X}\} \quad (7)$$

When the fluid pressure effect is added to Eqn. (4),

$$[M]\{\ddot{X}\} + [K]\{X\} = \{f_s\} \quad (8)$$

where, $\{f_s\}$ is the forcing component because of fluid.

$$\{f_s\} = - \sum_{S_{fs}} \int_{\Gamma_e} [N_s]^T p d\Gamma_e = -[R_{fs}]^T \{p\} \quad (9)$$

After substituting $\{\bar{p}\}$, it yields

$$[M]\{\ddot{X}\} + [K]\{X\} + [R_{fs}]^T\{\bar{p}\} = \{0\} \quad (10)$$

Substituting $\{\bar{p}\}$ from Eqn. (7), Eqn. (10) is

$$\begin{bmatrix} [K] & \mathbf{0} \\ \mathbf{0} & \frac{[E_l]}{\rho_f} \end{bmatrix} \begin{Bmatrix} \{\ddot{X}\} \\ \{\ddot{\bar{p}}\} \end{Bmatrix} + \begin{bmatrix} [K][M]^{-1}[K] & [K][M]^{-1}[R_{fs}]^T \\ [R_{fs}][M]^{-1}[K] & \frac{[G_{ls}]}{\rho_f} + [R_{fs}][M]^{-1}[R_{fs}]^T \end{bmatrix} \begin{Bmatrix} \{X\} \\ \{\bar{p}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad (11)$$

The coupled Eqn. (10) is solved by using Jacobi iterative method to assess the frequencies.

4. Result and Discussion

The first six natural frequencies of lock gates are evaluated for clamped and simply supported boundary conditions. The lock gate and the stiffener's modeling are processed, keeping the minimum thickness of 6 mm as per IS Codal provisions. The outstand of the stiffener is considered as $14t_s \epsilon$ (IS: 800 - 2007) [13], t_s is the thickness of stiffener and ϵ is yield stress ratio $\sqrt{\left(\frac{250}{f_y}\right)}$, f_y is yield stress (250 MPa). The natural frequencies are presented in the non-dimensional form as,

$$\Omega = \omega b^2 \sqrt{\frac{\rho_p t_p}{D_p}} \quad (12)$$

where, ω = natural frequency (Hz), b = width of lock gate, ρ_p = density of lock gate = 7850 kg/m³, t_p = thickness of the lock gate, D_p = flexural rigidity of the lock gate = $\frac{Et_p^3}{12(1-\nu^2)}$, where, E = Elastic modulus = 2.0×10^5 N/mm², ν = Poisson's ratio = 0.3.

A convergence study is carried out to decide the optimum mesh size. An unstiffened square lock gate of width (B) 1.0 m, and thickness (t_p) 6 mm is considered. The non-dimensional frequencies are plotted for both the boundary conditions when there is no fluid, as illustrated in Fig. 2.

It is found that the values are coming very close after the mesh division of 5×5 and 4×4 for clamped and simply supported boundary conditions, respectively. So, the mesh size of 5×5 for clamped and 4×4 for simply supported boundary conditions are considered. A parametric study is carried out to determine the effect of surrounding fluid on the gate's frequencies by stiffening the gate and the results are presented through some examples.

Example 1: The non-dimensional fundamental frequency of square unstiffened lock gate of B 1.0 m and B/t_p 150 is evaluated. The fluid's far boundary is varied, keeping the width (b) and depth (h_0) of the fluid equal to the lock gate's height. The results of the non-dimensional fundamental frequency are shown in Fig. 3 for different boundary conditions.

It is noticed that when the far fluid boundary is changed, the value of frequency converges after the length of the fluid domain equal to the height of the lock gate. Therefore, the fluid length (L) is truncated to 1.0 m.

