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Semi-Analytical Finite Element Method for the Analysis of Guided Wave Dispersion in the Pre-stressed Composite Plates

Akshay Prakash Kalgutkar [1, *], Sauvik Banerjee [2]

[1] Department of Civil Engineering, Research Scholar, Indian Institute of Technology Bombay, Mumbai, 400 076, India
[2] Department of Civil Engineering, Professor, Indian Institute of Technology Bombay, Mumbai, 400 076, India

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Abstract

Ultrasonic guided wave techniques offer an accurate and efficient procedure for damage monitoring in the structures. To develop reliable damage monitoring systems, it is essential to have a thorough understanding of the quantitative nature of ultrasonic guided waves that can be transmitted in composite laminates. In the present paper, the Semi-Analytical Finite Element (SAFE) method is employed due to its efficiency in the treatment of wave propagation problems involving complex materials and geometry. SAFE method is considered for the analysis of dispersion behaviour of guided waves in composite laminates by accounting the effect of in-plane load. The present study considers an infinite width plate such that the cross-section of the waveguide is modelled using 3 noded isoparametric 1-D elements representing the thickness of the plate. Equation of motion is formulated by using Hamilton's equation. Finally, various parametric studies are carried out. The study includes analysing the effect of wave velocity in the plate subjected to applied in-plane load, and understanding the effect of dispersion characteristics on the direction of propagation (slowness curve). The study shows that at the lower frequency thickness product, the group velocity of the wave increases with the increase in the applied tensile load and vice-versa.

Keywords: SAFE, Guided wave, Dispersion curve, slowness curve, Composite material, Prestressed plate

1. Introduction

Composite materials find their application in many engineering industries, especially in the field of mechanical, aerospace, marine and civil engineering due to their favourable engineering properties over conventional materials., However, these materials are highly susceptible to the hidden flaw that may arise due to mechanical damage and the service-related defects, which are caused by fatigue, impact of the foreign bodies or due to other unexpected events. These defects may grow to a critical size, which becomes unstable, causing the catastrophic failure of the entire structure. Therefore, it becomes essential to inspect the material during its manufacturing and overhaul period. Most of the inspection techniques are time-intensive and often requires the partial dismantling of the structure for evaluating the material condition in the critical structures, thereby increasing the cost of the inspection. But in the recent years, guided wave damage detection technique has become a more popular method in the non-destructive evaluation and the structural health monitoring (SHM) community due to its advantage of rapid evaluation over a large distance at the minimal expenses.

Due to anisotropic property and layered structure of composite materials, the guided wave propagation phenomenon becomes complex as the dispersive behaviour of the wave depends upon the direction of propagation. Therefore, the calculation of the guided wave dispersion curve and associated mode shapes becomes a great challenge. Different approaches have been used by many researchers to solve the dispersion equation. Over hundreds of years, the theoretical formulation has been carried to study the dispersion behaviour. Dispersion study for rods was carried by Pochhammer [1] and Chree [2]. Later, Rayleigh and lamb studied dispersive behaviour in the plates. Some of the other earlier works include the studies conducted by Mindlin [3], Onoe [4], Gazis [5], Victorov [6], Graff [7], Auld [8] and Achenbach [9]. Later, Mal [10] made a theoretical study through recurrence relation using the Thomson-Haskell approach.

Further to model and study the wave propagation behaviour in the complex geometry, prestress structure etc, the use of the analytical method becomes difficult. So apart from the analytical method, many researchers have worked on the numerical method to study the wave propagation behaviour. To study the dispersive behaviour in the honeycomb composite sandwich panel, waveguide finite element was employed by Baid et.al. [11]. Transfer matrix method (TMM) was used to solve the wave propagation problem in the layered medium [12], [13]. Later, Huber and Sause [14] used this method to study the dispersive behaviour in the composite laminated plate and developed a software named as Dispersion Calculator. In the case of the large frequency-thickness (f-d) products, this method reported numerical

*Corresponding author. Tel: +917795641186; E-mail address: akshay_kalgutkar@iitb.ac.in

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instability. To avoid this instability, reformulation of the TMM equations was proposed by Wang and Rokhlin [15]. The proposed method was coined as the Stiffness Matrix Method (SMM). The stable method known as Stiffness Transfer Matrix Method (STMM) was proposed by Kamal and Giurgiutiu [16], which aimed at overcoming the drawbacks of TMM and SMM. Later, Knopoff [17] was the first to develop the Global matrix method (GMM). Banerjee and Pol [18] have used this method to study the transient wave response in the composite honeycomb sandwich plate subjected to the surface excitation. Gavric [19] had made use of the Finite element method to study the wave propagation behaviour in rail. Later, Chitnis et.al. [20] have made use of a SAFE method to study the wave motion in fibre-reinforced composite laminates and sandwich plate. They conducted the study by taking into account the various displacement theories by assuming the variation in axial and transverse displacement along the plate thickness.

