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Static Analysis of thin Rectangular Plate Resting on Elastic Foundation Using Modified Vlasov Model

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Abstract

A four-nodded rectangular plate bending element based on Kirchhoff theory resting on elastic foundation using modified Vlasov model. For evaluate the two soil parameter for Vlasov foundation the modulus of elasticity and Poisson ratio of the soil is assumed constant from top surface to the top of bottom rigid base. All the deformation stiffness matrix of plate and subsoil are evaluated using finite element method. A Matlab code is developed and convergence study is carried out and then some realistic cases of plate under static load on elastic foundations are solved to determine the static response. The results, thus obtained, are compared, with the available results obtained by other researchers. It behaves extremely well for thin plates and convergence rate is high.

Keywords: Modulus of sub-grade reaction, elastic foundation, finite element and modified Vlasov.

1. Introduction

Analyses of plates on elastic foundations have wide applications in aerospace, civil and mechanical engineering. Developing more realistic foundation models and simplified methods to solve this complex soil-structure interaction problem are very important for safe and economical design. Majority of the problems cannot be solved by theoretical approach, led use the numerical techniques like finite element method.

In analysis of plates resting on the elastic foundation using the Winkler model, a single parameter model neglects the shear deformations between closely spaced elastic springs.

Widely used Winkler model main discrepancies are the discontinuity in the soil displacement between the soil under the structure and that outside the structure. To overcome the discrepancies of Winkler model two-parameter foundation models developed by Hetenyi [1], Filonenko Borodich [2], and Pasternak [3] provide for the displacement continuity of the soil medium by adding of a second spring which interacts with the first spring of the Winkler model. Vlasov and Leont'ev [4] developed a new concept on two-parameter model that has the advantage of determining soil parameters depending on soil material properties, modulus of elasticity and Poisson's ratio (E_s,ν_s) and the thickness of the subsoil (H) by introducing a third parameter, γ, to characterize the vertical deformation profile within the soil continuum [5]. Vallabhan and Das [6] determined the, γ parameter as a function of the characteristic of the structure and the foundation using an iterative procedure and named this model as modified Vlasov model .The γ parameters depend on the properties of the soil, the depth of the soil or rigid base and the structure as well as the type and magnitude of the loading. The analysis of plates resting on elastic foundations is the great interest of researcher and vast area of various research studies. Buczkowski and Torbacki [8], developed an 18- node isoparametric interface element of zero-thickness that account for shear deformation of the plate, details analyzed thick plates resting on two-parameter elastic foundation. Daloglu and Ozgan [9] developed an iterative method to determine the subsoil depth affected from the load on the plate resting on elastic foundation using stress distribution within the subsoil depending on the loading and dimension of the plate. Ozgan and Daloglu [10] investigate in details the effect of transverse shear strains on thin and the thick four-nodded and eight-nodded Mindlin plate resting on elastic foundation using modified Vlasov model. Turhan [14] studied in details thin plate resting on elastic foundation using modified Vlasov model using FEM. W.T. Straughan [11] studied in details thin plate resting on elastic foundation using modified Vlasov model using FDM.Mishra and Chakrabarti [12] investigated shear and contact effects on the behavior of rectangular plates resting on tensionless elastic foundation using finite element method. Celik and Saygun [13] developed a finite element formulation for plates on elastic foundation incorporating the shear deformations in the behaviour of the plate, and the effect of subsoil is considered as a combination of elastic bending and shear deformation of the soil.

In the present paper, a four-nodded rectangular plate bending element based on Kirchhoff theory resting on elastic

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foundation using modified Vlasov model analysis by finite element method. Convergence rate, accuracy and applicability of the present formulation for static analysis of plate on Vlasov foundation are demonstrated through number of numerical examples.

2. Methodology

2.1 Development of the theory of Vlasov model

The total potential energy in the soil-structure system may be

$$\Pi = \Pi_p + \Pi_s + V \quad (1)$$

Where,

Π_p = the strain energy stored in the plate.

Π_s = the strain energy stored in the soil, and

V = potential energy of the external loads.

In this model, the loads act on the plate domain in the lateral direction only and can consist of uniformly distributed loads on full plate or patch loads, line loads, concentrated loads, moments and any combination.

The strain energies of the plate and soil may be written as

$$\Pi_p = \frac{1}{2}\int_\Omega \left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x\,\partial y}\right)[D]\left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x\,\partial y}\right)^T dxdy \quad (2)$$

$$\Pi_s = \frac{1}{2}\int_0^H \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}\right) dxdydz \quad (3)$$

$$V = -\frac{1}{2}\int_\Omega q\, w dxdy \quad (4)$$

Where, w = the vertical displacement of the plate i.e. displacement in z - direction,

q = the applied distributed load,

σ, τ = normal and shear stress in the elastic foundation,

ε, γ = normal and shear strain in the elastic foundation,

H = depth of the subsoil,

Ω = domain of the plate,

[D] = Plate rigidity matrix.

$$[D] = \frac{Eh^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix} \quad (5)$$

E = the modulus of elasticity of the plate,

h = thickness of the plate, and

ν = Poisson's ratio of the plate.

