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A mixed-finite element approach for full-field residual stress estimation

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Abstract

In this paper, a full-field residual stress identification technique has been presented via a mixed finite element approach from limited measured detail. Being an inverse problem, complete identification of residual stresses from incomplete measurement is always suffered by inherent ill-posedness. To control this issue, the identification problem has been formulated as an optimization problem via a Tikhonov type cost functional. The first part of this cost functional is associated with an energy norm of mismatch between predicted and measured residual stress. The second part represents the regularization term. A Lagrangian is then considered to satisfy the equilibrium constraint over the region of interest. The incorporation of Lagrangian with aforementioned cost functional provides a form which is very similar to the standard Hellinger-Reissner (HR) functional except the discrepancy term between the data. The minimization of this formulation leads to the solution of two-field mixed variational form associated with residual stress and Lagrangian variable. To show the efficacy of the proposed procedure a two-dimensional welded specimen is considered where the weld bead is assumed to be spatially varying in all direction for residual stress identification. Line scan details are also presented to examine the deviation between the actual and predicted residual stress in the heat affected zone and at the region away from the weld bead. The results indicate that the mixed finite element formulation has the capability to provide the complete information of residual stress despite of having very few information of measurement.

Keywords: Residual stress identification, Inverse problem, Mixed finite element formulation

1. Introduction

The study of identification of residual stress field has gain a profound attention since long days in the field of engineering. The primary reason for this is the unavoidable existence of self-equilibrated residual stress in newly manufactured or processed material that leads to the alteration of structural behaviour such as strength, stability and durability under subsequent in-service loaded situation. It is well observed that residual stress is essentially introduced by means of various inelastic deformations or eigenstrain [1] caused due to one or several conditions. These conditions are usually associated with spatial temperature gradient during localized heating [2], uneven cooling of hot-rolled or cold-formed material surface [3,4], non-uniform surface peeing with external substance [5], transformation of material phase [6] etc. Deterioration of fatigue strength and initiation of shrinkage, distortion, dimensional instability etc. are serious concern in structural health assessment of any component and these factors are intensified with the presence of initial residual stress. Among a vast range of manufacturing and material processing activities, welding has a widespread application in joining structural components. Despite this usefulness residual stress introduced due to the non-uniform temperature distribution, may modify in-service material performance through the generation of micro or macro crack under different loading condition [7]. So, knowledge of actual distribution and magnitude of residual stress over the entire region of interest is a basic requirement in structural health assessment and material performance. A diverse range of identification techniques have already been mentioned so far in several studies where attempts are mainly devoted to capture spatial distribution of residual stress in different geometrical shaped components by means of either experimental observation or numerical simulations of entire processing steps. Least square based eigenstrain reconstruction method is a well-established approach where knowledge of one dimensional varying eigenstrain estimated with an inverse analysis is employed in residual stress estimation [8]. It can also directly be approximated without the information of eigenstrain (source of self-equilibrated residual stress) distribution with a reverse stress analysis where stress satisfies the equilibrium condition implicitly [9]. However, the inherent dependence on the amount of estimated inverse co-efficient in both these approaches poses a limitation in full residual stress estimation for complex geometrical element. Finite element (FE) simulation of material processing is a useful approach for complete residual stress estimation [10] but the availability of exhaustive material characterisation and execution time is a matter of concern. Moreover, the profile of self-equilibrated stress is influenced by the nature of material processing and fabrication [11] along with distinct process

