

Acoustic Analysis of Partially Flexible Cavity with Opening

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Abstract

In this study, the SPL at the boundary and within the domain of a rectangular-shaped container with a flexible laminated composite panel with opening have been investigated. Finite Element Analysis (FEM) for the flexible panel has been done and coupled with the acoustic domain using the Boundary Element Method (BEM) through the mobility relation. A MATLAB program has been developed to find flexible panel behavior by FEM and BEM to calculate the sound pressure level for a cavity. Eight-noded isoparametric serendipity elements have been used to model the boundary. A pressure-velocity formulation has been adopted to model the acoustic domain with radiation impedance for window boundary. It has been shown that the presence of a thin flexible plate and opening, drastically changes the SPL pattern inside and at the boundary compared to a rigid cavity due to the relative movement of flexible panel and energy dissipation through the opening by radiation.

Keywords: Interior Acoustic, Flexible Cavity, Opening, Radiation, Finite Element Method, Boundary Element Method

1. Introduction

Acoustic analysis of the flexible laminated composite cavity is very important because of its wide range of applications in our society. Noise and vibration are significant considerations for the comfort of operators and passengers in surface vehicles, off-road vehicles, aircraft, and ships. These vehicles can be ideally represented by a box structure with a vibrating wall and opening, forming an acoustic cavity. Excessive sound pressure level (SPL) in vehicles causes serious ill effects on different physiological and mental conditions of the driver as well as passengers. Many times, the cavity is made up of flexible laminated composites material to make it lighter and economic. Hence proper care must be taken concerning their vibroacoustic characteristics and noise reduction ability.

Acoustic analysis of the composite cavity can provide fundamental insight into the physical understanding and effectively guide for acoustic design or noise control for complex sound fields. As far as studies of acoustic cavities are concerned, two classes of problems are involved, i.e., closed and open cavities, which are both of engineering interest and fundamental importance in acoustics. The acoustic behavior in an enclosure can be significantly influenced by the acoustic boundary of the enclosure. The position of an opening in a cavity and presence of a flexible wall in the boundary can alter the sound pressure level inside and the outside of the cavity, affecting the drivers, passengers as well as the neighborhood. Hence, there is a need for regulating the sound pressure level inside the cavity.

Previously, many research works had been carried out for simple rigid closed acoustic cavity problems [1-14]. Morse *et*

al. [1] obtained the theoretical sound pressure solution for a rigid box type cavity. But in real life, the acoustic enclosures are not perfectly rigid, rather they have a flexible wall with general boundary conditions like absorbing material and opening. Numerical methods, such as the boundary element method (BEM) and finite element method (FEM), have been used to obtain the acoustic quantities of interest. Solutions of coupled interior exterior acoustic problems have been obtained by Seybert *et al.* [15] using the BEM, which can handle continuity conditions at the interface between two domains. They considered only a rigid cavity in their analysis. Recently, Koch [16] used FEM for finding the acoustic resonances in rectangular open cavities by solving the Helmholtz wave equation. Niyogi *et al.* [17], had calculated acoustic pressure in the laminated cavity using coupled FEM-BEM. Nowadays many researchers developed analytical methods for calculating complex boundary problems to avoid large computational time and storage of coupled FEM-BEM. V. Jayachandran *et al.* [18], Kim *et al.* [19-20], Venkatesham *et al.* [21], Shi *et al.* [22], Jin *et al.* [23] and Wang *et al.* [24] had analyzed a box-type structure by both analytical and experimental method and validated the results with the numerical method using structural and acoustic mode shape function. They observed similar characteristics of SPL pattern and modal radiation efficiencies for a box structure with a flexible panel and window boundary condition.

In this study, the SPL at the boundary and within the domain of a rectangular-shaped container with a flexible laminated composite panel with opening have been investigated. Finite element (FEM) analysis for the flexible panel has been done

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and coupled with the acoustic domain using the boundary element method (BEM) through mobility relation. Eight-noded isoparametric serendipity elements have been used to model the boundary. A pressure-velocity formulation is adopted to model the acoustic domain with radiation impedance for window boundary. It has been shown that the presence of a thin flexible plate and opening drastically changes the SPL pattern inside and at the boundary compared to a rigid cavity due to the relative movement of flexible panel and energy dissipation through the opening by radiation.