Vlasov suggested that the functions of u, v and w can be expressed as u (x,y,z) = 0; v (x,y,z) = 0; w (x,y,z) = w (x,y) φ (z). Where φ (z) is the function describing the variation of the function w from the top of the soil to its bottom such that φ (0) = 1 and φ (H) = 0. It is further assumed that the thickness of the plate is small, so that the displacements of the surface of the soil are equal to the displacements of the middle surface of the plate, w(x,y,0) = w(x,y).

Using the strain-displacement equations of elasticity (Timeshenko) and the mentioned assumptions, total potential energy of the plate-soil system can be written as

$$\Pi = \frac{1}{2}\int_\Omega \left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x\,\partial y}\right)[D]\left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x\,\partial y}\right)^T dxdy + \frac{1}{2}\int_\Omega [kw^2 + 2t(\nabla w)^2]\, dxdy - \frac{1}{2}\int_\Omega q\, w dxdy \quad (6)$$

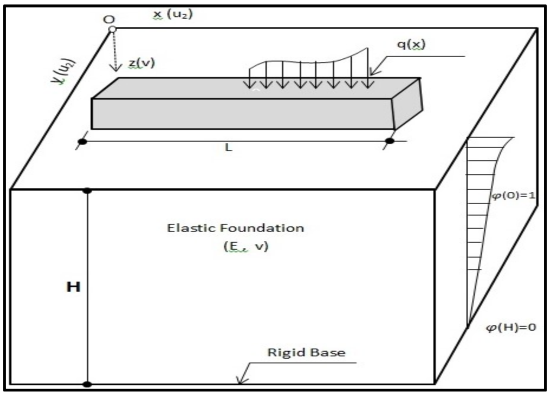

Figure 1 Finite plates resting on two parameters Vlasov foundation

Where k and 2t are the soil parameters defined as

$$k = \frac{E_0}{(1-\nu_0{}^2)}\int_0^H \left(\frac{d\varphi}{dz}\right)^2 dz \quad (7\text{ a})$$

$$2t = \frac{E_0}{2(1+\nu_0)}\int_0^H \varphi^2(z)\, dz \quad (8\text{ a})$$

$$E_0 = \frac{E_s}{(1-\nu_s{}^2)} \text{ and } \nu_0 = \frac{\nu_s}{(1-\nu_s)}$$

$$\varphi(z) = \frac{\sinh\gamma(1-\frac{z}{H})}{\sinh\gamma}$$

E_s, ν_s = Young's modulus of elasticity and Poisson's ratio of soil.

E_0, ν_0 = Effective modulus of elasticity and Poisson's ratio of soil

$$k = \frac{E_s(1-\nu_s)\gamma}{(1+\nu_s)(1-2\nu_s)H}\left(\frac{\sinh\gamma\cosh\gamma + \gamma}{2\sinh^2\gamma}\right) \quad (7\text{ b})$$

$$\&\ 2t = \frac{E_s H}{2\gamma(1+\nu_s)}\left(\frac{\sinh\gamma\cosh\gamma - \gamma}{2\sinh^2\gamma}\right) \quad (8\text{ b})$$

For thin layer variation of

$$\varphi(z) = \left(1 - \frac{z}{H}\right)$$

$$\therefore\ k = \frac{E_s(1-\nu_s)}{H(1+\nu_s)(1-2\nu_s)} \quad (7\text{ c})$$

$$and\ 2t = \frac{E_s H}{6(1+\nu_s)} \quad (8\text{ c})$$

γ parameter denotes the vertical deformation within subsoil. Using variational principles and minimizing the total potential energy of Equation (6) by taking variations in w and φ yields [14]

$$\delta\Pi = \int_\Omega (D\nabla^4 w - 2t\nabla^2 w + kw - q)\delta w dxdy + \int_0^H \left(-m\frac{\partial^2\varphi}{\partial z^2} + n\varphi\right)\delta\varphi dz + \text{boundary condition} = 0 \quad (9)$$

$$\text{Where } m = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{E}\, w^2 dxdy \text{ and}$$

$$n = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_s\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] dxdy$$

$$\overline{E} = \frac{E_s(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)} \text{ and } G_s = \frac{E_s}{2(1+\nu_s)}$$

Since the variations δw and δφ are not equal to zero, the terms in the parentheses and boundary conditions must be equal to zero.