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parameter [12-15]. This makes the material processing simulation based stress estimation technique complicated. Nevertheless, residual stress modelling in simple as well as complex geometrical shape [16] is possible nowadays with the advancement of various finite element tools. Acoustic characteristic obtained via an ultrasonic stress measurement can be incorporated with FE model to capture through thickness residual stress in three-dimensional body [17]. Full field residual stress can also be estimated without process simulation via a finite element algorithm using only the measurement of surface residual stress [18]. Recently, an analytical residual stress estimation process based on elasto-plastic analysis with an arbitrary heat source model is proposed where use of finite element model can be completely excluded [19]. As experimental residual stress measurement technique invasive stress relaxation based or non-invasive diffraction-based process [20, 21] are usually followed. However, through thickness residual stress distribution can be obtained with the use of a semi-invasive focused-ion beam and digital image correlation setup [22]. Nevertheless, the choice of test location and preparation of test set-up posed a restriction on capturing complete residual stress profile. Hence, from incomplete measured details analysis of complete residual stress profile seeks an inverse optimization technique. In this paper, we have explored a complete residual stress identification process via a non-iterative mixed finite element approach in an inverse sense. It is noteworthy to mention that the estimation of stress with well-established displacement based finite element method [23] strictly depends on the approximation of displacement profile. Thus the order of polynomial representing the displacement essentially influences the estimated stress. However, in case of mixed finite element approach [24] the stress can be independently approximated and the stability of the solution depends on number of stress variable. In section 2, detail formulation for residual stress identification with a numerical experiment is shown. The inverse reconstruction process is also mentioned here. Reconstructed result and necessary discussion associated with proposed technique is demonstrated in section 3. Finally, we conclude this paper in section 4.

2. Formulation for residual stress identification

One or several eigenstrain associated with non-uniform inelastic deformation distribution, discrepancy in thermal expansion etc. generally introduce residual stress that presents in a self-equilibrated state within a domain in the absence of any force. Let, $\bar{\Omega}$ is a bounded domain with $\bar{\Omega} = \Omega \cup \Gamma$. Now if any eigenstrain ($\boldsymbol{\varepsilon}^*$) which is noncompatible with the domain presents, the non-uniformly varying residual stress can be obtained via a finite element solution after applying necessary boundary condition. The governing equilibrium equation associated with residual stress distribution can be expressed as

$$div\boldsymbol{\sigma} = 0 \quad \text{in} \quad \Omega \tag{1a}$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{n} = 0 \quad \text{on} \quad \Gamma \tag{1b}$$

$$\text{with} \quad \boldsymbol{\sigma} = \mathbb{D}:\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^*\right) \tag{1c}$$

$$\text{where} \quad \boldsymbol{\varepsilon} = \nabla^s\boldsymbol{u} = \frac{1}{2}\left(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^T\right) \tag{1d}$$

where $\boldsymbol{\sigma}$ denotes residual stress, $\boldsymbol{n}$ is outward unit normal, $\boldsymbol{\varepsilon}$ denotes compatible total strain which is symmetric, $\boldsymbol{u}$ is displacement and $\mathbb{D}$ is fourth order elasticity tensor.

2.1 Numerical experiment with welding

Fusion welding is one of the superior fabrication processes and it can extensively be used in joining different parts of any structural system. This process is executed with melting and solidification of parent and filler material. However, a huge amount of differential temperature distribution evolves during welding through a complex thermo-mechanical process. This non-uniformly varying temperature profile over the whole domain not only alters the microstructure of the parent metal but also contributes in residual stress formation. Again, the existence of two mutually perpendicular weld beads may sometimes be present in a region especially for repaired material. However, to idealize the aforementioned process we have explored a two-dimensional weld model of IN 718, a nickel based super alloy which consist of two mutually perpendicular weld zones. The schematic diagram of the specimen is shown in figure 1. The overall dimension of specimen is 120 x 120 mm^2. Here, only single component of eigenstrain is considered to represent both weld zones that generate residual stress. The eigenstrain distribution for weld 1 is taken as

$$\varepsilon_{xx}^* = -0.0015\cos\left(\frac{\pi y}{8}\right) \quad \text{with} \quad -4 \le y \le 4 \tag{2}$$

For weld 2, distribution of eigenstrain is considered as

$$\varepsilon_{yy}^* = -0.0015\cos\left(\frac{\pi x}{8}\right) \quad \text{with} \quad -4 \le x \le 4 \tag{3}$$

The width of both the eigenstrain region is taken as 8 mm.