2. Theoretical Formulation

The governing equation of a time-harmonic acoustic problem is given by the reduced wave (Helmholtz) equation,

$$\nabla^2 p + k^2 p = 0 \quad (1)$$

Here, p is the acoustic pressure and k is the wavenumber. Assuming the surface is discretized into M number of eight-noded surface elements, the discretized form of boundary integral equation (2) is given as

$$C(p)p(P) + \sum_{m=1}^M \sum_{l=1}^8 \int_{-1}^{+1} \int_{-1}^{+1} \frac{\partial p^*}{\partial n}(P, Q) N_l(\xi_1, \xi_2) p_l J(\xi_1, \xi_2) d\xi_1 d\xi_2 = \sum_{m=1}^M \sum_{l=1}^8 \int_{-1}^{+1} \int_{-1}^{+1} [-i\omega \rho p^*(P, Q)] N_l(\xi_1, \xi_2) v_l j(\xi_1, \xi_2) d\xi_1 d\xi_2 \quad (2)$$

Each node of the BEM mesh is used once as an observation point and a boundary element equation is generated. Upon assembly of these equations, the system equation for the acoustic enclosure is found in the form of a set of linear algebraic equations as

$$[H]\{p\} = [G]\{u_{an}\} \quad (3)$$

Here, $[H]_{nn \times nn}$ is a square matrix due to full assembly, while $[G]_{nn \times 8 \times ne}$ remains a rectangular one, being not fully assembled (nn = total number of node, ne = total number of element). Thus, the imminent objective is to get the unknown quantities to the left-hand side, inside vector $\{p\}$, and send the known quantities inside vector $\{u_{an}\}$.

2.1. Boundary Conditions for opening and Flexible panel

Physically there may be three types of boundary conditions for the acoustic cavity problem.

- Where the fluid particle velocity along the normal to the surface is specified. This is also known as the Neumann Boundary condition.
- An interacting zone where neither pressure nor the fluid particle velocities are known.
- Leakage zone, a window for example through which energy escapes into outer space.

The radiation impedance function at the leakage zone may be defined as follows:

$$Z_r = \frac{\text{Pressure at local node } i \text{ of element } k \times \text{Area of element } k}{\text{Velocity at local node } i \text{ of element } k} \quad (4)$$

According to Kinsler *et al.* [5] the radiation impedance $Z_r (= R_1 + i X_1)$, for the circular openings, has been calculated with respect to wave number (ka) and shown in Fig. 1.

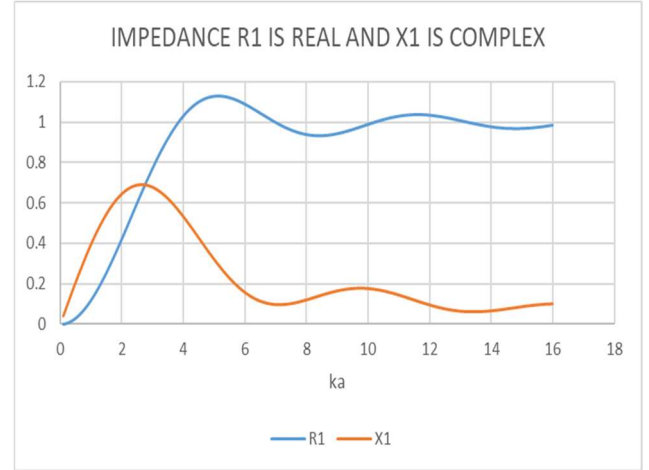


Fig.1 Radiation Impedance for the circular opening

For interacting boundary, modal data has been calculated from free vibration analysis. First order transverse shear deformation based on Yang-Norris-Stavsky (YNS) theory [25] has been used along with rotary inertia of the material. The stiffness matrix of the plate element is in the form

$$[K]_e = \int [B]^T [D] [B] dA \quad (5)$$

where, $\{\epsilon\} = [B] \{\delta_i\}$

$\{\epsilon\}$ being the strain vector, and $\{\delta_i\}$ the nodal displacement vector. $[B]$ is the strain displacement matrix and $[D]$ is the stiffness matrix for orthotropic materials.