These days, many researchers are focusing towards the use of the guided wave to study the propagation behaviour of the wave in the stress environment. To carry out this work, thorough knowledge of the dispersion characteristics of the guided wave is essential. Developing the methodologies to study the guided wave propagation in the pre-stressed structural element is the subject of the current research in the SHM field. In the case of guided wave propagation in the prestressing or deformed solid, Biot [21] was first to develop a mathematical formulation to solve such a problem. With the application of the Murnaghan theory to the isotropic structure, Hughes and Kelly [22] found out the velocities of the elastic wave in the stressed solids or acoustoelasticity. Similarly, Toupin [23] performed a study on the arbitrary symmetry structure. Qu and Liu [24] conducted the wave propagation study in the stressed layered media by generating the dispersion curve. Further, Lematre [25] developed the theory to understand the wave propagation in the stressed piezoelectric plate structure. Mott [26] found out that by adjusting or altering the material's elastic properties, the guided wave propagation in a cylinder can be revised to incorporate the axial load effect. Later to understand the effect of stress on the phase velocity of guided modes in a strand, Dubuc et.al. [27] used an acoustoelastic theory for a rod.

Apart from the theoretical studies, few researchers have used the SAFE method for analysing the dispersive characteristics in the pre-stressed structures. Work of Chen [28] and Loveday [29,30] shows the influence of the axial load on the guided wave propagation in the arbitrary cross-section structures by employing a SAFE method. Marzani [31] studied the wave propagation in the viscoelastic axisymmetric waveguides. Later, Mazzotti [32] and Yang et.al. [33] carried out the work to understand the behaviour of the pre-stressed viscoelastic waveguides under the action of the axial loading. Treyssede [34,35] examined the wave behaviour in the helical beam and seven-wire strands which is the more complex structures. Ma et.al. [36] studied the behaviour of the feature guided wave in the axial stressed stringer using the acoustoelastic theory combined with the SAFE method. Yan and Yuan [37] used the SAFE method to study the characteristics of the shear wave mode by considering the phase at which wave converts from the evanescent wave modes to the propagating wave modes in an isotropic plate. Later, Hanfei et.al. [38] studied the behaviour of the wave in the delaminated plate by the application of horizontal shear wave, as it is less dispersive when compared to lamb wave for the large frequency range.

From the above literature, it can be seen that only a few researchers have used the horizontal shear wave (SH wave) to assess the condition of the structure. Due to its less dispersive behaviour when compared to other wave modes, it has lots of potential in the present time.

The present study deals with the SAFE method by considering the in-plane and transverse displacement in the thickness direction to investigate the phenomenon of wave propagation in pre-stressed laminated composite plates. The present work aims at studying the dispersive behaviour of the propagating wave in the pre-stressed uni-directional laminated composite plate. The effect of propagating (steering) angle on the wave velocity is also discussed in this work.

2. Mathematical framework

The SAFE method is used to depict the wave behaviour in the plate subjected to initial in-plane load. In the SAFE method, the Finite Element Method (FEM) is combined with the analytical expression. FEM is used to describe the displacement field in the waveguide cross-section, while the displacement field, along the propagation direction is expressed analytically by a complex exponential function $e^{(ikx)}$ where k is the wavenumber of the Lamb wave which can be seen from the Fig. 1(a).

The obtained m^{th} eigenvalue k_m of the eigensystem denotes the wavenumber of the m^{th} resonance mode and further, the dispersion curve is depicted. In the present case, the cross-section of the infinite width plate is sub discretised using 3-noded 1-D bar element as shown in Fig. 1 (b) and wave that propagates in the longitudinal x-direction is described by an orthogonal function $e^{(ikx)}$.

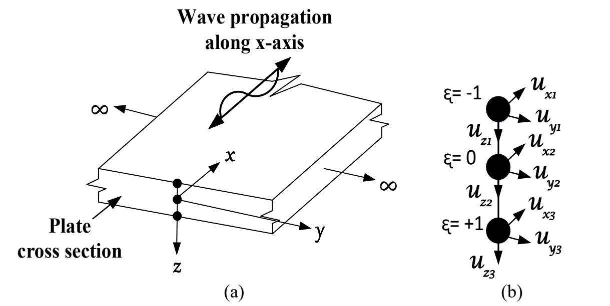


Fig. 1. General Wave-guide cross-section (shown with 1D element along the thickness direction), (b) Degrees of freedom for a 1D three nodes element.

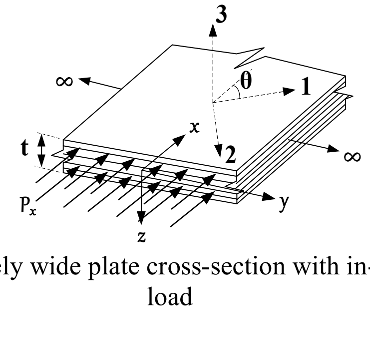


Fig. 2. Infinitely wide plate cross-section with in-plane edge load

To model the wave propagation in the elastic waveguides, Finite element formulation was developed by Gavric [19] and applied this method to understand the wave propagation characteristics in the rails. Similar works have been carried out by the other researchers. Damljanovic´ and Weaver [39] used a transformation to achieve symmetric eigenvalue problem comparable to that presented by Gavric´ [38] as shown in Eq. (1).

$$\left[K_1+kK_2+k^2K_3-\omega^2M\right]\hat{Q}=0 \tag{1}$$

Where, $$K_1=T^T\left(\bigcup_{e=1}^{n_{el}}\int_{-1}^{1}B_1^TC_{(e)}B_1d\xi\right)T \tag{2}$$

$$-iK_2=T^T\left(\bigcup_{e=1}^{n_{el}}\int_{-1}^{1}\left(B_1^TC_{(e)}B_2-B_2^TC_{(e)}B_1\right)d\xi\right)T \tag{3}$$

$$K_3=T^T\left(\bigcup_{e=1}^{n_{el}}\int_{-1}^{1}B_2^TC_{(e)}B_2d\xi\right)T \tag{4}$$

$$M=T^T\left(\bigcup_{e=1}^{n_{el}}\int_{-1}^{1}N^T\rho_{(e)}Nd\xi\right)T \tag{5}$$

Here T is the Hermitian matrix such that $T.T^T = I$.