Therefore the field equation in the domain, Ω, can be written as

$$D\nabla^4 w - 2t\nabla^2 w + kw = q \quad (10)$$

$$\nabla^2 = \text{Laplacian operator} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^4 = \text{Bi} - \text{harmonic operator} = \nabla^2\nabla^2 = \frac{\partial^4}{\partial x^4} + \frac{2\,\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4}$$

Outside the plate domain, the field equation is

$$-2t\nabla^2 w + kw = 0 \quad (11)$$

The second expression within the parentheses in Equation (9) is the field equation for the deformation pattern of the soil in the vertical direction. The equation is

$$-m\frac{\partial^2\varphi}{\partial z^2} + n\varphi = 0 \quad (12)$$

The Solution of Eq. (12) with the boundary conditions φ (0) = 1 and φ (H) = 0 yields

$$\varphi(z) = \frac{\sinh\gamma(1 - \frac{z}{H})}{\sinh\gamma} \quad (13)$$

2.2 Finite element formulation

It has four corner nodes and each node is associated with three degrees of freedom.

$$u(x,y,x) = -z\frac{\partial w_0}{\partial x};\ v(x,y,z) = -z\frac{\partial w_0}{\partial y}$$

and $w(x,y,z) = w_0(x,y)$.

The nodal displacement at i^{th} node

$$\{\delta_i\} = \left\{ w_i \quad \left(\frac{\partial w}{\partial x}\right)_i \quad \left(\frac{\partial w}{\partial y}\right)_i \right\}^T$$

The element displacement vector is defined as $\{d_e\} = \{d_1 d_2 d_3 d_4\}^T$For four nodded elements.

The element is based on thin plate theory. Hence, it is sufficient to prescribe variation of transverse displacement w on element region.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w_0}{\partial x^2} = -z\chi_x;\ \varepsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\varepsilon_{yy} = -z\frac{\partial^2 w_0}{\partial y^2} = -z\chi_y;\ \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\gamma_{xy} = -z\left(\frac{\partial^2 w_0}{\partial xdy} + \frac{\partial^2 w_0}{\partial xdy}\right) = -2z\frac{\partial^2 w_0}{\partial xdy}$$

$$\gamma_{xy} = -z\chi_{xy};\ \varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$$

$$\begin{Bmatrix}\chi_x\\ \chi_y\\ \chi_{xy}\end{Bmatrix} = \begin{bmatrix}\frac{\partial^2 w_0}{\partial x^2}\\ \frac{\partial^2 w_0}{\partial y^2}\\ 2\frac{\partial^2 w_0}{\partial xdy}\end{bmatrix} = \begin{bmatrix}\frac{\partial^2 N}{\partial x^2}\\ \frac{\partial^2 N}{\partial y^2}\\ 2\frac{\partial^2 N}{\partial xdy}\end{bmatrix}\{w_0\}$$

$$\therefore\ [B_{bi}] = \begin{bmatrix}\frac{\partial^2 N}{\partial x^2}\\ \frac{\partial^2 N}{\partial y^2}\\ 2\frac{\partial^2 N}{\partial xdy}\end{bmatrix};$$

$[B_b] = [B_{b1} B_{b2} \dots\dots\dots B_{b16}]$for four nodded elements.

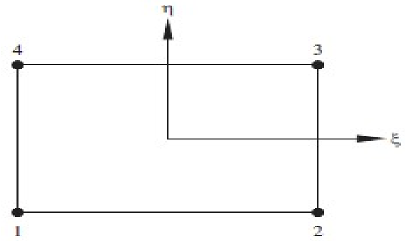

Figure 2 PBR4 plate elements

$$C_{11} = \frac{E}{(1-\nu^2)} and G = \frac{E}{2(1+\nu)};$$

$$C_{22} = C_{11}; C_{33} = C_{11}; C_{12} = \nu C_{11}; C_{13} = C_{12};$$
$$C_{21} = C_{12}; C_{23} = C_{12}; C_{31} = C_{12};$$
$$C_{32} = C_{12}; C_{44} = G;$$

$$\therefore\ [C_b] = \begin{bmatrix}C_{11} & C_{12} & 0\\ C_{21} & C_{22} & 0\\ 0 & 0 & C_{44}\end{bmatrix};\ D_b = \int_{-\frac{h}{2}}^{\frac{h}{2}} z[C_b]\,dz$$

$$[D_b] = \frac{Eh^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}$$

Hence bending stiffness matrix,

$$[K_b] = [B]^T[D_b][B];$$

$$[K_b] = \int_{-1}^{1}\int_{-1}^{1}[B_b]^T[D_b][B_b]|J|dsdt$$

Following usual steps, the bending is expressed as

$$[K_b] = \sum_{j=1}^{2}\sum_{i=1}^{2} W_i W_j |J| [B_b]^T[D_b][B_b]$$

From these equations, it is observed that full 2 × 2 Point Gauss-Legendre-type quardature is adopted for bending stiffness.