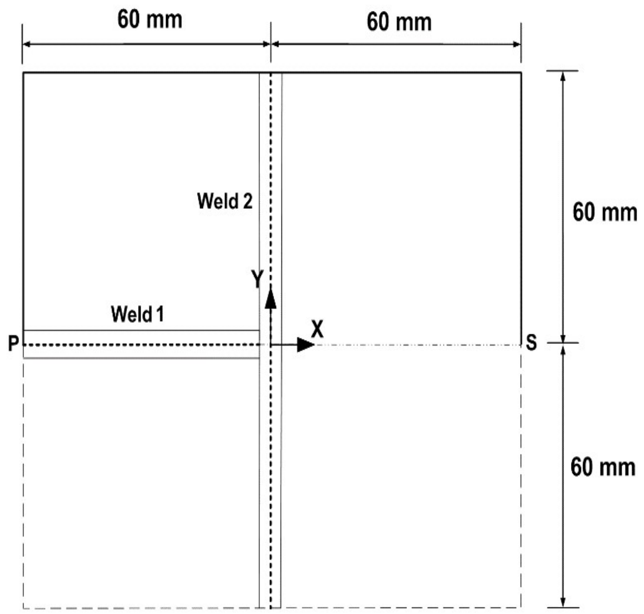

Figure 1: Schematic diagram of a 2D weld model with two mutually perpendicular weld region

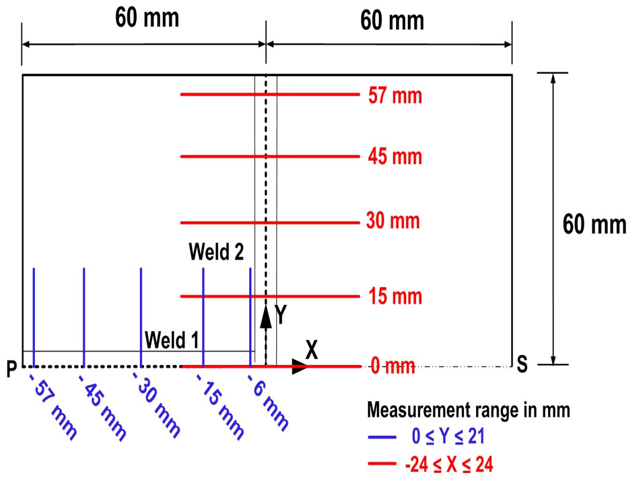

Figure 2: Measurement zones of residual stress for reconstruction of weld model.

The modulus of elasticity and Poisson's ration are 200 GPa and 0.29 respectively. Here the longitudinal stress distribution for overall sample is considered along X direction whereas transverse distribution is considered along Y direction. To brief the reconstruction scheme, we have only considered the upper part of the specimen. A symmetric boundary condition is implemented along PS line. We discretize entire domain with 120 x 60 mesh and obtain residual stress for the given eigenstrain distribution and boundary condition. The measurement is taken through line scan as shown in figure 2. For each weld zone, 5 lines are considered for measurement of all stress. The line scan around weld 1 is taken from 0 mm to 21 mm along transverse direction whereas these are taken from -24 mm to 24 mm around weld 2 along longitudinal direction with step size of 3 mm for both cases. Now the synthetic residual stress data captured in the measured locations are employed in our reconstruction algorithm.

2.2 Inverse reconstruction of residual stress estimation

Estimation of residual stress requires the experimental measurement of residual elastic strain or stress which is actually sparse in nature. This unavailability of complete measured data transforms the identification problem in an ill-posed inverse problem. Towards this, we have solved a Tikhonov type optimization problem that is subjected to the equilibrium equation (1). For a limited set of measured residual stress captured from measured domain Ω^m such that $\Omega^m \subseteq \bar{\Omega}$, we present the regularized cost functional as

$$\psi(\sigma,\beta)=\psi_1+\psi_2 \tag{4a}$$

with

$$\psi_1=\frac{\beta}{2}\int\left(\sigma-\sigma^m\right):\mathbb{D}^{-1}:\left(\sigma-\sigma^m\right)d\Omega^m \tag{4b}$$

$$\psi_2=\frac{1}{2}\int\sigma:\mathbb{D}^{-1}:\sigma d\Omega \tag{4c}$$

Here, ψ_1 is associated with energy norm of discrepancy between estimated and available measurement of residual stress. Complementary strain energy is presented with ψ_2 in the same way. We consider a nonnegative penalization parameter β as the inverse of regularization parameter which essentially controls the solution of this ill-posed problem. To obtain residual stress, we minimize equation (4) which is subjected to the governing equilibrium equation (1).