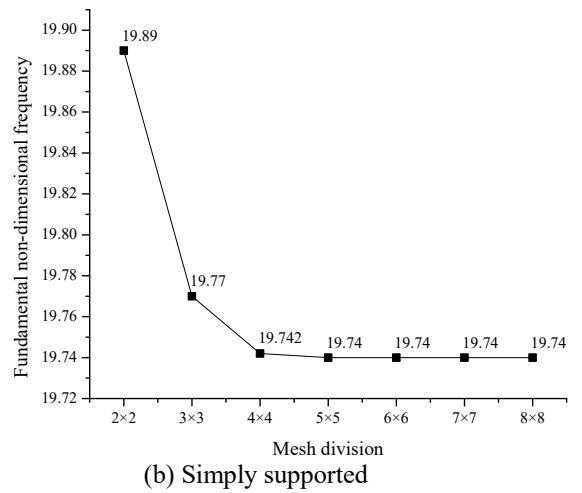
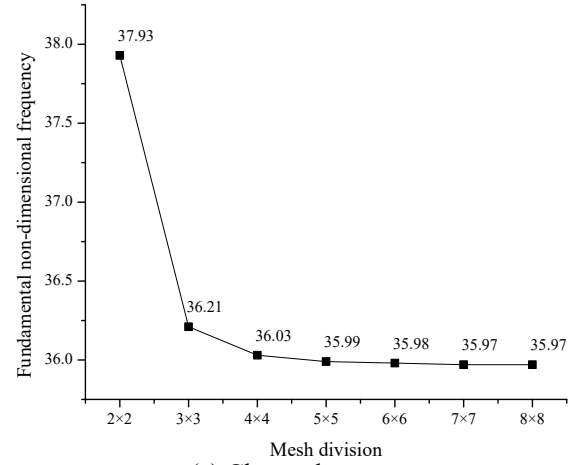
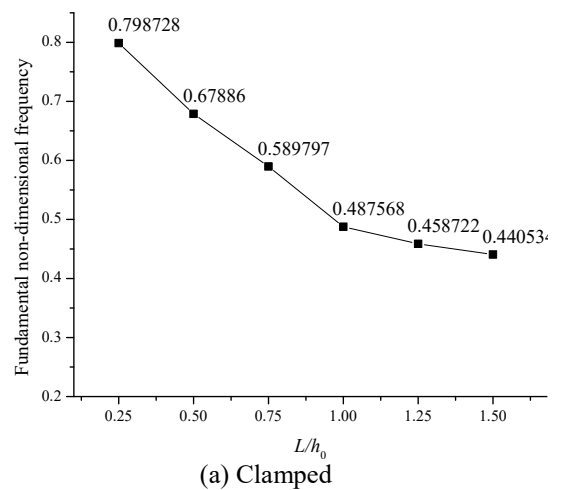
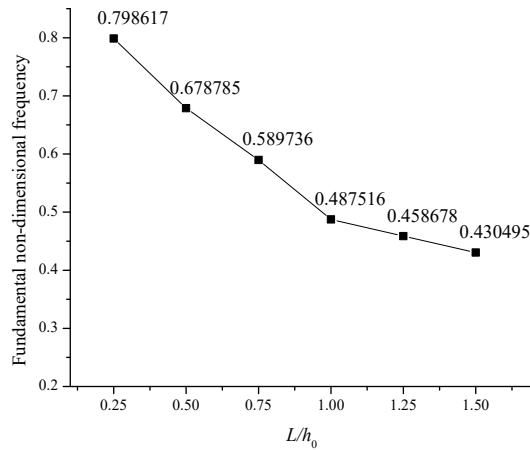


Fig. 2 Convergence study



Example 2: In this example, the unstiffened lock gate's non-dimensional fundamental frequency is determined for both the boundary conditions in the presence of surrounding fluid having the linearised free surface condition. An unstiffened square lock gate of width (B) 1.0 m and width-to-thickness ratio (B/t_p) 150 is considered with fluid height, length and width as 1.0 m. The percentage reduction in frequencies in the influence of surrounding fluid is compared to when there is no fluid, as shown in Fig. 4. It is seen that the frequencies of the lock gate decrease when the surrounding