The mass matrix of the plate element is given by

$$[M]_e = \int_{A_e} [N]^T [\rho] [N] dA \quad (6)$$

$[\rho]$ being the density matrix functions and $[N]$ is shape factor matrix functions. Eight-noded isoparametric plate elements with five degrees of freedom per node have been implemented in the present computations. Folded plate transformation [26-27] has been used. Finally, the governing equation can be written as

$$([K] - \omega_n^2 [M]) = 0 \quad (7)$$

From this equation, natural frequency and mode shape data have been calculated. This data is then used in the mobility matrix [17] required for the structural-acoustic coupling.

$$\{\dot{d}\} = [\Omega(\Phi)] \left[\text{diag} \left(\frac{2\Omega\omega_k \xi_k + i(\omega^2 - \Omega^2)}{(\omega_k^2 - \Omega^2)^2 + 4(\Omega\xi_k\omega_k)^2} \right) \right] \Phi^T F_0 e^{i\Omega t} \quad (8)$$

3. Numerical Results

In this investigation coupled FEM and BEM have been adopted to calculate the sound pressure levels at the boundary and the domain of the rectangular cavity. A MATLAB program has been developed to calculate the modal data in FEM which has been used in the Boundary Element Method to calculate acoustic pressure for a single domain with the flexible wall with the opening.

3.1. Validation Study

The formulations discussed in section 2 and the outcomes of the model has been validated with published results available in the literature. A cavity of dimensions $L_x=1.5$ m, $L_y=0.3$ m, and $L_z=0.4$ m has been adopted in the study of Venkatesham *et al.* [21]. Two cases have been considered. First, the cavity is assumed as fully rigid. The

sound pressure level in dB at domain point P ($0.4L_x$, $0.5L_y$, $0.5L_z$) is found out and compared with [21] in Fig. 2. In the second case, the cavity is bounded by a flexible aluminium panel of 5 mm thickness except at the left and right face which is considered as rigid. A constant oscillatory velocity 0.01 m/s is specified on the cavity wall $x=0\text{ m}$ as the acoustic source. The material properties of the flexible panel are: mass density $\rho = 2770\text{ kg/m}^3$, Poisson ratio $\nu = 0.29$, and Young's modulus $E = 71\text{ GPa}$. The speed of sound is $c_0 = 340\text{ m/s}$ and air density is $\rho_0 = 1.225\text{ kg/m}^3$. The damping ratio has been chosen as 0.01 . The sound pressure level at P has been calculated and compared with [21] in Fig. 3. From Fig. 2 it is clear that the rigid or uncoupled result matches very well except at the peaks and the coupled result with flexible plate matches fairly well with [21] at frequency range up to approximately 240 Hz . Venkatesham *et al.* [21] used analytical method for their study whereas here numerical analysis using coupled FEM and BEM has been used with coarse meshing ($10 \times 5 \times 5$) to optimize the time of computation and memory storage.

3.2. Case Study

Similar cavity model of dimension $1.8\text{ m} \times 0.6\text{ m} \times 0.6\text{ m}$ as shown in Fig. 4 has been taken to investigate the sound pressure level at the boundary ($1.8, 0.4, 0.4$) at the domain ($0.9, 0.3, 0.3$). The left and right-side faces have been taken as rigid and the other four faces have been considered as the flexible panel made with the graphite-epoxy composite material of lamination $(0/90/90/0)$ with 5 mm thickness. The material properties used are as given $E_1 = 130\text{ GPa}$, $E_2 = 9.5\text{ GPa}$, $G_{12} = G_{13} = 6.0\text{ GPa}$, $G_{23} = 3.0\text{ GPa}$, $\nu_{12} = 0.23$, $\rho = 1600\text{ Kg/m}^3$, speed of sound is $c_0 = 340\text{ m/s}$ and air density is $\rho_0 = 1.2\text{ kg/m}^3$ has been taken. A window of dimension 0.2 m square is placed centrally at the right face.