To satisfy the equilibrium constraint we choose a Lagrangian as

$$\mathcal{L}(\sigma,\boldsymbol{w})=\psi(\sigma,\beta)+\int\sigma:\boldsymbol{\varepsilon}(\boldsymbol{w})d\Omega \tag{5}$$

where $\boldsymbol{w}$ is Lagrange multiplier. If the discrepancy part is neglected, Lagrangian is similar to the two-field Hellinger-Reissner type mixed variational formulation [25].

Now following first order optimality condition of Lagrangian as

$$\left(\frac{\partial\mathcal{L}}{\partial\sigma},\delta\sigma\right)=0 \tag{6a}$$

$$\left(\frac{\partial\mathcal{L}}{\partial\boldsymbol{w}},\delta\boldsymbol{w}\right)=0 \tag{6b}$$

we obtain following equations

$$\beta\int\delta\sigma:\mathbb{D}^{-1}:\sigma\,d\Omega^m+\int\delta\sigma:\mathbb{D}^{-1}:\sigma\,d\Omega+\int\delta\sigma:\boldsymbol{\varepsilon}(\boldsymbol{w})d\Omega =\beta\int\delta\sigma:\mathbb{D}^{-1}:\sigma^m d\Omega^m \tag{7a}$$

$$\int\boldsymbol{\varepsilon}(\delta\boldsymbol{w}):\sigma d\Omega=0 \tag{7b}$$

We want to approximate residual stress and Lagrangian variable independently. To implement this, we discretize the domain and follow a mixed finite element formulation [24] where full approximation of residual stress and Lagrangian variable in element level is achieved by considering following approximation

$$\sigma=H\hat{\alpha};\quad \boldsymbol{\varepsilon}(\boldsymbol{w})=B\hat{w};\quad \sigma^m=N\hat{\sigma}^m \tag{8a}$$

$$\delta\sigma=H\delta\hat{\alpha};\quad \boldsymbol{\varepsilon}(\delta\boldsymbol{w})=B\delta\hat{w};\quad \boldsymbol{w}=M\hat{w};\quad \delta\boldsymbol{w}=M\delta\hat{w} \tag{8b}$$

Now we can express equation (7) as

$$\begin{bmatrix} G & J^T \\ J & 0 \end{bmatrix}\begin{Bmatrix} \hat{\alpha} \\ \hat{w} \end{Bmatrix}=\begin{Bmatrix} F \\ 0 \end{Bmatrix} \tag{9}$$

where

$$G=\beta\int H^T\mathbb{D}^{-1}H\,d\Omega^m+\int H^T\mathbb{D}^{-1}H\,d\Omega \tag{10a}$$

$$J=\int B^T H\,d\Omega \tag{10b}$$

$$F=\beta\int H^T\mathbb{D}^{-1}N\hat{\sigma}^m d\Omega^m \tag{10c}$$

Following element level static condensation, we obtain residual stress and Lagrangian variable as follows

$$\{\hat{\alpha}\}=G^{-1}\left(F-J^T\hat{w}\right) \tag{11a}$$

$$\left(JG^{-1}J^T\right)\{\hat{w}\}=\left(JG^{-1}F\right) \tag{11b}$$

When $N_{\hat{\alpha}}$ and $N_{\hat{w}}$ are the number of residual stress and Lagrangian variable, the solution stability is achieved in following condition

$$N_{\hat{\alpha}}\geq N_{\hat{w}} \tag{12}$$

3. Reconstruction and discussion

In this section we are trying to estimate full field residual stress for a two-dimensional weld model shown in figure 1 following aforementioned formulation. It can be noted here that the distribution of eigenstrain is taken in such a way that its spatial variation exists in all direction. We consider 40 x 20 Pian-Sumihara quadrilateral element