(b) Simply supported

Fig. 3 Non-dimensional fundamental frequency of unstiffened lock gate

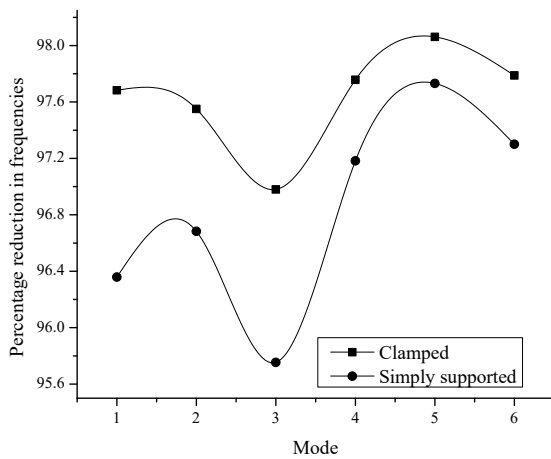
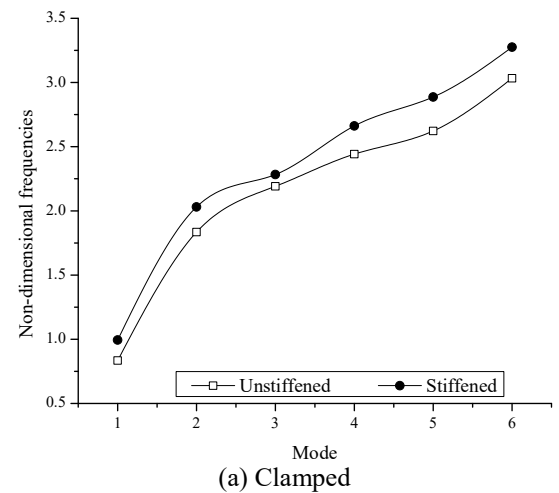


Fig. 4 Reduction in frequencies

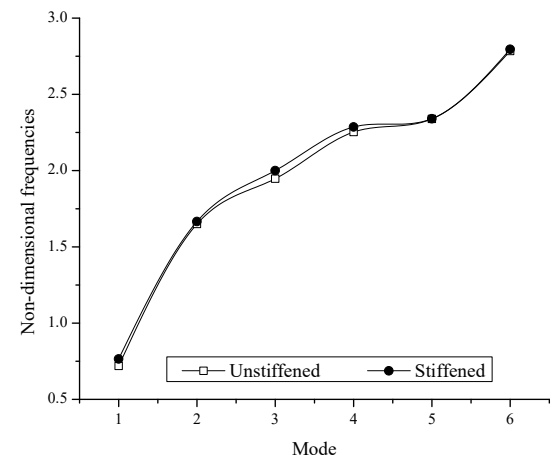
fluid influences it. The decrement is almost 96 percent, which is a matter of concern. So, the lock gate needs to be stiffened to enhance the gate's frequencies and is discussed in the next examples.

Example 3: A square stiffened lock gate of width (B) as 1.0 m and width-to-thickness ratio (B/t_p) as 150 and a stiffener of 1.0 m length, 84 mm outstand and 6 mm thick is considered. The results of the stiffened lock gate are compared with the unstiffened lock gate. To maintain the constant volume of material in both unstiffened and stiffened lock gates, the stiffened lock gate's thickness is reduced, following the minimum thickness criteria as per Codal provision. The non-dimensional frequencies are demonstrated in Fig. 5 for both the boundary conditions. It is found that when the plate is stiffened, free vibration frequencies are increased due to the presence of the fluid. The increment in frequencies is more for clamped boundary condition compared to simply supported condition.

Example 4: A square stiffened lock gate is considered for investigation. The results of the stiffened lock gate are compared with the unstiffened lock gate. The increment in the first six frequencies is demonstrated in Fig. 6. It is observed that the frequencies are increased in both the boundary conditions when the lock gate is stiffened.