The kinetic energy is used to derive the mass matrix (M) whereas, strain energy is used to derive the stiffness matrix, which depends on the wavenumber (k).

The application of an initial load leads to an additional term in the strain energy which in turn add a term to the stiffness matrix.

2.1. Plate with initial in-plane load

The effect of axial load for the analysis of the rods and the rails has been carried out by using the SAFE method [29]. Therefore, the same concept can be applied to study the dispersion behaviour of the plate subjected to initial in-plane load.

Potential energy per unit volume can be written as the sum of the strain energy associated with small amplitude elastic wave (in terms of σ) and the work performed by the initial stress (in terms of $\sigma^{(0)}$) which is shown as

$$U=\frac{1}{2}\sigma^T.\varepsilon^{(e)}+\sigma^{(0)^T}.\varepsilon^{(e)} \tag{6}$$

As the in-plane load is applied only in the x-direction, the initial stress developed due to this load $\sigma_{xx}^{(0)}$ is retained. Therefore, the strain energy due to the initial in-plane load is written as

$$\sigma_{xx}^{(0)}.\varepsilon_{xx}=\sigma_{xx}^{(0)}.\frac{\partial u_x}{\partial x}+\sigma_{xx}^{(0)}.\frac{1}{2}\left[\left(\frac{\partial u_x}{\partial x}\right)^2+\left(\frac{\partial u_y}{\partial x}\right)^2+\left(\frac{\partial u_z}{\partial x}\right)^2\right] \tag{7}$$

The term $\sigma_{xx}^{(0)}.\frac{\partial u_x}{\partial x}$ vanishes, when the Hamiltonian variation of the waveguide is taken as in [40] or when the Lagrange equation is applied as in [41]. Then by substituting the displacement interpolation function, an additional strain energy term is yield. Hence, Eq. (7) can be expressed as

$$K_4=\frac{\sigma_{xx}^{(0)}}{\rho}T^T\left(\bigcup_{e=1}^{n_{el}}\int_{-1}^{1}N^T\rho_{(e)}Nd\xi\right)T=\frac{\sigma_{xx}^{(0)}}{\rho}M \tag{8}$$

Adding Eq. (8) in the governing equation, the Eq. (1) is rewritten as

$$\left[K_1+kK_2+k^2(K_3+K_4)-\omega^2M\right]\hat{Q}=0 \tag{9}$$

Eq. (9) has two variables, the wavenumber (k) and circular frequency (ω). Therefore, this equation can be solved by specifying the frequency ω and then solving for the wavenumber k. Hence, Eq. (9) is changed to a linear form as

$$\left[A-k.B\right]\begin{bmatrix}\hat{Q}\\ k\hat{Q}\end{bmatrix}=0 \tag{10}$$

The Eq. (10) is the eigenvalue problem with the size of 2M. The obtained eigenvalue k_m is the m[th] wavenumber and the corresponding eigenvector represent the modal displacement. The phase velocity of the m[th] mode of the frequency ω is given by $C_{pm}=\omega/k_m$. Based on the characteristics of the displacement profile obtained in terms of eigenvector, the wave is categories as Lamb wave, Shear wave or Mixed wave as proposed by Bartoli [39].

The group velocity is given by the equation $C_g=d\omega/dk$. But in the SAFE method, the group velocity of the propagating mode can be directly derived from the Eigen solution of the phase velocity, which was proposed by Finnveden [42] and Bartoli [43].

$$C_g=\frac{\partial\omega}{\partial k}=\frac{\hat{Q}_L^T\left(K_2+2k\left(K_3+K_4\right)\right)\hat{Q}_R}{2\omega\hat{Q}_L^TM\hat{Q}_R} \tag{11}$$

where, $\hat{Q}_L$ and $\hat{Q}_R$ are the left and right eigenvectors.

3. Results and Discussions

In this section, the dispersion equation which is formulated by using the SAFE method is solved numerically by considering the multi-layered laminated plate subjected to initial in-plane load. To perform the numerical solution, the SAFE formulation is implemented in MATLAB 2018a program. The resulting dispersion curve is presented in terms of the phase velocity and the group velocity. The results are validated with the results obtained from the Global matrix method by Banerjee and Pol [18] and the Dispersion Calculator software developed by Huber [14], which is shown in Fig. 3. It is necessary to know that while using the Finite Element (FE) approach, the convergence of the solution is important to get a more accurate solution. This convergence issue is handled by increasing the number of elements. The study showed that by discretising the plate's cross-section by the 12 elements, the result gets converge. Therefore, in further studies, the results are obtained by discretising the plate's cross-section by 12 elements.

Throughout the paper, the dispersive behaviour of the transversely isotropic composite plate which is subjected to initial in-plane load is studied by considering the properties shown in Table 1 unless otherwise specified.

