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Analysis of Smart Functionally Graded Beams combined with piezoelectric material using Finite Element Method

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Abstract

The present work devotes the static and vibration behaviour of Smart Functionally Graded (SFG) Beams combined with piezoelectric material. In this work, Lead Zirconate Titanate (PZT-4) is taken as the piezoelectric material which is commercially available. The efficiency of piezoelectric material integrated onto the functionally graded (FG) beams is evaluated. A finite element model is made for the FG beam combined with piezoelectric material considering first order theory (FSDT). The material properties of the FG beam is graded along the direction of its thickness following power law distribution method. Simulation models are also made employing finite element software (ANSYS) for the functionally graded beams. Different boundary conditions as well as and power law indexes are taken for analysis. Validation of results is presented with the results available in earlier research publications. Effects of gradation of material properties on static and dynamic behaviour of smart functionally graded beam are studied. Efforts are also made to examine the performance of piezoelectric patches towards control of deflections and vibrations.

Keywords: Smart functionally graded beam; piezoelectric material; power law; finite element method

1. Introduction

The Functionally graded materials (FGMs) are the advanced composite materials having microscopic inhomogeneous characteristics [1]. Functionally graded materials was first developed by Japanese scientists in 1981 for application to high temperature environment in the region of about 2000 K. Mostly these materials are made from the mixture of ceramics which have high thermic resistance and metal constituents gives strength as well as toughness to the structure. Gradual variation of volume fraction of constituent material gives smooth variation and graded properties to the FG structures with respect to spatial coordinates [2-4]. Applications of functionally graded beam type structures are wide and include civil as well as aerospace structural components. Also these structures play a vital role when subjected to high temperature environment.

The researchers extensively studied in the area of static as well as dynamic characteristics of FG structural elements. A smart structure normally consists of a passive load bearing part such as beams, plates or shells which are the host structures along with an active part which may act as sensor or actuator [5-7].

Piezoelectric material is one of the most easily available smart materials which are utilized for active control of the smart structures. These Piezoelectric materials have both converse and direct effects and are used as actuators and sensors in recent few decades [8-10]. When the host structure is a functionally graded structure, the resulting structure is known conventionally as a smart FG structure. Free vibration of functionally graded beams was studied by Alshorbagy et.al [11] using finite element approach. Different material gradation, boundary conditions and slenderness ratios are taken for dynamic analysis of FG beams considering the constituent material property variation both in axial and transverse direction. Talha et.al [12] carried out free vibration and static analysis of FG plates using higher order shear deformation theory. The results were obtained for various thickness ratios of FG plates. Different volume fraction indexes are considered to analyze the bending and vibration behaviour of plates. Armin et al [13] analyzed the static as well as the dynamic behavior of FGPM beam. Effect of gradation of material, different loading pattern and boundary conditions on deflection, natural frequencies and dynamic behavior was investigated. Li et al. [14] analyzed the static and dynamic behaviour of functionally graded cantilever beam. Numerical results were presented in graphical form which indicates that the influence of power law and gradient index on the deformations and stresses are quite sensitive in the direction of material gradation. Vibration control analysis of FG plate combined with piezoelectric material as actuator was carried out [15] and the effect of volume fraction index and piezoelectric component on vibration control was obtained.

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2. Finite element modelling

A smart functionally graded beam combined with the piezoelectric material on its top surface is shown in Fig. 1. From the figure, L, b and h represent respectively the length, width and thickness of the smart FG beam. Whereas, L_p and h_p represent the length and thickness of the piezoelectric material respectively.

The properties of constituent material of the FG beam vary along the thickness direction and based on power law as explained below.

$$P_z = (P_U - P_L)\left(\frac{z}{h}+\frac{1}{2}\right)^k + P_L$$

where P_U and P_L denote material properties of FG structures respectively at top most and bottom most surfaces, h represents the total thickness of structure and k represents the non-negative power-law index.

Kinematics of axial deformations is explained using first order shear deformation theory. The displacement along axial as well as transverse direction of the FG beam can be expressed as:

$$u(x,z,t) = u_0(x,t) + z\,\theta_x\,(x,t),$$
$$w(x,z,t) = w_0(x,t) + z\,\theta_z\,(x,t) \qquad (1)$$

The Constitutive equations for FG beam and the piezoelectric layer are explained.