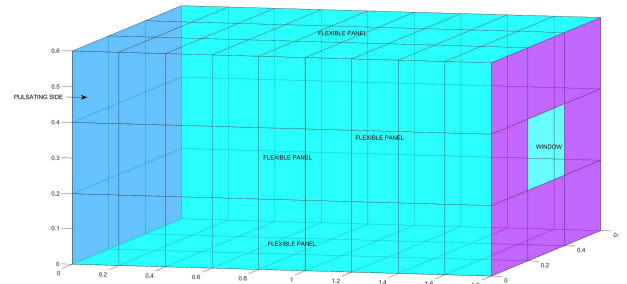


Fig.4. The model with a flexible panel and window boundary

Frequency and modal data for free vibrations have been calculated using FEM. The first thirty natural frequencies have been shown in Table 1. In the mobility relation first, forty natural frequencies have been used. The pulsating velocity at the left side wall has been taken as 0.001 m/s . In the case of window boundary, the radiation impedance has been calculated by numerical integration for a circular baffle plate according to Kinsler *et al.* [5] as shown in Fig. 1. Using the radiation impedance relation, the sound pressure level has been evaluated first and then the velocity terms have been extracted.

The sound pressure level in dB at Boundary ($1.8, 0.4, 0.4$) and the domain ($0.9, 0.3, 0.3$) has been shown in Fig. 5 and Fig. 6 considering the rigid cavity without window and compared with analytical value [18]. The variation of SPL due to the presence of the window also has been shown here and a shift in peaks has been observed in both diagrams. At the boundary, the peak value reduces to a large extent (Fig. 5).

Table 1. First Twenty Natural Frequency in Radians

Mode 1	345.45	Mode 11	654.77	Mode 21	1081.09
Mode 2	378.31	Mode 12	779.05	Mode 22	1109.36
Mode 3	454.21	Mode 13	781.57	Mode 23	1113.44
Mode 4	507.62	Mode 14	783.39	Mode 24	1124.63
Mode 5	508.06	Mode 15	787.53	Mode 25	1211.46
Mode 6	561.28	Mode 16	826.42	Mode 26	1279.01
Mode 7	562.03	Mode 17	938.08	Mode 27	1310.42
Mode 8	585.91	Mode 18	966.61	Mode 28	1319.85
Mode 9	640.58	Mode 19	976.28	Mode 29	1342.34
Mode 10	643.02	Mode 20	1034.40	Mode 30	1433.13

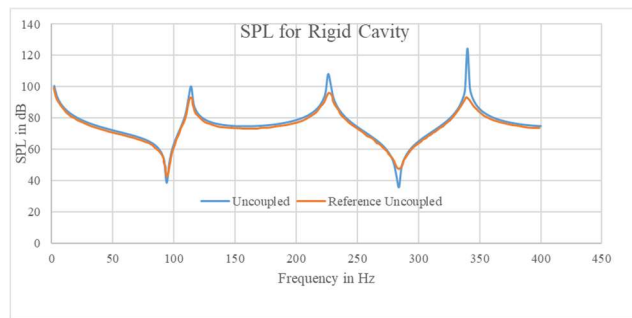


Fig.2. Comparison for a closed rigid cavity at Domain point ($0.6\text{ m}, 0.15\text{ m}, 0.2\text{ m}$)

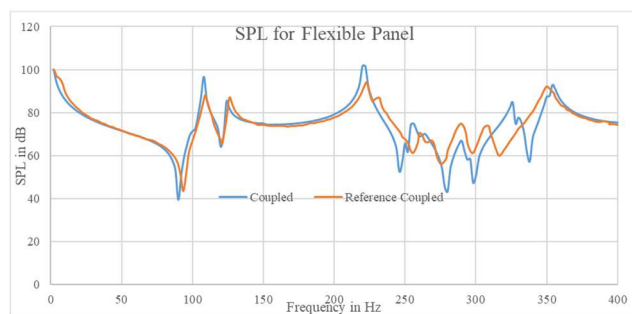


Fig.3. Comparison for the flexible cavity at domain point ($0.6\text{ m}, 0.15\text{ m}, 0.2\text{ m}$)