Table-1 Engineering properties of the composite plate

Engineering Parameter	
E_1	144.646 GPa
E_2, E_3	9.636 GPa
$\upsilon_{12}, \upsilon_{13}$	0.29927
υ_{23}	0.284742
G_{12}, G_{13}	6 GPa
G_{23}	4 GPa
ρ	1550 kg/m^3

3.1. Validation study

To validate the computational strategies used in this paper, unidirectional composite plate with the plies oriented at 0° and the properties mentioned in Table. 1 is considered. The phase velocity obtained from the SAFE is compared with numerical matrix-based solution [18] and the Dispersion Calculator software program [14]. The result is plotted in Fig. 3.

It can be seen from Fig. 3 that phase velocity obtained from the SAFE is identical to the one obtained from the Dispersion Calculator irrespective of the mode. However, the velocity of the S0 mode, which is obtained from the SAFE method is consistent with the analytical calculation up to the frequency thickness product (f.d) of 1.3 MHz-mm and further at the higher frequency thickness product, velocity deviates by the small amount from the analytically obtained velocity. In the case of the phase velocity of the A0 mode, the SAFE result begins to diverge by a small amount at lower frequency thickness product itself, which is shown in Fig. 3. Thus, we can conclude that the SAFE results are in excellent agreement with the Dispersion Calculator results. While the velocities are well agreed with analytical results up to a f.d of 1.3 MHz-mm and a further increase in the f.d, the result begins to diverge.

3.2. Case Study

In this section, the dispersive behaviour of the guided wave in the laminated composite plate which is subjected to the initial in-plane load is studied. The main objective of the study is to understand the variation in the wave velocity due

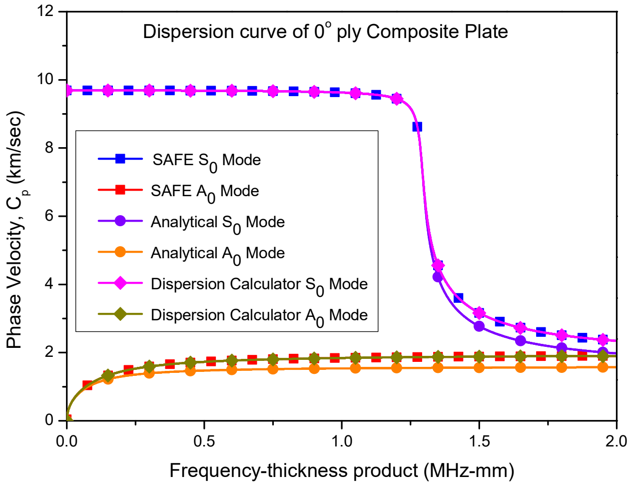


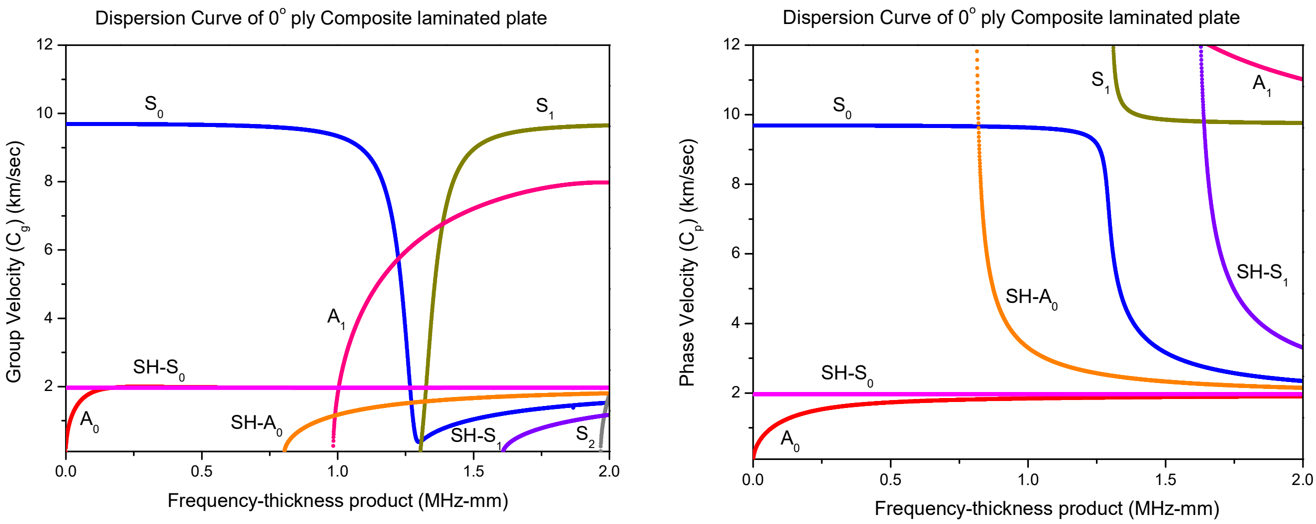
Fig. 3. Dispersion curve using SAFE for 1-mm thin Composite plate.

to the effect of the initial load that is applied on the plate and also to study the dispersive nature of the lamb wave and shear wave propagating in different steering angles. In the present study, the load is presented in terms of the strain experienced by the plate. In this paper, the analysis of the unidirectional composite laminated plate whose plies are orientated at 0° is carried out. It is important to note that only the propagating wave modes are considered in the study and evanescent wave mode (pure imaginary wavenumber) or attenuated wave (complex wavenumber) is not involved in the study.