Expression for potential energy is given below.

$$P_e = \int (\{\mathcal{E}_b\}^T\{\sigma_b\} + \{\mathcal{E}_s\}^T\{\sigma_s\})dv - \int P\,w\,dA \qquad (2)$$

Expression for Kinetic Energy is given as

$$T_k^e = \frac{1}{2}\{d_t^e\}^T[Me]\{d_t^e\} \qquad (3)$$

Governing equations of motion for an element is obtained by using the Hamilton's Principle.

Then the elemental governing equations are assembled in the straight forward manner into the global space to obtain the global equations of motion as follows:

$$[M]\{\ddot{X}_t\} + [K_t]\{X_t\} + [K_{tr}]\{X_r\} = \{F\} + \{F_{tp}\}V \qquad (4)$$
$$[K_{tr}]^T\{X_t\} + [K_r]\{X_r\} = \{F_{rp}\}V \qquad (5)$$

For the analysis of smart functionally graded beam, a simulation model is made in ANSYS (APDL environment). Shell 281 (8 node) element is chosen for the discretization of the model, which is most suited to analyze thin to moderately thick structures with lay-up options and possesses the ability for modeling composite and functionally graded structures. The substrate FG beam considered here for analysis is composed of steel (metal) as bottom layer and alumina (ceramic) as upper layer. The material properties of the constituent materials of FG beam are mentioned in Table 1. The functionally graded beam of length 1 m, width 0.1 m and thickness 10 mm is considered

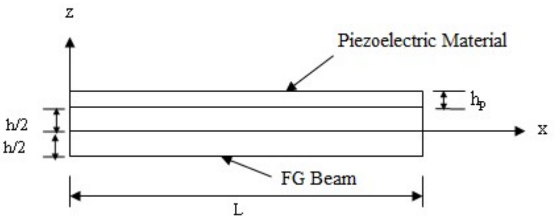


Fig. 1. Functionally Graded Beam combined with piezoelectric material patch.

for analysis. Thickness of piezoelectric layer is taken as 1.5 mm. Clamped-Free (CF), Clamped-Clamped (CC) and Simply Supported (SS) boundary conditions are considered for evaluating the numerical results of the smart functionally graded beam. A simulation model for FG beam in ANSYS software is shown in fig. 2. Variation of Young's modulus (E) and density (ρ) along the thickness of the FG beam is shown in fig. 3 for different power indexes.

The material properties considered here for the piezoelectric material (PZT-4) for evaluating the results are explained in Table 2.

Table 1. Properties of FG beam constituent materials [19]

Material	E(GPa)	ρ (Kg/m^3)	υ
Steel (SUS304)	201.04	8166	0.32
Alumina (Al_2O_3)	349.55	3800	0.26

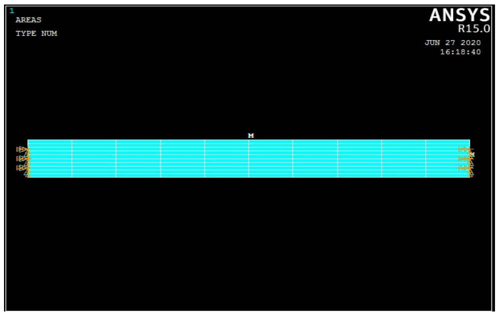


Fig. 2 Simulation model of FG beam using ANSYS software

Table-2 Material properties for PZT-4

C_{11}=139 GPa, C_{12}=77.8 GPa, C_{22}=139 GPa,
$C_{44} = C_{55}$ = 30.6 GPa, C_{66} = 30.6 GPa,
$e_{11} = e_{12}$=0 C/m^2, e_{13}= -5.2 C/m^2, e_{16} = 12.6