Considering a structural element which has a differential area 'dA' in contact with the foundation the lateral deflection of area 'dA' normal to the foundation is, w = $[N_f]\{d\}$

The strain energy U_r in a linear spring is given by eq. $= \frac{1}{2}kw^2$

$U_r = \frac{1}{2}\int kw^2\,dA;$ k is the soil first parameter.

$$w^2 = w^T w w^2 = \{d\}^T[N_f]^T[N_f]\{d\}$$

$$Strainenergy U_r = \frac{1}{2}\int k\{d\}^T[N_f]^T[N_f]\{d\}\,dA$$

$$U_r = \frac{1}{2}\{d\}^T[K_f]\{d\}$$

In which the foundation stiffness matrix for the element is,$[K_f] = \int k[N_f]^T[N_f]\,dA$

$$[K_f] = kab\int_{-1}^{1}\int_{-1}^{1}[N_f]^T[N_f]\,dsdt \quad (14)$$

$$[K_f] = k\int_{-1}^{1}\int_{-1}^{1}[N_f]^T[N_f]\,|J|dsdt \quad (15)$$

If the problem deals with a plate on elastic foundation, $[N_f]$ is identical to the shape function matrix [N] of the plate.

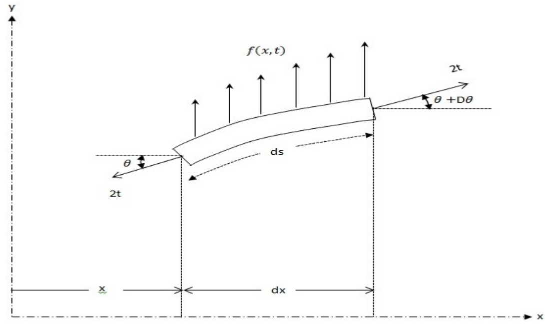

Figure 3 the length of a differential element 'dx'in the deformed position, 'ds', can be expressed as

$$ds = \sqrt{(dx)^2 + (dw)^2} = dx\sqrt{1 + \left(\frac{dw}{dx}\right)^2}$$

$$ds = dx\left[1 + \left(\frac{dw}{dx}\right)^2\right]^{\frac{1}{2}} = dx\left[1 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]$$

$$\therefore ds - dx = \frac{1}{2}\left(\frac{dw}{dx}\right)^2 dx$$

Strain energy stored by foundation parameter '2t' is given by

$$U = \frac{1}{2}\int_{-a}^{a} 2t\left(\frac{\partial w}{\partial x}\right)^2 dx$$

$$U = \frac{1}{2}\int_{-a}^{a} 2t\{d\}^T\left[N_f{'}\right]^T\left[N_f{'}\right]\{d\}\, dx$$

$$U = \frac{1}{2}\{d\}^T[K_{ex}]\{d\}$$

$$where[K_{ex}] = \int_{-a}^{a} 2t\left[N_f{'}\right]^T\left[N_f{'}\right]dx; S = \left[N_f{'}\right]$$

Similarly for y-direction

$$where\left[K_{ey}\right] = \int_{-b}^{b} 2t\left[N_f{'}\right]^T\left[N_f{'}\right]dy; R = \left[N_f{'}\right]$$

$$\therefore Stiffnessforshearparameter, 2tis$$

$$[K_e] = 2t\iint [S^T S + R^T R]dA$$

$$[K_e] = 2t\int_{-1}^{1}\int_{-1}^{1}\left(\frac{1}{a^2}\left[\frac{\partial N}{\partial s}\right]^T\left[\frac{\partial N}{\partial s}\right] + \frac{1}{b^2}\left[\frac{\partial N}{\partial t}\right]^T\left[\frac{\partial N}{\partial t}\right]\right)|J|dsdt$$

$$[K_e] = 2tab\int_{-1}^{1}\int_{-1}^{1}\left(\frac{1}{a^2}\left[\frac{\partial N}{\partial s}\right]^T\left[\frac{\partial N}{\partial s}\right] + \frac{1}{b^2}\left[\frac{\partial N}{\partial t}\right]^T\left[\frac{\partial N}{\partial t}\right]\right)dsdt$$

The element load vector for a plate due to transverse distributed load of q per unit area acting top of the plate

$\{f\} = q\int_{-1}^{1}\int_{-1}^{1}[N]^T\ |J|dsdt.$

$Forcevectorforapointload$

$\{f\} = [N_d]^T[F]$

The element load matrix for n nodded plate due to transverse distributed load of q per unit area acting top of the plate

$$\{q_i\} = q\sum_{i=1}^{2}\sum_{j=1}^{2} W_i W_j |J| N_i$$

Integration is carried out using 2 × 2 Gauss-Legendre integration.

