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Two Stage SEREP Method for Condensation of Space Frame Structure for Dynamic Analysis

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Abstract

Analysis of frame structure can be done by the computer analysis, but there is requirement of simple and easy model which can give accurate result or with tolerable accuracy. The process of reducing the size of matrix is called condensation. It needs to be converted into system or model having less number of degrees of freedom as huge computational effort is required to analyze big models. Also, the models which requires analysis multiple times, for example, optimization of structural system, condensation is very useful. The single stiffness value can represent the stiffness of the whole story (elastic, rectangular frame with evenly distributed load), which give the responses accurately or tolerable accuracy. With this we can create a single lumped mass system of the frame structure which come very handy in further analysis of structure. For finding the single stiffness value, there are various approach like by using Approximate Story Stiffness Formulas, Static condensation, *System Equivalent Reduction Expansion Process (SEREP)* etc. Approximate Story Stiffness formulae don't give very accurate result. To overcome its shortcoming Static Condensation and SEREP methods are used. Static Condensation gives better result but there is significant difference in the responses. Especially for space frame structures it is difficult to directly condense in to single degrees of freedom at each floor level. Hence, the present study proposes two stage SEREP method for condensation of spaceframe structure. Firstly, space frame is reduced to single degrees of freedom at each node, then it is further reduced to single degree of freedom at each floor. The proposed method gives very less (acceptable) error in comparison of natural frequencies.

Keywords: SEREP, Condensation, Space frame, Natural frequency, Dynamic Analysis

1. Introduction

Condensation of model is required when the model has huge degree of freedom. It needs to convert into system or model having a smaller number of degrees of freedom as huge computational effort is required to analyze big models. Also, the models which requires analysis multiple times, for example, optimization of structural system, condensation is very useful. Analysis of frame structure can be done by the computer analysis, both dynamic and static. The single stiffness value can represent the stiffness of the whole story (elastic, rectangular frame with evenly distributed load), which give the responses accurately or tolerable accuracy. With this we can create a single lumped mass system of the frame structure. Large stiffness matrix requires large memory and it is difficult to predict the effect of one degree of freedom on the complete structure. The process of reducing the matrix is called condensation. It is done by identifying slave degree of freedom (degrees of freedom which has no effect on the response) and by omitting them. This can be done by various method like by using approximate stiffness formulae, Guyan Reduction (Static Condensation), SEREP etc. Reduced Matrix come very handy in analyzing different type of structure like building, aero plane, cars, any mechanical tool etc. Rather than analyzing all degrees of freedom, you choose active degrees of freedom and analyze them.

Muto tried to formulate simple and direct formulas for finding stiffness of story by applying slope deflection method [1]. It is based on assumption that all story will take equal shear force, and both ends of all members will undergo equal rotation. Also, Muto taken base fixity into account. These assumption needs to be considered while using Muto's formulae. Muto's formula gives accurate result for intermediate story [1]. Schultz presented a formula by taking effect of unequal height of adjacent frame, effect of top and bottom frame and base fixity effect of low-rise structure [2]. Schultz introduced correction factor related to it. But the formulas are limited to rectangular frame with fixed base and only the flexural deformation are considered [2]. Correction factors are also presented for nine-story building. Hosseini and Imagh tried to condense the large frame structure to one bay-one story frame structure [3]. This method does not take

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in to account of any correction factor due to base fixity and due to discontinuity of top frame [3]. This method also considered only flexural deformation. Condensation technique is presented using Lateral force-deformation method based on the displacement vector calculated during the design and analyses of frame structure, at each story [4]. Design earthquake loads are used to calculate story stiffness (as ratio of cumulative shear force to lateral displacement between two consecutive story) [4]. Here, stiffness value depends on the distribution of lateral force. Stiffness value of story changes by changing the distribution. However, all the above methods have some or other assumption which are not hold good for all buildings. Each method holds good only if assumptions hold good.

There are various mathematical techniques for condensing the matrix like Static Condensation, Dynamic Condensation, System equivalent reduction expansion process (SEREP), Improved Reduced System etc. In all these methods, the results depend on the choice of degree of freedom which are active. Response of reduced system by Static Condensation generally gives higher value. SEREP gives the reduced system which are identical to the original system [5].