Case Study 1: Dispersion Behaviour of wave in the laminated composite plate.

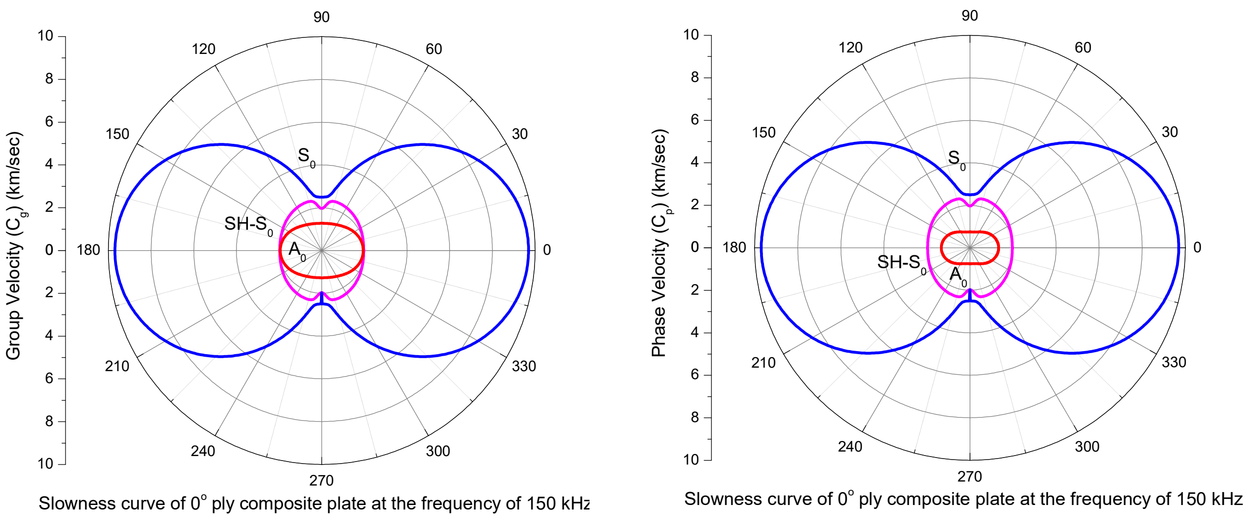
Before analysing the wave behaviour in the preloaded plate, it is necessary to study the behaviour of the propagating wave in the unloaded plate. The study is carried out by plotting the dispersion curve of the wave propagating along the longitudinal direction of the fiber (i.e. 0°). Fig. 4 shows the dispersion curve representing the variation of the group velocity and the phase velocity for the frequency thickness product up to 2 MHz-mm. In the present study, apart from analysing the dispersion characteristics of the Lamb wave, the horizontal shear wave behaviour is also considered. Understanding the behaviour of the horizontal shear wave is very much essential in the present scenario. From Fig. 4, it can be seen that the group velocity and the phase velocity of the fundamental symmetric mode of the horizontal shear wave are non-dispersive for a large range of f.d. Due to this unique characteristic, many researchers are carrying out different studies using the S-H wave.

The present study shows that the S0 mode of Lamb wave is initially less dispersive upto the frequency thickness product of 1 MHz-mm and then the wave becomes highly dispersive i.e. there is a sharp change in the velocity for the frequency thickness product in-between 1 MHz-mm to 1.25 MHz-mm. In the case of A0 mode, the wave is highly dispersive at the lower frequency thickness product upto 0.2 MHz-mm and with the further increase in the f.d, the A0 mode wave propagates with steady velocity. As seen from the discussion above, the SH-S0 mode is non-dispersive i.e. the wave propagates with a uniform velocity of about 2 km/sec irrespective of the f.d. On the other hand, the presence of the S1 mode at the frequency thickness product of 1.3 MHz-mm propagates with the rapidly increasing group velocity upto a frequency thickness product of 1.5 MHz-mm and further, the wave propagates with the constant velocity. However, the phase velocity of this mode decreases speedily with the increase in the frequency thickness product upto 1.5 MHz-mm and then it propagates with constant speed. Similar behaviour is observed in the case of the A1, SH-S1, SH-A0 and S2 wave mode, which are developed in the composite plate. Also, it is observed that at the higher frequency thickness product, lots of modes are generated which creates a difficulty in the identification of the developed modes. Due to this problem, it is irrelevant to study the characteristics of the wave at a higher frequency thickness product. But most of these generated wave modes propagate with the velocity of Rayleigh wave at higher f.d product which is seen by the converges of the velocities in the plot.



(a) (b)

Fig. 4. Dispersion curve showing the variation of the (a) group velocity (b) phase velocity



(a) (b)

Fig. 5. Slowness curve representing the (a) group velocity (b) phase velocity at a frequency of 150 kHz

Fig. 5 shows the slowness curve of the waves in the 0° ply oriented composite plate studied at the operating frequency of 150 kHz. It is most important to plot the slowness curve, especially in the case of anisotropic media due to the dependency of the elastic properties with respect to the direction. Hence, this will significantly affect the speed of the wave propagating in different directions.