$\{f\} = \{q_1 q_2 \dots\dots\dots.q_{12}\}^T$ (16)

For four nodded elements.

Similarly a typical sub-matrix for foundation parameter corresponding to i - th node is

$$\left[K_{fi}\right] = k\sum_{i=1}^{2}\sum_{j=1}^{2} W_i W_j |J| N_i{}^T N_j$$

$\left\{K_f\right\} = \left\{K_{f1}K_{f2} \dots\dots\dots.K_{f12}\right\}^T$ (17)

For four nodded elements.

And a typical sub-matrix for foundation second parameter corresponding to i-th node is

$$\left[K_{e_i}\right] = 2t\left(\sum_{i=1}^{2}\sum_{j=1}^{2} W_i W_j |J| \frac{dN}{dx}_i + \sum_{i=1}^{2}\sum_{j=1}^{2} W_i W_j |J| \frac{dN}{dy}_i\right)$$

Stiffness matrix of four nodded elements

$[K_e] = [K_{e1}K_{e2} \dots\dots\dots..K_{e16}]$ (18)

$[K] = [K_b] + \left[K_f\right] + [K_e]$ (19)

$Henceobtain[x] = [K]^{-1}\{f\}$(20)

Equivalent Boundary Forces at the Nodes

Outside the plate domain for a rectangular plate with dimensions of 2a in the x direction and 2b in the y direction., the equivalent boundary forces due to the infinite soil domain on the plate boundary (Turhan [14]). There are two type of stiffness to be considered. One is axial stiffness related to the displacement of the plate boundary in the transverse direction, and the other type is a rotational stiffness related to the rotation of the plate at its edge. Thus the effect at boundary other than corner of the soil region is modelled by adding equivalent to the stiffness term and for a corner region, as shown in Figure 4. Minimizing the energy with respect to displacement at that Point, the equivalent corner reaction is, R=1.5tw_c.

$Stiffnessforcornernode = \frac{3}{2}t$ (21)

Thus the effect at the corner node of the soil region is modelled by adding 3t/2 to the stiffness term representing the corner displacement. Vertical and rotational reaction forces for a continuous boundary can be obtained by

$Stiffness = a\sqrt{2kt};$ (22)

$RotationalStiffness = a\frac{1}{2}2t\sqrt{\frac{2t}{k}}$ (23)

Where 'a' is the tributary length of respectivenode.

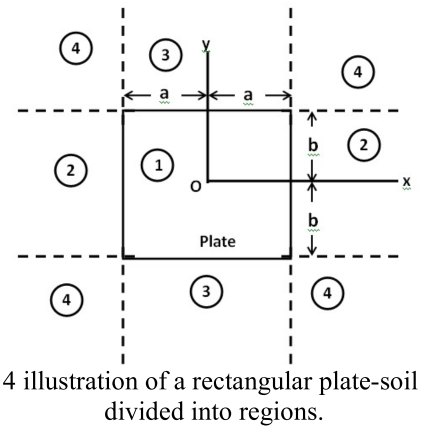

Figure 4 illustration of a rectangular plate-soil surface divided into regions.

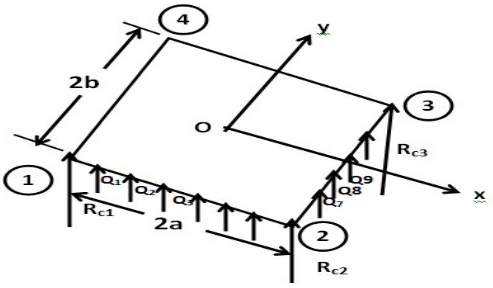

Figure 5 Forces on the boundary nodes.

$Q = a\sqrt{2t\,k}\,w$ and $R = 0.75 \times 2t \times w_c$

Where w and w_c is the deflection of the respective node.