The present study focuses on static condensation of 3D frame structure. Comparative study has been done between the approximate formulae for finding story stiff-ness for the space frame structure (three story), static condensation and SEREP method for the condensation of space frame to lumped mass system. A novel method of two stage SEREP method is proposed for more accurate condensation of space frame. Comparison is carried between static condensation after SEREP and SEREP followed by SEREP (two stage SEREP). For frame structure, direct condensation from (72x72) to (3x3) is not yielding the accurate result. So, it is alternatively done by first reducing it to the (12x12) by using SEREP (first stage), then further reduce (12x12) to (3x3) using again SEREP (second stage). In this method, the original system is first converted to system with less DOF at each node and then reduced to lumped mass system with one DOF at each floor.

2. Mathematical Formulation

2.1. Approximate stiffness formulae

Muto has formulated some approximate stiffness formulae. Here, assumptions are that the frame is infinite array of members and shear force is taken by all columns equally. Story stiffness can be found by summation of stiffness of all columns [1].

Stiffness of column calculated by taking an effect of beams at top and bottom of column is given by:

For story other than first story,

$$K_c = \left(\frac{12E_cI_c}{H^3}\right)\left(\frac{\sum k_{ga}+\sum k_{gb}}{4k_c+\sum k_{ga}+\sum k_{gb}}\right) \quad (1)$$

For Story 1,

$$K_c = \left(\frac{12E_cI_c}{H^3}\right)\left(\frac{k_c+\sum k_{ga}}{4k_c+\sum k_{ga}}\right) \quad (2)$$

Σk_{ga} and Σk_{gb} = Sum of flexural stiffness of girders forming joint at top and bottom of column; E_c, I_c, k_c and H are modulus of elasticity, second moment of inertia, stiffness and height of column respectively.

Alternate formulation [2] Approach 1: Alternate approach is proposed for taking in account the different behavior of story which are not close to top and bottom [2]. For isolated frames i.e. stories which are not adjacent to top or bottom frame, stiffness of story (K_s) is given by:

$$K_s = \left(\frac{24}{H^2}\right)\left(\frac{1}{\frac{2}{\sum k_c}+\frac{1}{\sum k_{ga}}+\frac{1}{\sum k_{gb}}}\right) \quad (3)$$

For story 1, the term $\frac{1}{\sum k_{gb}}$ is neglected and Σk_c is sum of stiffness of all columns.

Approach 2: To take in account the impact of direction of loading, the following approach is formulated. If the frame is laterally loaded in y-direction, the stiffness of beams parallel to x-direction is neglected.

$$K_s = \left(\frac{24}{H^2}\right)\left(\frac{1}{\frac{2}{\sum k_c}+\frac{1}{\sum k_{ga}}+\frac{1}{\sum k_{gb}}}\right) \quad (4)$$

For Story 1, the term $1/\Sigma K_{gb}$ is neglected to take in account base fixity.

$k_{ga} = 2k_y$ and $k_{bg} = 2k_y$

Approach 3: To try new approach, here, for calculating stiffness of story 1, Σk_{gb} is not neglected**.**

$$K_s = \left(\frac{24}{H^2}\right)\left(\frac{1}{\frac{2}{\sum k_c}+\frac{1}{\sum k_{ga}}+\frac{1}{\sum k_{gb}}}\right) \quad (5)$$

Approach 4: Alternate formulation with correction factor [2] For boundary stories (stories that are top and bottom, and adjacent to first story) correction factor to be applied because of the fixed base. Effect of fixed base propagate till the adjacent storey. Given formulas are for nine storey frame structure.

$$K_s = \left(\frac{24}{H^2}\right)\left(\frac{1+C_S}{\frac{2}{\sum k_c}+\frac{1}{\eta_a\sum k_{ga}}+\frac{1}{\eta_b\sum k_{gb}}}\right) \quad (6)$$

Where,

$$C_1 = \frac{\sum k_c}{22\sum k_{ga}}, \qquad C_2 = \frac{\eta_b\sum k_c}{32\sum k_{gb}}, \qquad C_t = \frac{-\sum k_c}{55\sum k_{ga}}$$

S = Storey number (For example: 1, 2, 3 etc.)

η_a and η_b are correction factor.

The above formula will reduce the structure such that there will be only one node at one storey. This are easy to use but all these formulas are formulated keeping 2-D frame structure in focus.

2.2. Condensation Techniques

There are various approaches with the help of which we can condense the stiffness matrix. For example, Static condensation, dynamic condensation, Improved reduced system, System Equivalent Reduction Expansion Process and Hybrid Reduction.