From Fig. 5, it can be observed that the S0 mode wave propagates with the maximum velocity in the direction, along which the plies are aligned. On the other hand, the wave propagates with the minimum velocity in the direction normal to the alignment of the fiber. Variation in the wave velocity of the SH-S0 mode is such that, the wave travels with the maximum velocity in the direction closer to the transverse direction of the fibres and then the velocity suddenly decreases when the wave propagates along the transverse direction of the fibres. It is also noted that the SH-S0 mode is less dispersive than S0 and A0 mode irrespective of the direction of the wave propagation (steering angle) when it is operated at a frequency of 150 kHz. Also, the A0 mode wave travels with the maximum velocity along the longitudinal direction of the fibre. This shows that the wave velocity of the different modes depends mainly upon the elastic properties of the plies and its orientation.

Finally, it can be summarised that S0 wave mode propagates with a greater velocity along the longitudinal direction of plies and with a lower velocity along the transverse direction of the plies. On the other hand, the opposite behaviour is observed in the case of the SH-S0 mode as the wave propagates with a minimum velocity along the longitudinal direction of the fibre. In the case of A0 mode, the wave propagates with maximum velocity along the longitudinal direction of the plies of the surface lamina.

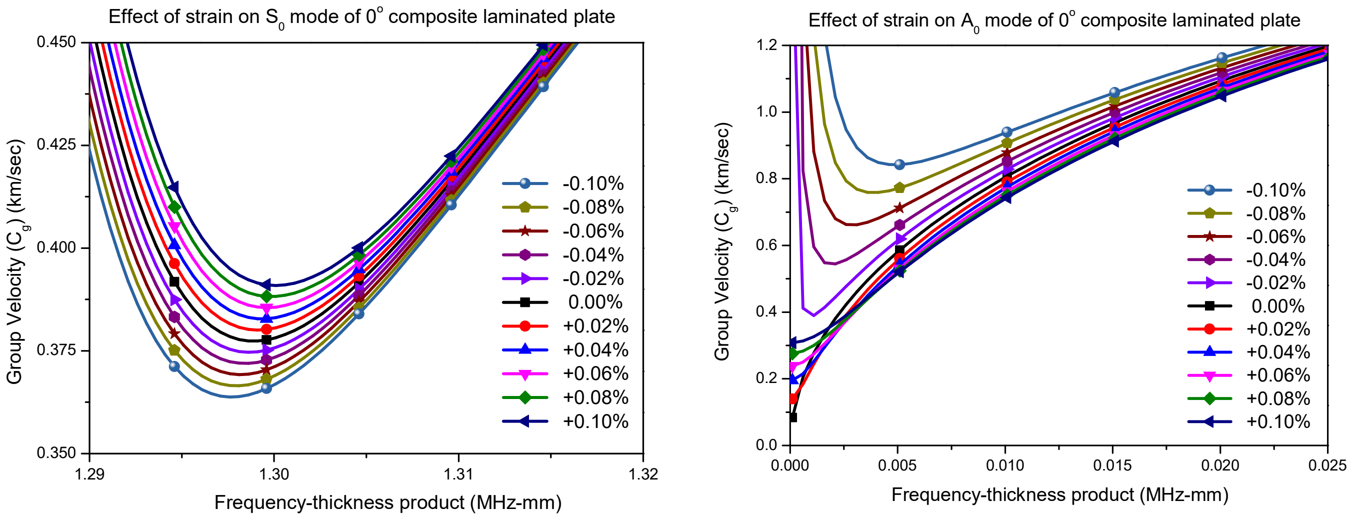


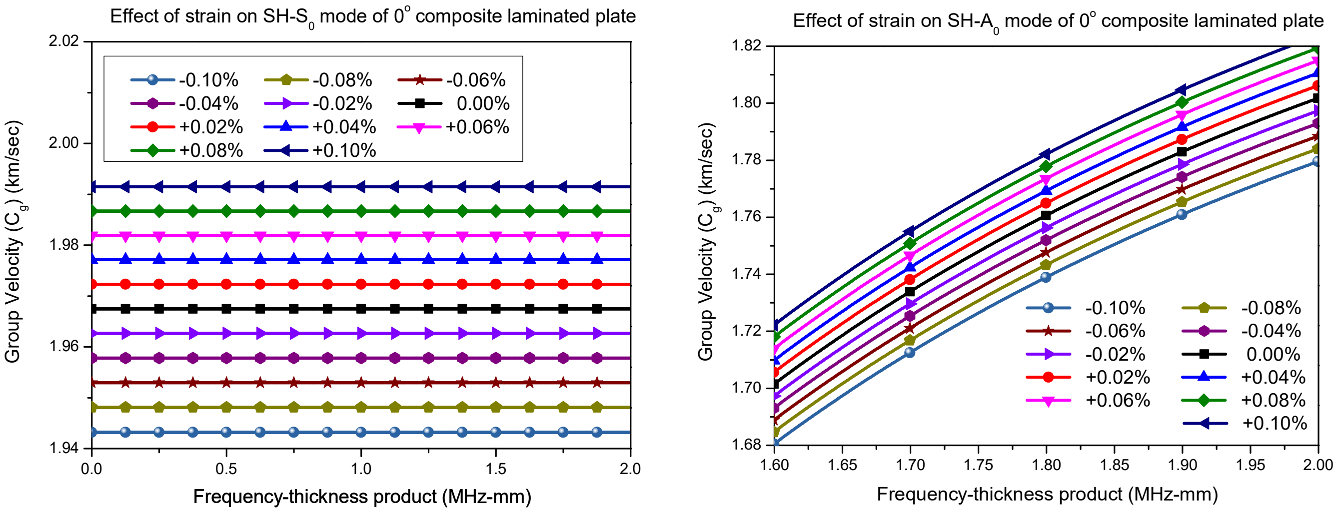
Fig. 6. Effect of in-plane load on the group velocity of the (a) S0 mode wave (b) A0 mode wave in 0º composite laminate

Case Study 2: Effect of the in-plane load on the velocity of different wave modes in the laminated composite plate.