2.3 The iterative procedure

In this model, the solution technique is an iterative process which is dependent upon the value of the γ parameter. Therefore, γ is initially set equal to one and φ is calculated. These values are used for the computation of the values of sub-grade reaction, k, and soil shear parameter, 2t, from Eqs. (7a) and (8a). With these values of k and 2t, the total coefficient matrix of the plate-soil system is constructed and the set of simultaneous equations is solved to find the displacements at discrete Points in the plate. Next the value of γ is calculated using the plate displacement values found in the previous step. A comparison between this calculated value of γ and the initial by assumed γ or previously calculated γ is then made. If the difference between the two successive γ values is within a prescribed tolerance, the analysis is stopped. Otherwise, iteration is performed and the process is repeated until convergence is obtained. Therefore, the mode shape parameter γ may be calculated at the end of any analysis step in terms of vertical displacements of the foundation-subsoil system. For more details of Modified Vlasov model is available in [4, 12, 14 and 15]

Now how obtained the γ parameter

$$\left(\frac{\gamma}{H}\right)^2 = \frac{1-2v}{2(1-v)}\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\nabla w)^2 dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(w)^2 dxdy} \qquad (24)$$

The integrals in equation (24) may be calculated in terms of the nodal displacements of the subsoil shear element within plate domain is

$$\int_{-b}^{b}\int_{-a}^{a}(\nabla w)^2 dxdy = \sum_{i=1}^{n}\frac{1}{2t}[w]^T[K_e]\,[w] \qquad (25)$$

and $$\int_{-b}^{b}\int_{-a}^{a}(w)^2 dxdy = \sum_{i=1}^{m}A[w_i]^2 \qquad (26)$$

Where n, m and A represent the element number, the node number and the tributary area of node i, respectively.

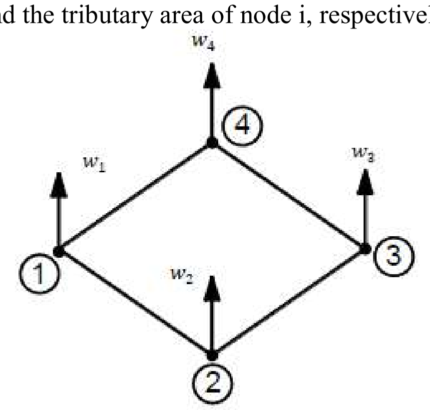

Figure 6 4 Nodded subsoil shear element

By putting 2t = 0 the foundation model reduced to the well known Winkler model (single parameter model) .By putting 2t = 0 and k = 0 the foundation model become applicable to plate without foundation.

For more details of Modified Vlasov model is available in [4, 9, 10 and 11]

3. Results and discussion

3.1 Convergence study and test the formulation

After the test of the present formulation and simultaneously a convergence study the mesh size of 14 ×14 is decided for a reasonable result.

3.2 Validation work

An example has been chosen from the study done by Ozgan K, Daloglu AT [11] for validation of the present formulation. The results are presented in Fig. 7 - 12 for uniformly distributed load and concentrated load case. As seen from the curves for displacements are in excellent agreement with the results given in [11] get fairly closer to each other for any loading cases as the depth of the subsoil increases in case of displacement and for bending moments get fairly closer to each other and in excellent agreement with the results given in [11] for any loading case.

The same example is considered by Buczkowski R, Torbacki W [8] but only subsoil depth H = 15.24 m different B/h ratio from 2 to 10^6 i.e. thin to thick plate limit for uniformly distributed loading case only the curves for displacements Fig. 10 are in excellent agreement with the results given in Ref [8].

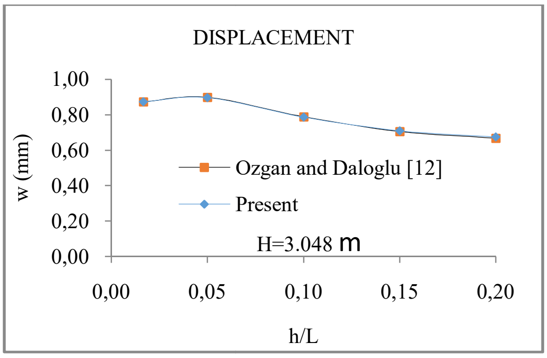

Figure 7 Comparison of deflection *w* of a free plate with uniformly distributed load

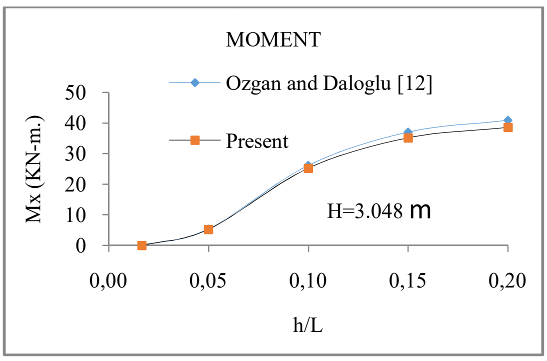

Figure 8 Comparison of bending moment M_x of a free plate with uniformly distributed load