Static Condensation:

This technique is used to omit the unwanted or unnecessary degrees of freedom (DOF). Unwanted degrees of freedom mean those degree of freedom even if it releases it will not have any or very less impact on the response of the frame structure. But here we must be very critical which DOFs we have to choose as master DOF [5]. To get accurate result we have choose right master DOF.

Summary of method [5][6]:

Stiffness matrix

$$[K_n]\{x_n\}=\{F_n\} \quad (7)$$

By identifying master and slave DOF and changing the form of equation (7),

$$\begin{bmatrix}[K_{aa}] & [K_{ad}]\\ [K_{da}] & [K_{dd}]\end{bmatrix}\begin{Bmatrix}x_a\\ x_d\end{Bmatrix}=\begin{Bmatrix}F_a\\ F_d\end{Bmatrix}$$

(8)

Here, suffix ‘a’ corresponds to active or master DOF and suffix ‘d’ corresponds to dependent or slave DOF. x and F are displacement and force vector.

Reduced stiffness (K_R) and reduced mass matrix (M_R) is given by:

$$[K_R] = [T_S]^T[K_n][T_S], \quad [M_R] = [T_S]^T[M_n][T_S] \quad (9)$$

Where, K_n and M_n are global stiffness and mass matrix. T_s is transformation matrix.

$$[T_s] = \begin{bmatrix}[I]\\ [t_s]\end{bmatrix} = \begin{bmatrix} & [I] \\ -[K_{dd}]^{-1} & [K_{da}]\end{bmatrix} \quad (10)$$

SEREP (System Equivalent Reduction Expansion Process):

The System Equivalent Reduction Expansion Process (SEREP) relies on a finite element model or analytical model from which an Eigen solution is obtained for developing the mapping between the full set of 'n' finite element DOF and the reduced set of 'a' DOF [5].

The modal transformation equations can be written as:

$$\begin{Bmatrix}\{x_a\}\\ \{x_d\}\end{Bmatrix} = \{x_n\} = [U_n]\{p\} = \begin{bmatrix}[U_a]\\ [U_d]\end{bmatrix}\{p\} \quad (11)$$

$\{x_n\}$ = All degrees of freedom

$\{x_a\}$ = Master degrees of freedom

$\{x_d\}$ = Omitted or Slave degrees of freedom

$[U_n]$ = Eigen vector matrix

$\{p\}$ = model displacement

$[U_a]$ = Eigen vector matrix corresponding to active degree of freedom considered

$[U_d]$ = Eigen vector matrix corresponding to dormant degree of freedom not considered

Reduced stiffness (K_R) and reduced mass matrix (M_R) is given by:

$$[K_R] = [T_U]^T[K_n][T_U], [M_R] = [T_U]^T[M_n][T_U] \quad (12)$$

where,

$$[T_U] = [U_n][U_a]^g, \ [U_a]^g = ([U_a]^T[U_a])^{-1}[U_a]^T$$

Here, the transformation matrix is obtained from eigenvector matrix, the reduced matrix is exact as the vectors are preserved exactly in the reduced model. This means that the required vectors can be easily preserved exactly in reduced system [5].

In the present study, MATLAB has been for implementing the SEREP [5] and the model mass and stiffness are taken from SAP. All the translational in *y*-direction is assigned as active DOF. Then set of modes required for the reduced model is selected. After that, by using the Eigen vector, the reduced stiffness and reduced mass matrix is calculated.

Below is program written in MATLAB for SEREP [5]:

```
activedof = [“set of active degree of freedom”];
modes = [“set of modes required”];
Ua = EigenVectors(activedof, modes);
Un = EigenVectors(:, modes) ;
Uag = pinv(Ua);
T = Un*Uag ;
MR = T'*M*T;  % Reduced mass matrix (MR)
KR = T'*K*T;    % Reduced stiffness matrix (KR)
```

3. Numerical Study

The consistent stiffness matrix (72×72) of a 3-storey space frame model to be reduced using different condensation technique is as shown in Fig.1. It clearly shows the assumed *x, y* and *z* direction and all node numbers. The base is assumed to be fixed. Different methods are used for condensation to check which method will give best result. The reduction of matrix is done by using the approximate
stiffness formula, static condensation and SEREP. The frame structure is modeled in SAP software and the consistent stiffness and mass matrices are extracted for condensation. The parameters of model are listed in Table 1. Using SAP, the first ten mode shapes of the frame structure are identified. Also, the translational mode shapes in *y*-direction and corresponding natural frequencies are calculated using lumped mass (Eq.12).