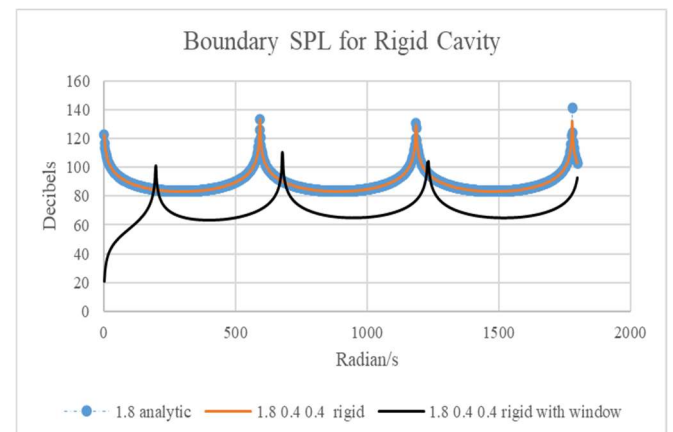


Fig 5. Boundary SPL for rigid cavity

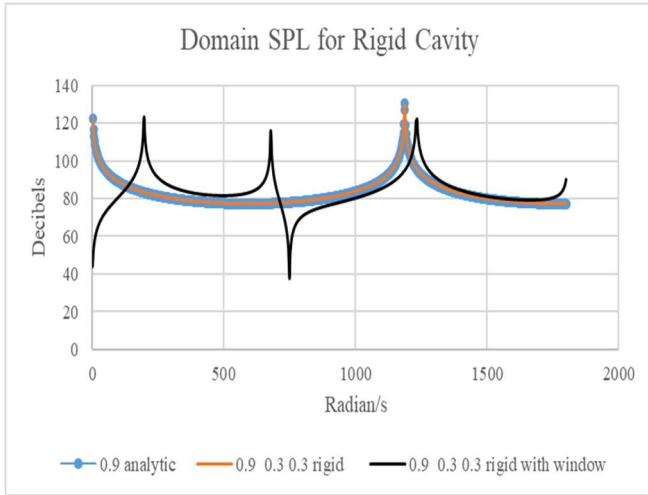


Fig 6. Domain SPL for rigid cavity

Fig. 7 and Fig. 8 represent the SPL at the boundary and the domain considering a flexible panel without a window. It is seen that the SPL pattern changes drastically due to the interaction between a flexible panel with an acoustic domain. The first peak shifts to the left at 542 radian/s in Fig. 7. Some intermediate kinks are visible both in Fig. 7 and Fig. 8. At 1310 radian/s, the SPL reduces to 14.7 dB at the boundary. A similar reduction in SPL is prominent in Fig. 8 also.