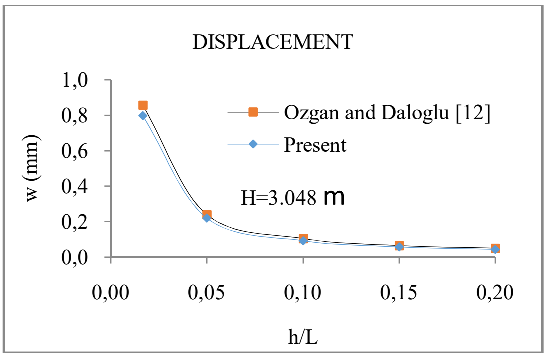

Figure 9 Comparison of deflection *w* of a free plate with concentrated load

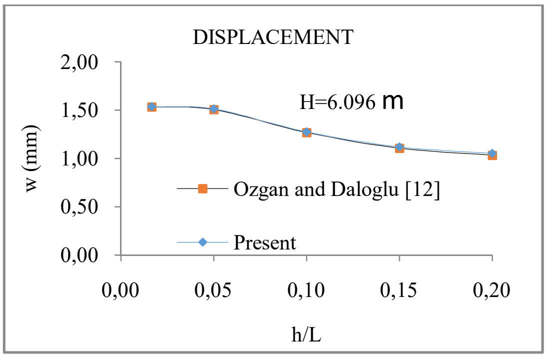

Figure 10 Comparison of deflection *w* of a free plate with uniformly distributed load

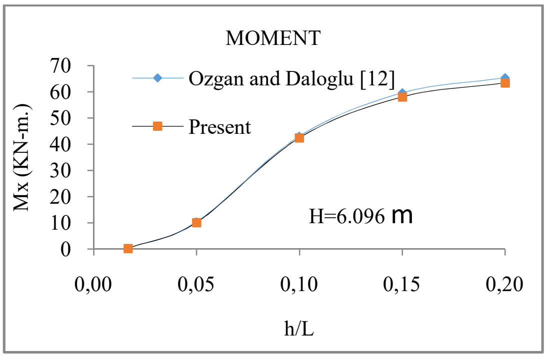

Figure 11 Comparison of bending moment $M_x$of a free plate with uniformly distributed load

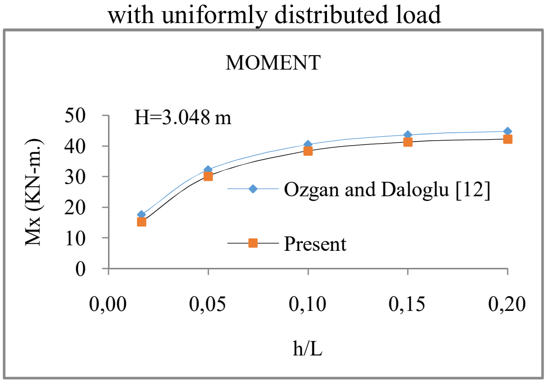

Figure 12 Comparison of bending moment $M_x$of a free plate with concentrated load.

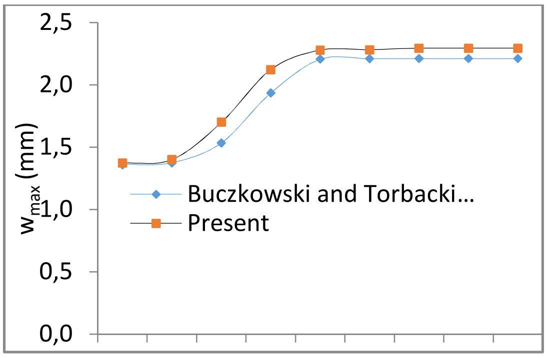

Figure 13 Comparison of deflection *w* of a free plate with uniformly distributed load.

3.3 The effectiveness of the formulation

The previous example is considered with free boundary conditions. The properties of the plate-soil system are the same as before. The ratio of the plate thickness to the length of the shorter side of the plate is taken as 0.001, 0.002, 0.01, and 0.02. The example is solved by thin plate theory to show the effect of several subsoil depth, and thickness of the plate on displacements and bending moments. The displacement of the plate always decreases with increasing h/L ratio for a constant value of H for any loading cases while the bending moment of the plate increases as presented in Figs. 14–20, the curves get fairly closer to each other as the value of h/L increases.