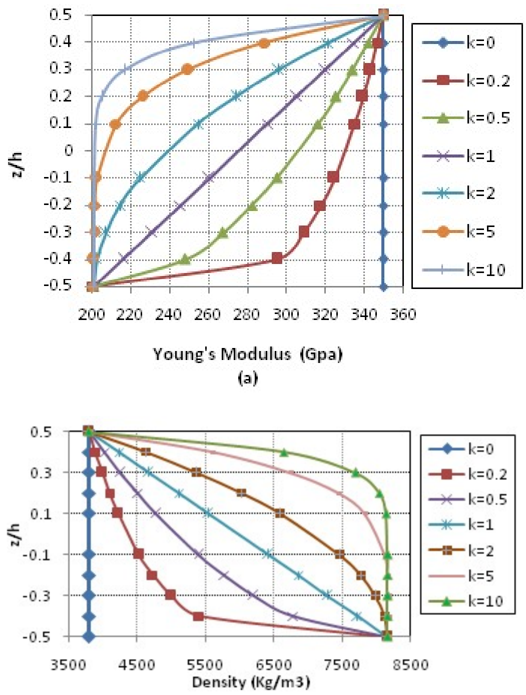


Fig. 3 Variation of (a) Young's modulus (b) density through beam thickness

3. Results and Discussions

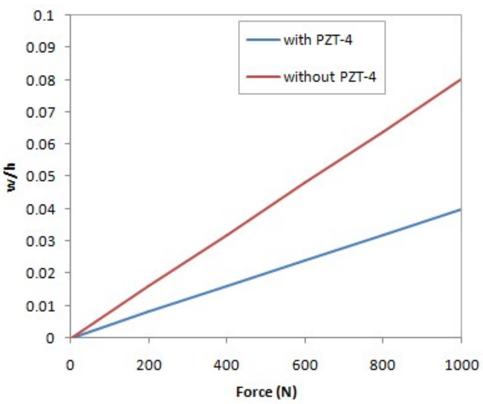
In this section, the static and dynamic response of the FG beam is studied and the numerical results are computed using the simulation model. First to validate, non-dimensional central deformations are obtained by applying suitable boundary conditions in ANSYS and the results are compared with published results [17]. The geometry and material properties of the problem considered here are taken as per the reference. This comparison is presented in Table 3 and shows the validation of the model.

For static analysis, the non-dimensional centre deflections for functionally graded (FG) beam without and with PZT-4 layer are calculated and presented in fig. 4 which shows the performance of piezoelectric layer (PZT-4) on control of deflection of FG beams studied.

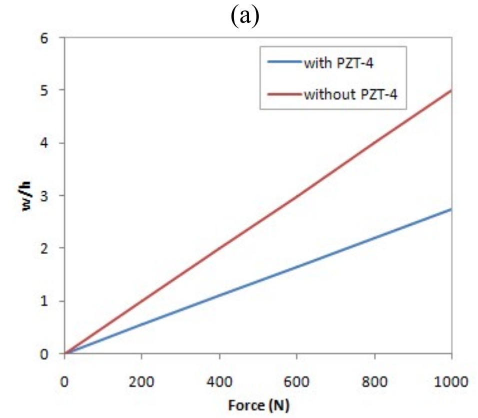
Table- 3. Center deflection (ώ) of functionally graded beam subjected to uniformly distributed loads. ($ώ = \frac{100 w E_m h^3}{q L^4}$)

BC	Source	k=0	k=1	k=2	k=5	k=10
SS	Ref [17]	3.1654	6.2594	8.0677	9.8281	10.9381
	Present	3.1562	6.2496	8.0233	9.7986	10.7287
CC	Ref [17]	0.8501	1.6179	2.1151	2.7700	3.1812
	Present	0.8432	1.6085	2.0595	2.6213	3.0156
CF	Ref [17]	28.7555	57.3323	73.6482	88.2044	97.4151
	Present	28.7243	57.2765	73.5896	87.8521	96.6541

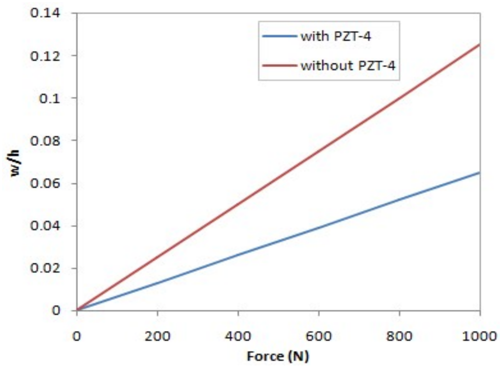
BC: Boundary Conditions, SS: Simply supported, CC: Clamped-Clamped, CF: Clamped-Free (L/h=5)



(a)



(b)



(c)

Fig. 4 Maximum non-dimensional centre deflection for (a) CC, (b) CF and (c) SS functionally graded beams without and with PZT-4 layer.

Next, free vibration analysis is carried out for the FG beam with and without combined PZT-4 layer. Numerical results are calculated for different boundary conditions as well as various power-law indexes. For validation, non-dimensional natural frequencies are calculated using finite element simulation model (ANSYS) and the results are compared with the available literature [11]. The comparison is presented in table 4. Table 5 and 6 explains first three non-dimensional frequencies for functionally graded (FG) and smart functionally graded beam (SFG).