Lumped Mass Matrix (without Plate/Slab)

$$[M] = \begin{bmatrix}5.346697 & 0 & 0\\ 0 & 5.346697 & 0\\ 0 & 0 & 4.112857\end{bmatrix} \ in\ Kg$$

Table 1: Material and geometric properties of Space frame model

Density	7849 Kg/m^3
Young’s Modulus	200 GPa
Poisson’s ratio	0.3
Yield Stress	344.7 MPa
Effective Yield Stress	344.7 MPa
Yield Stress	344.7 MPa
Plan Dimension in *x*-direction	0.6 m
Plan Dimension in *y*-direction	0.8 m
Height of each storey	0.6 m
Area of all members (Square cross section)	131 mm^2
Moment of Inertia of all members	7305 mm^4
Mass of Storey1 and Storey2 (without plate)	5.35 Kg
Mass of Storey 3 (without plate)	4.11 Kg

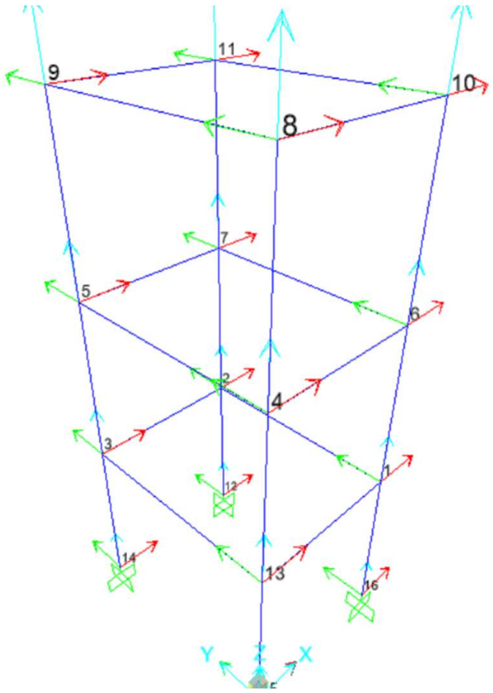

Fig.1: Space frame model with node numbers

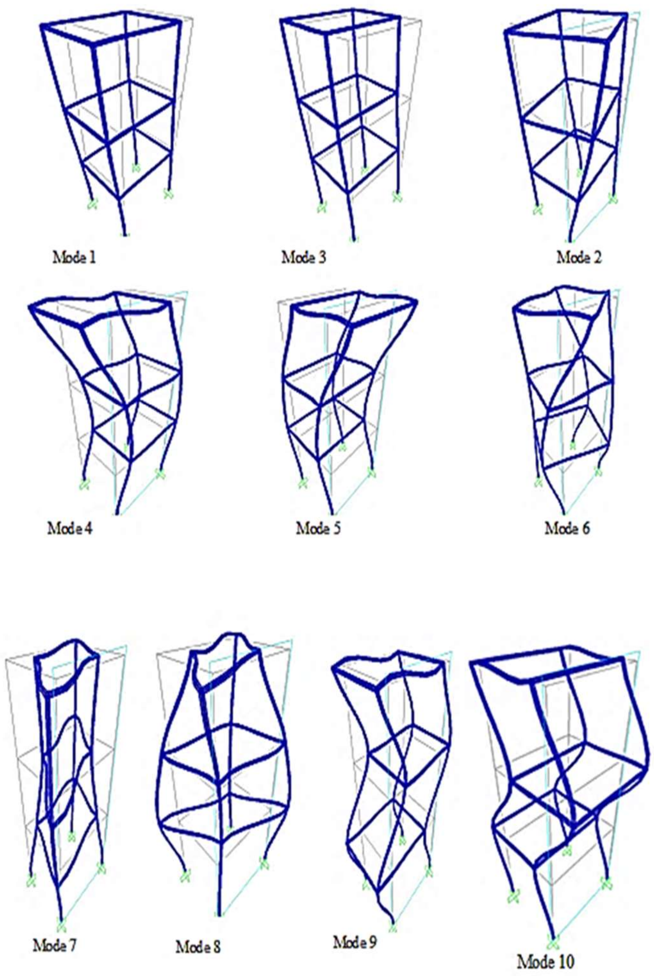

Fig. 2: First 10 modes of the frame

Table 2: Frequency of first 10 modes.