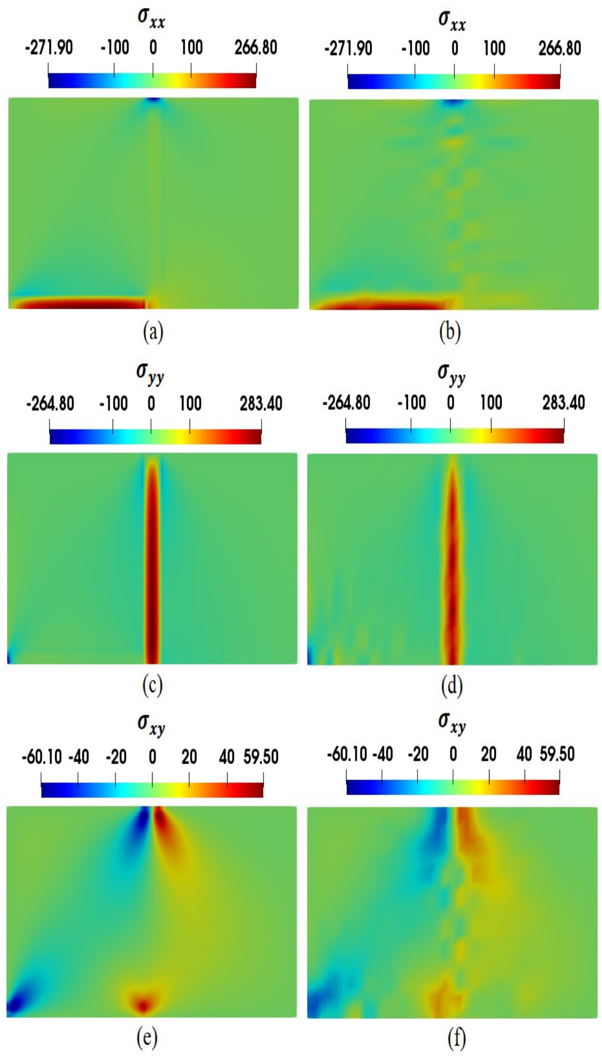

Figure 3: Target and reconstructed distribution of residual stress for two-dimensional weld model having two weld zones. (a) and (b) longitudinal stress. (c) and (d) transverse stress. (e) and (f) shear stress

[26] to reconstruct the self-equilibrated stress. The primary reason to select this element is to satisfy the equation (12). This element is quite successful to provide more accurate solution than that of conventional quadrilateral element which is used in displacement based finite element approach. The reconstruction is performed with only 121 measurement points captured from the lines as shown in figure 2. The target and reconstructed residual stress profile are shown in figure 3.

It is quite obvious that σ_{xx} is predominant in weld 1 zone whereas the existence of σ_{yy} is significant in weld 2 zone. The result indicates that the proposed reconstruction method can successfully estimate the residual stress over the entire domain despite the availability of very few measurements. As the inverse reconstruction is governed by correct choice of the penalization parameter, we choose an optimal penalty parameter as 1.0 x 10^{-3} on the basis of least discrepancy between measured and reconstructed stress data. It is quite natural that tensile residual stress exists in the weld zone.