In this section, the study describing the effect of the in-plane load on the velocity of the different fundamental wave modes are carried out. The study is conducted by representing the load in term of percentage of strain developed on the plate. In this analysis, positive strain indicates the use of the tensile load on the plate while the negative strain depicts the plate subjected to the compressive load.

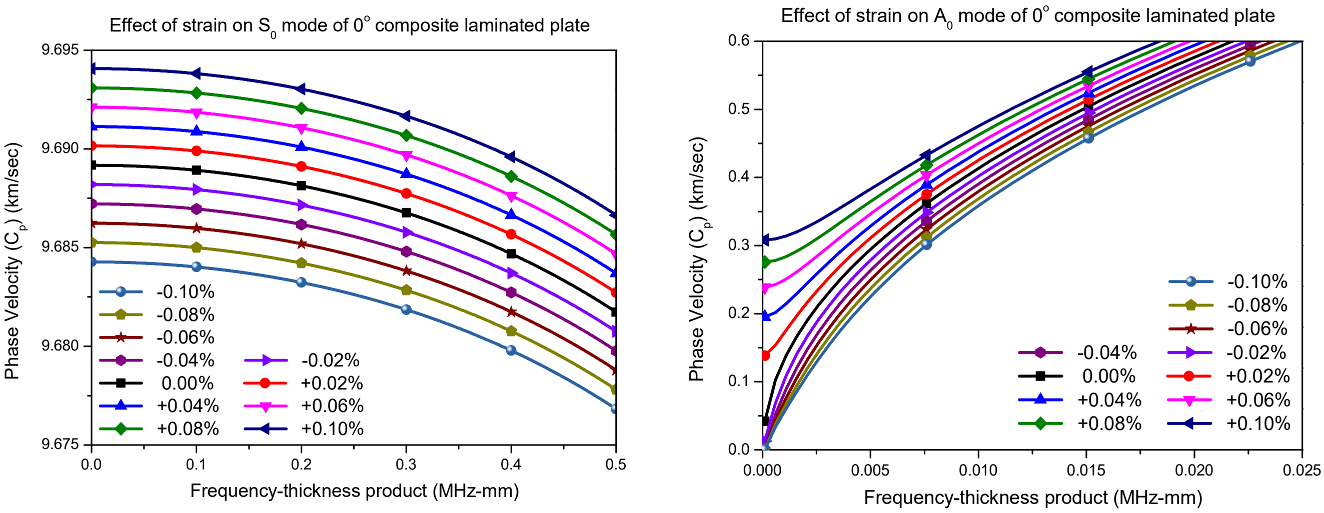
Fig. 6 shows the influence of the group velocity of the Lamb mode due to the application of preload in the composite plate. The study is carried out by considering a portion of frequency thickness product from Fig. 4, where the wave velocity changes rapidly. It is observed from Fig. 6 (a) that the velocity of the S0 mode wave in the plate subjected to the initial load has a significant effect in the region where the wave velocity reduces quickly. Further, loading has a minor effect when the velocity begins to increase. Fig. 6 (b) shows the effect of the load on the group velocity of the A0 mode at the lower frequency thickness product. The study indicates that for a frequency thickness product up to 2-3 kHz-mm, the velocity of the A0 mode increases by a small amount with the increase in the tensile load applied to the plate. Further, at the higher frequency thickness product, it is seen that the velocity of the wave in a preloaded plate subjected to larger tensile load is less than the velocity of the wave in a plate subjected to a smaller tensile load. Also, in the plate subjected to compressive load, the velocity of the A0 mode wave reduces rapidly at the lower value of frequency thickness product and further, the velocity increases with the increase in the frequency thickness product. This propagating behaviour of a wave mode is significantly seen in the plate subjected to a larger compressive load. It is also pointed out that larger the compressive strain, greater will be group velocity of A0 mode wave. It is interesting to note that the velocity of the wave in a plate subjected to compressive load is greater than the velocity of the wave in the plate subjected to tensile load irrespective of the frequency. On the other hand, the SH-S0 wave propagates with a steady velocity even after the influence of the preload on the plate which is seen from Fig. 7 (a). However, the velocity by which the wave propagates is significantly affected due to the application of the loading. Similar behaviour is observed in the SH-A0 mode wave with the small variation in the velocity due to the influence of loading as seen in Fig. 7 (b). Unlike the A0 Lamb mode wave, the velocity of the SH mode wave in the plate subjected to tensile will be higher than the velocity of the wave in the plate subjected to compression.