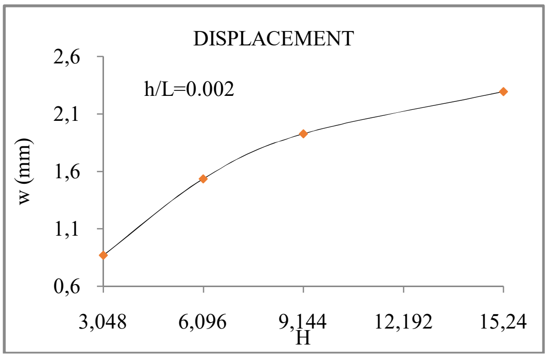

Figure 14 Changes of deflection, w of a free plate with uniformly distributed load

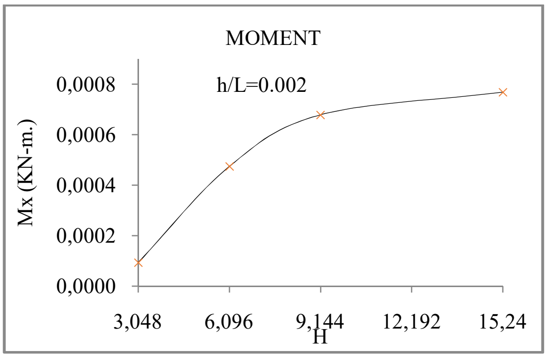

Figure 15 Changes of bending moment $M_x$of a free plate with uniformly distributed load

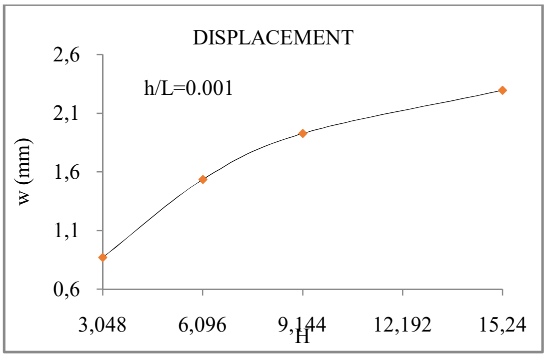

Figure 16 Changes of deflection, w of a free plate with uniformly distributed load

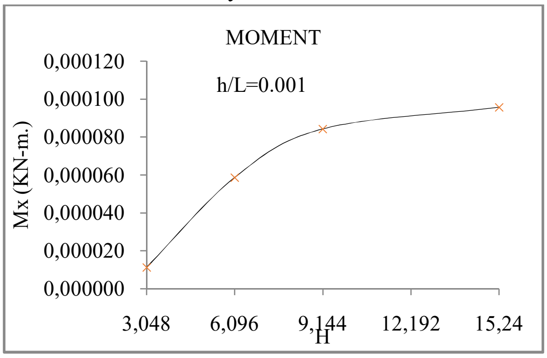

Figure 17 Changes of bending moment $M_x$of a free plate with uniformly distributed load

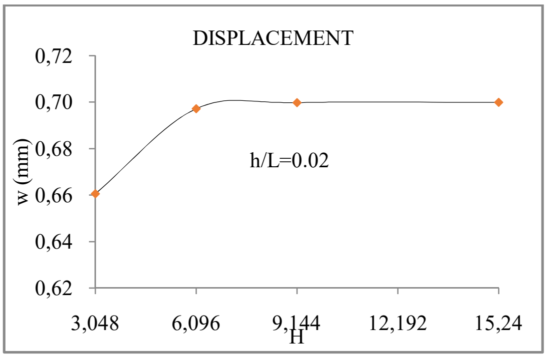

Figure 18 Changes of deflection, w of a free plate with concentrated load

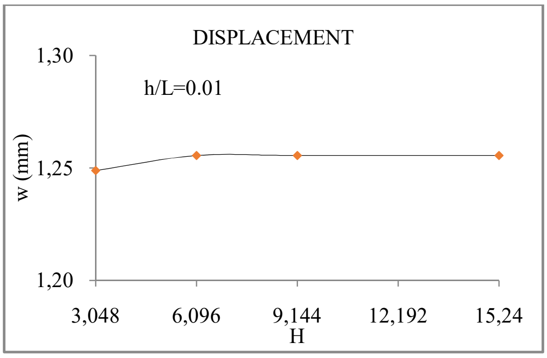

Figure 19 Changes of deflection, w of a free plate with concentrated load

5. Conclusions

The accuracy and the efficiency of theelementfor different subsoil depth and different load cases and then a parametric study are performed. The effect of subsoil depth and thickness of plate on the displacement is larger for concentrated load case than for distributed load case, and this effect increases as h/L ratio increases for any subsoil depth. The observations indicate that the effect of the thickness of plate on the behaviour of the plate bending is always smaller for free plates.

The presented examples show some of the advantages of the suggested approach for numerical solution of a plate on an elastic foundation. It gives opportunities for:

- Application of various loads at an arbitrary Point or a region on the plate;
- The approach can be performed on a thin plate effectively and efficiently;
- The plate and the soil medium stiffness can vary smoothly along the plate's length.

Disclosures

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