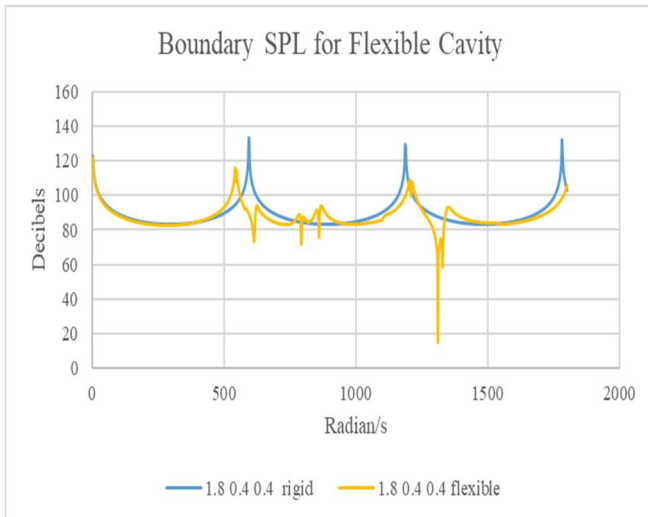


Fig.7. Boundary SPL for rigid and flexible cavity without window

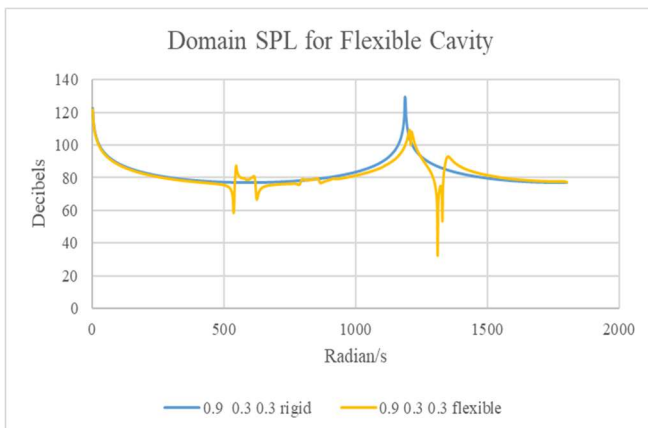


Fig.8. Domain SPL for rigid and flexible without window

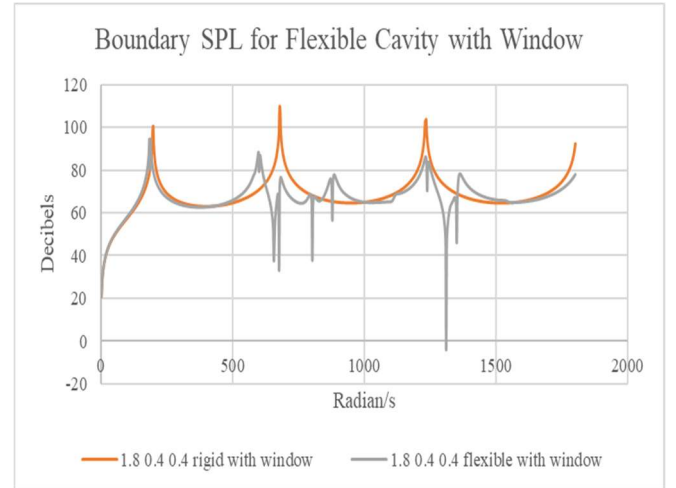


Fig.9. Boundary SPL for a rigid and flexible cavity with window

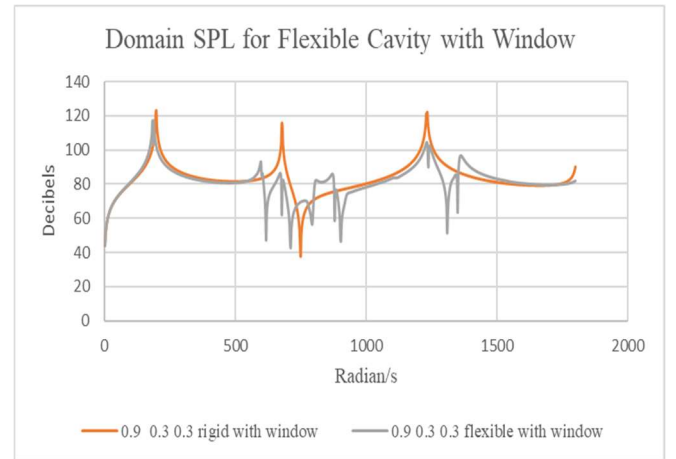
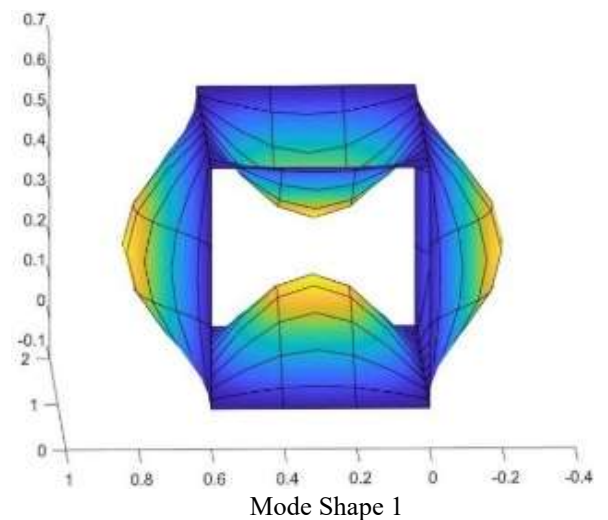


Fig.10. Domain SPL for rigid and flexible cavity with window

Fig. 9 and Fig. 10 represent the SPL at the boundary and the domain considering a flexible panel with a central window at the right face. Due to interaction with the flexible panel, there are intermediate kinks seen in Fig. 9 and 10. It is prominent that the presence of a window reduces the peak SPL values. The first three mode shapes for the flexible cavity have been plotted in Fig. 11.