Next, nonlinear transient vibration analyses for clamped-free FG beam is carried out considering time dependent load which indicates the dynamic characteristics of the structure. For this, a sinusoidal loading of 100 sin (π/4) t for a time period of 0.5 s with time step 0.001 s is considered. FG beam of length (L) 500mm and width (B) 100mm is taken for analysis. The thickness of PZT-4 layer and substrate FG beam are taken respectively as 5 mm and 20 mm. The FG beam considered here with 60% Length of PZT- layer on its top. The force is applied at the free end of the smart functionally graded beam. Figure 5 and 6 shows the transient vibration response of the FG beam without piezoelectric layer

Table- 4. Frequency (λ) for FG beams for various power index (k)

L/h	Sources	modes	k=0.2	k=1	k=5
20	Ref [11]	λ_1	4.2315	4.0359	3.9075
	Present (ANSYS)	λ_1	4.2281	4.0156	3.8966
50	Ref [11]	λ_1	4.2333	4.0377	3.9092
	Present (ANSYS)	λ_1	4.2296	3.9664	3.8897

Table- 5. Non dimensional frequency of FG beams (without PZT-4 layer)

BC	k=0.2			k=1			k=5		
	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
CF	4.08	25.01	37.32	4.18	26.50	40.8	4.33	27.04	42.64
CC	26.8	70.23	81.8	27.11	74.63	87.16	28.41	78.32	90.62
SS	17.0	44.62	77.6	21.23	46.37	83.04	22.30	47.59	89.37

Table 6. Non dimensional frequency for SFG beams (with PZT-4 layer)

BC	k=0.2			k=1			k=5		
	λ_1	$\lambda 2$	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
CF	5.12	25.78	37.67	4.59	26.92	41.33	4.77	28.32	42.86
CC	27.32	71.21	82.06	27.99	75.01	87.49	28.90	78.96	91.05
SS	17.51	45.29	77.84	22.37	46.89	84.12	22.76	48.16	90.23

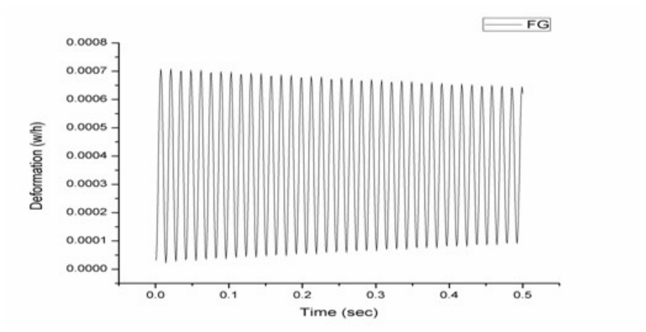


Fig. 5. Transient vibration response of Clamped-Free FG Beam without piezoelectric patch.

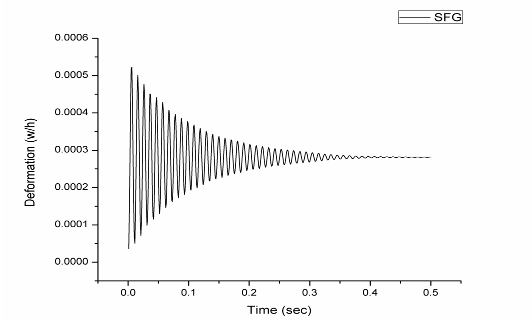


Fig. 6 Transient vibration response of Clamped-Free FG Beam with piezoelectric patch.

and with piezoelectric layer (SFG beam). This clearly shows that the piezoelectric (PZT-4) patch significantly controls the amplitude of vibrations thus enhance the damping behavior of FG beam.

Disclosures

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4. Conclusions

Finite element (FE) model is developed for the smart FG beams combined with piezoelectric material (PZT-4) patch. Hamilton's Principle is used along with FSDT for modeling the structure. Simulation model is also made using ANSYS software to analyze the static and dynamic behaviour of the functionally graded beams combined with piezoelectric material under mechanical loading conditions. Different boundary conditions and power-law indexes are considered for evaluations of numerical results. It is seen that the PZT-4 patches perform satisfactorily to control the deflections of smart FG plates. The numerical results also indicate that the non-dimensional frequencies increases as the power-law index increases for all boundary conditions and for a particular power-law index the natural frequency for SFG beams are higher as compared to FG beams. Also it is noticed that the piezoelectric material patch significantly controls the amplitude of vibrations thus enhance the damping behavior of FG beam.

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