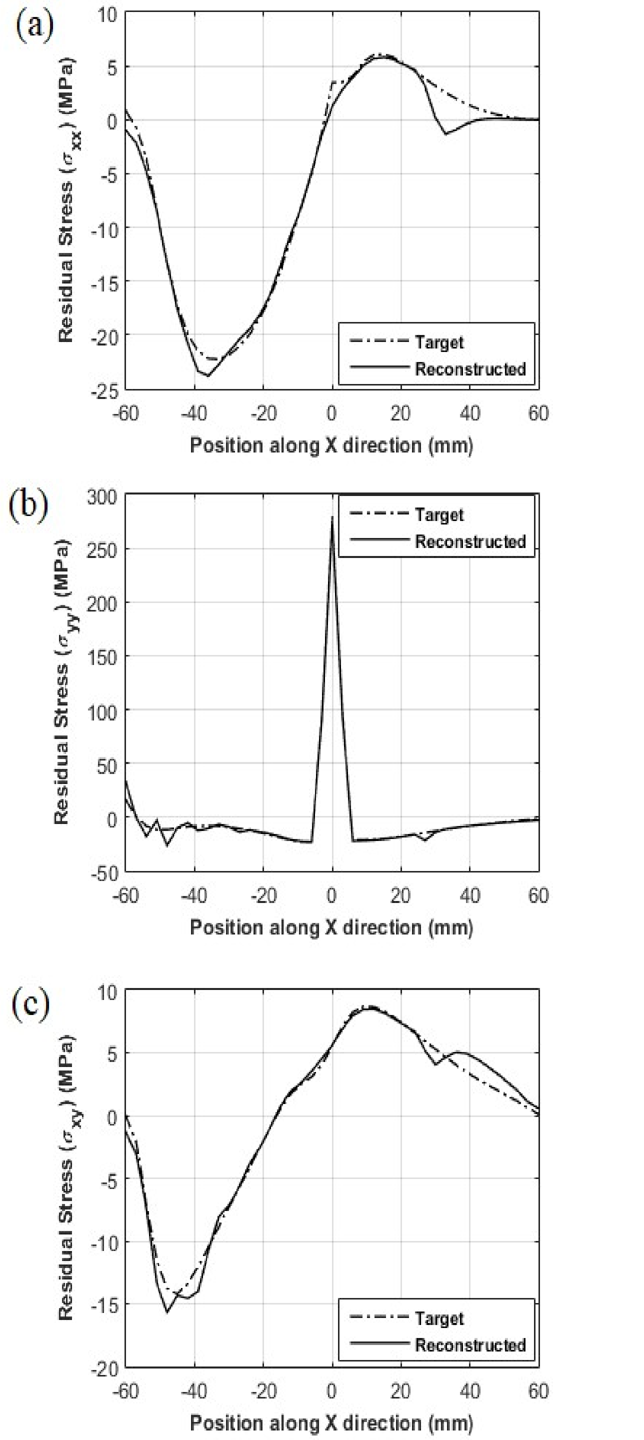

Figure 4: Details of line plot along Y=15 mm for comparison of target and reconstructed distribution of residual stress. (a) longitudinal stress. (b) transverse stress. (c) shear stress

However, high compressive residual stress presents in the boundary at the region of weld. High rate of change of temperature which is significant in the boundary region than that of the non-boundary zone due to the unavailability of constraint may cause this compressive stress. A line plot is shown in figure 4 at a position of Y = 15 mm from the PS line to compare the residual stress in the region of weld bead (fusion zone), heat affected zone and away from weld zone. Both the stress pattern is quite consistent with each other. The presence of non-zero longitudinal residual stress in the zone away from weld is due to the existence of multiple weld zones. However, in the boundary this stress almost disappears. Similar pattern for shear stress is also observed. In case of transverse stress component, the stress profile is maximum in the weld zone and in the surrounding this almost evanesces. This plot consists of measurement as well as non-measurement points. There is a little mismatch between the target and reconstructed pattern at non-measured zone. However, this discrepancy can be eliminated with the choice of more measurement points as the amount of known stress data will increase in this case. There is indeed a practical difficulty in selection of many measurement points from the perspective of total duration of test, workmanship and preparation of individual location of test. In this context this method can be a good choice as a reconstruction method.

4. Conclusion

A residual stress identification via a mixed finite element based non-iterative inverse optimization process is explored. Independent approximation of residual stress and Lagrangian variable is obtained with the use of hybrid element. The solution obtained with this method does not require the information of the source of residual stress. Moreover, the numerical experiment reveals that the model can qualitatively estimate all the self-equilibrated stress components with little available information.

Disclosures

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