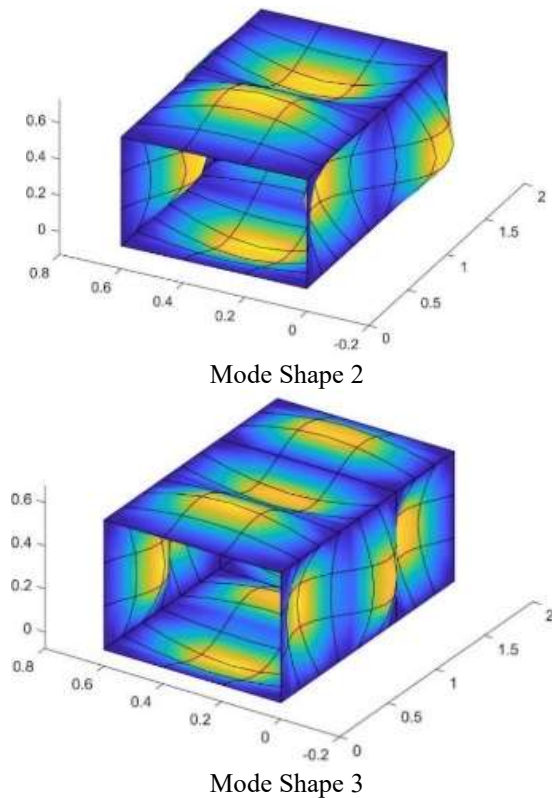


Fig.11. First three mode shapes of the flexible cavity

The boundary SPL has been plotted in four-dimension in figure 12a at 200 Hz. From this figure, it is seen that due to radiation through the window, the SPL at the opening is very low. Domain SPL contour at the XY plane at the mid-height has also been plotted in figure 12b. Alternate high and low-pressure zone are observed in the domain.

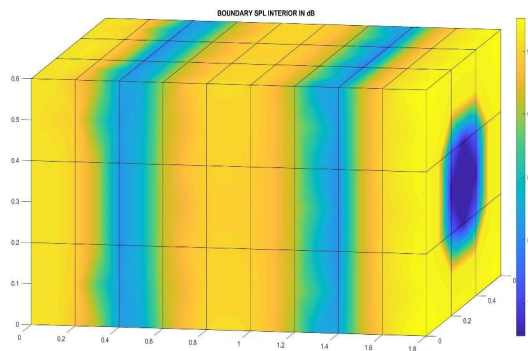


Fig.12a. Boundary SPL (in dB) plot at 200 Hz

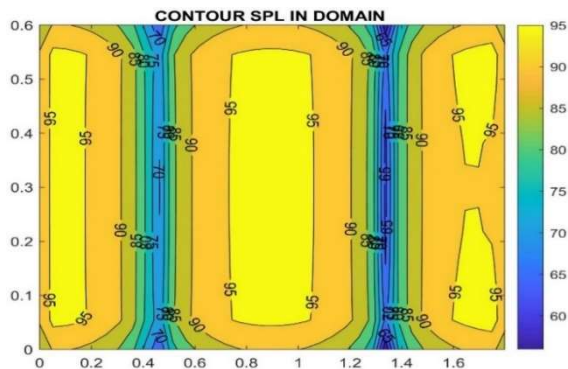


Fig.12b. Domain SPL (in dB) contour plot at Z=0.3m on XY plane for 200 Hz

4. Conclusion

The present paper offers a very general FEM-BEM analysis of interior acoustic problems within partly flexible laminated composite enclosures. Harmonic pulses generated by a surface piston have been applied to excite the domain. The YNS theory has been employed for the structural analysis so that moderately thick folded plates may be used in the analysis. It was shown how the presence of window shifts the peak SPL values both at the boundary and at the domain. Also, the peak value reduces to a large extent. The presence of a flexible panel also alters the SPL patterns at different points on the cavity.

Disclosures

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