Moreover, the study is conducted to examine the variation of phase velocity in the preloaded plate. Fig. 8 (a) shows that there is a constant change in the S0 mode phase velocity due to the load applied in the plate. Fig. 8 (b) point out that at the lower frequency thickness product of 0- 3 kHz-mm, A0 mode phase velocity increases with the increase in the tensile load and this variation is quite large from the velocity in the unloaded plate. On the other hand, the variation in the velocity of the wave in the plate due to the compressive load is quite small at very low frequency thickness product. Further, with the increase in frequency thickness product, the deviation in the A0 mode phase velocity is quite constant with the application of loading.



(a) (b)

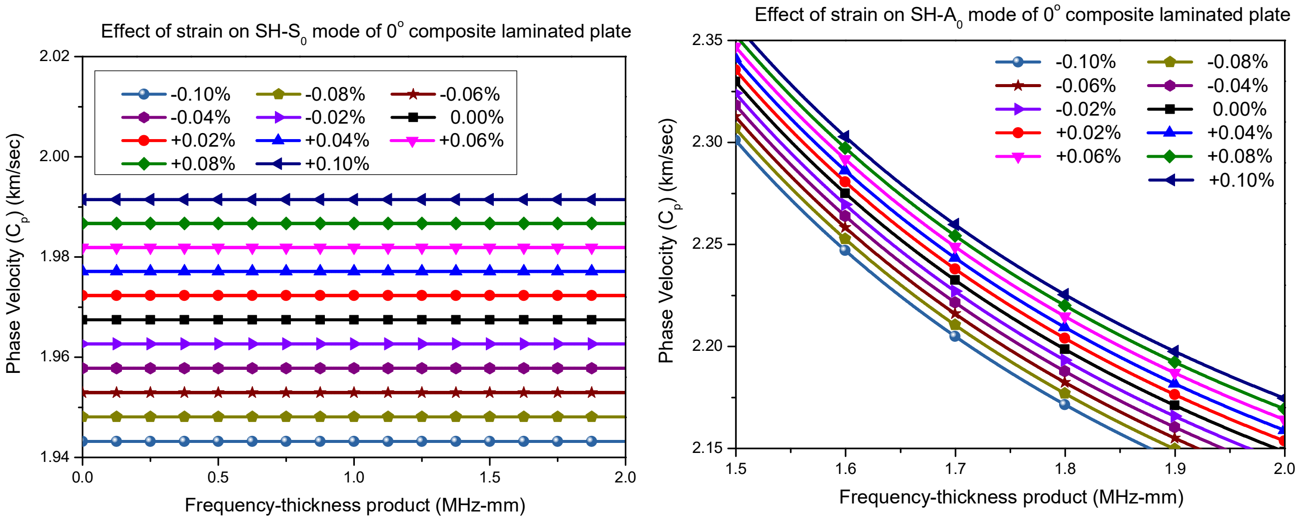
Fig. 7. Effect of in-plane load on the group velocity of the (a) SH-S0 mode wave (b) SH-A0 mode wave in 0º composite laminate



(a) (b)

Fig. 8. Effect of the in-plane load on the phase velocity of the (a) S0 mode wave (b) A0 mode wave in 0º composite laminate

The influence of loading on the phase velocity of the SH-S0 mode wave is shown in Fig. 9 (a). The study shows that no dispersive characteristic is observed in the case of SH-S0 mode as the phase velocity of the wave remains constant irrespective of the frequency thickness product which is alike the group velocity trend as observed in Fig. 7 (a). The investigation on the influence of the load on the phase velocity of the SH-A0 mode is carried out and the results are shown in Fig. 9 (b). The study shows a constant deviation of the velocity for a large range of frequency thickness product which is similar to the behaviour of the group velocity of the SH-A0 mode in the prestressed plate.



(a) (b)

Fig. 9. Effect of the in-plane load on the phase velocity of the (a) SH-S0 mode wave (b) SH-A0 mode wave in 0º composite laminate

4. Conclusions

The analysis to understand the dispersive behaviour of the wave propagating in the plate with infinite width using semi-analytical finite element method was extended to examine the effect of axial load.

From the present study, it can be concluded that the S0 Lamb wave mode is initially less dispersive for the lower frequency thickness product up to 1 MHz-mm and then the wave becomes highly dispersive for the frequency thickness product in-between 1 MHz-mm to 1.25 MHz-mm. Likewise, the SH-S0 mode wave propagates with a constant velocity irrespective of the exciting frequency of the wave. It is noted that at the higher frequency thickness product, A0 mode wave propagates with the velocity of the SH-S0 mode.

From the slowness curve, it is shown that S0 and SH-S0 mode wave velocity depends upon the equivalent elastic properties of the laminates, whereas the velocity of the A0 mode depends on the elastic properties of the surface lamina.

The introduction of the in-plane load in the composite plate has a significant influence on the dispersive characteristics of the A0 mode wave at the lower frequency thickness product. On the other hand, the velocity of the wave increases when the plate is subjected to tensile load and it gets reduced by the application of compressive load except for the group velocity of the A0 mode. Also, the application of the initial load on the composite plate does not influence the dispersive nature of the SH-S0 mode and it propagates with constant velocity irrespective of the excitation frequency.

Therefore, the obtained results can be used to design the testing conditions for guided waves-based inspection of loaded and unloaded composite plate.

Disclosures

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