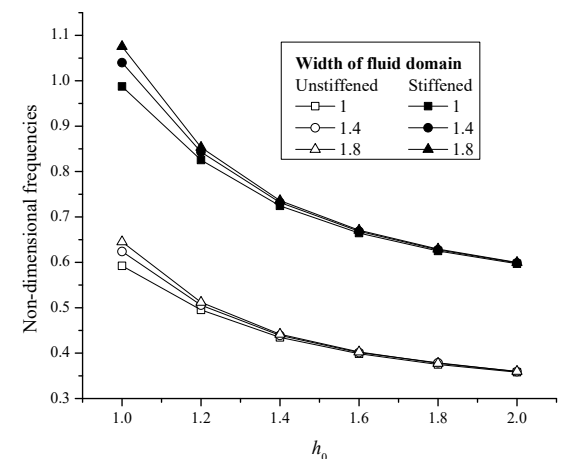


(a) Clamped

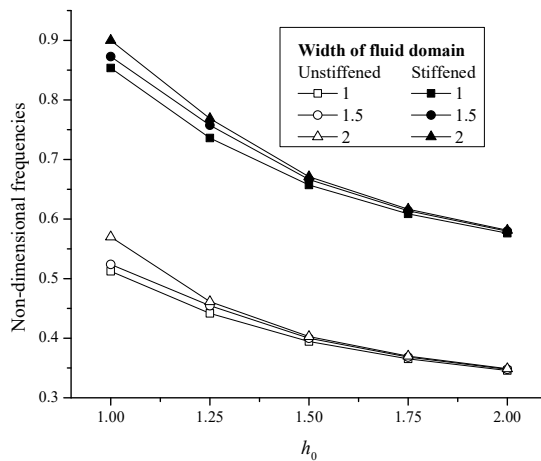


(b) Simply supported

Fig. 5 Non-dimensional frequencies



(a) Clamped



(b) Simply supported

Fig. 6 Non-dimensional frequencies

5. Conclusions

The natural frequencies are determined both in the absence and presence of fluid. Both the unstiffened and stiffened lock gates are considered for the study and the following conclusions are drawn.

- There is almost no variation in the fundamental frequency when the distance of infinite boundary of the fluid is greater than the height of the gate.
- The frequencies decrease in the presence of surrounding fluid, and reduction is almost constant, but about 96 percent.
- When the lock gate is stiffened, frequencies increase and the increment is almost twice compared to that of unstiffened lock gate keeping the almost same volume of material for both the gates.
- For both unstiffened and stiffened lock gates, frequencies slightly increase with the width of fluid. But it decreases for the corresponding fluid width when fluid's height is raised above the gate.

Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

References

1. Amabili M, Frosali G, Kwak MK. Free vibrations of annular plates coupled with fluids. *Journal of Sound and Vibration*, 1996; 19(5): 825-846.
2. Amabili M, Kwak MK. Vibration of circular plates on a free fluid surface: Effect of surface waves. *Journal of Sound and Vibration*, 1999; 226: 407-424.
3. Maity D, Bhattacharyya SK. Time domain analysis of infinite reservoir by finite element method using a novel far-boundary condition. *Finite Element Analysis and Design*, 1999; 32: 85-96.
4. Maity D. A novel far-boundary condition for the finite element analysis of infinite reservoir. *Applied Mathematics and Computation*, 2005; 170:1314-1328.
5. Zhou D, Cheung YK. Vibration of Vertical Rectangular Plate in Contact with Water on One Side. *Earthquake Engineering and Structural Dynamics*, 2000; 29: 693-710.
6. Pani PK, Bhattacharyya SK. Fluid-structure interaction effects on free vibration of a vertical rectangular lock gate using a near truncated boundary. *Institution of Engineers (India) - Structural Division*, 2006; 86: 187-194.
7. Pani PK, Bhattacharyya SK. Free vibration characteristics of a rectangular lock gate structure considering fluid-structure interaction. *Advances in Vibration Engineering*, 2008; 7(1): 51-69.
8. Pal P. Free-vibration analysis of liquid-filled containers using meshless local Petrov-Galerkin approach. *International Journal of Recent Trends in Engineering*, 2010; 3:1-5.
9. Pal P, Bhattacharyya SK. Sloshing in partially filled liquid containers - Numerical and experimental study for 2-D problems. *Journal of Sound and Vibration*, 2010; 329: 4466-4485.
10. Pal P, Singh RR, Singh DK. Free vibration frequencies of lock gate structure considering fluid structure interaction. *International Journal of Advanced Civil Engineering and Technology*, 2016; 1: 1-22.
11. Singh DK, Duggal SK, Pal P. Free vibration analysis of stiffened lock gate structure coupled with fluid. *Journal of Structural Engineering (Madras)*, 2018; 45: 1-9.
12. Singh DK, Pal P, Duggal SK. Free vibration analysis of lock gate structure. *Journal of Mechanics*, 2020.
13. IS: 800 (2007) Indian standard, general construction in steel — Code of practice, Bureau of Indian Standards, New Delhi.