Mode	Frequency (Hz)	Mode	Frequency (Hz)
1	11.646	6	41.808
2	12.497	7	49.635
3	13.662	8	62.348
4	37.476	9	64.954
5	39.269	10	65.832

Table 3: Comparison of natural frequencies between SAP and Lumped mass method

Lumped mass method		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	12.825	1	11.646	10.11
2	35.186	4	37.476	-6.11
3	49.189	9	64.954	-24.27

4. Result and Discussion

4.1. Approximate Stiffness Formulae:

Firstly, the stiffness of the story is directly calculated by the direct approximate stiffness formulae. These formulas are very easy to use, and it directly gives the stiffness of story.

By using Muto's formulae, i.e. eq. (1) and (2). Values of all parameters are taken from Table 1. Reduced Mass matrix will be same as lumped mass matrix of frame (eq. 12).

$$[K] = \begin{bmatrix} 306786.47 & -151511.11 & 0 \\ -151511.11 & 303022.22 & -151511.11 \\ 0 & -151511.11 & 151511.11 \end{bmatrix} \text{N/m}$$

Alternate approach 1, i.e. eq. (3): Different formulation is given for the story not adjacent to top and bottom. Though, there is no such story in our model, still we applied it in our case.

$$[K] = \begin{bmatrix} 458895.02 & -206502.76 & 0 \\ -206502.76 & 413005.52 & -206502.76 \\ 0 & -206502.76 & 206502.76 \end{bmatrix} \text{N/m}$$

Table 4: Comparison of natural frequencies between SAP and Muto's Formulae.

Muto's Formulae		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	12.825	1	11.646	10.12
2	35.187	4	37.476	-6.11
3	49.189	9	64.954	-24.27

Table 5: Comparison of natural frequencies between SAP and alternate approach 1.

Alternate Approach 1		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	15.701	1	11.646	34.82
2	42.382	4	37.476	13.09
3	57.9632	9	64.954	-10.76

Alternate approach 2, i.e. eq. (4): Since we have to find response only in y-direction, the effect of beams parallel to x-axis is neglected.

$$[K] = \begin{bmatrix} 227574.47 & -88501.18 & 0 \\ -88501.18 & 177002.36 & -88501.18 \\ 0 & -88501.18 & 88501.18 \end{bmatrix} \text{N/m}$$

Table 5: Comparison of natural frequencies between SAP and alternate approach 2.

Alternate approach 2		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	10.907	1	11.646	-6.35
2	29.073	4	37.476	-22.42
3	38.699	9	64.954	-40.42

Alternate approach 3, i.e. eq. (5): In this approach, the K_{gb} term for story 1 is also considered.

$$[K] = \begin{bmatrix} 413005.52 & -206502.76 & 0 \\ -206502.76 & 413005.52 & -206502.76 \\ 0 & -206502.76 & 206502.76 \end{bmatrix} \text{N/m}$$

Table 6: Comparison of natural frequencies between SAP and alternate approach 3.

Alternate approach 3		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	14.868	1	11.646	27.66
2	40.909	4	37.476	9.16
3	57.363	9	64.954	-11.69

Formulae with correction factor i.e. eq. (6): Different approach is taken to find the reduced matrix, considering the base fixity effect. Also, different approaches are taken to calculate stiffness of story.

Approach 1: For Story 1, ($1/\Sigma K_{gb}$) is not neglected.

$$[K] = \begin{bmatrix} 468444.47 & -229089.00 & 0 \\ -229089.00 & 422450.67 & -193361.67 \\ 0 & -193361.67 & 193361.67 \end{bmatrix} \text{N/m}$$

Table 7: Comparison of natural frequencies between SAP and alternate formulation with correction factor for approach 1

Formulae with correction factor with approach 1		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	15.734	1	11.646	35.09
2	41.453	4	37.476	10.61
3	58.699	9	64.954	-9.63

Approach 2: For Story 1, ($1/\Sigma K_{gb}$)is neglected and since we have to find response in y-direction, the stiffness of beams parallel to x-axis is neglected and eq. (6) is used. ($K_{ga} = 2*K_y$ and $K_{bg} = 2*K_y$)

$$[K] = \begin{bmatrix} 182093.70 & -90575.43 & 0 \\ -90575.43 & 177869.78 & -87294.35 \\ 0 & -87294.35 & 87294.35 \end{bmatrix} \text{N/m}$$

Table 8: Comparison of natural frequencies between SAP and alternate formulation with correction factor for approach 2

Formulae with correction factor with approach 2		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	9.86	1	11.65	-15.33
2	26.89	4	37.48	-28.24
3	37.72	9	64.95	-41.93

Approach 3: For Story 1 term $1/\Sigma K_{gb}$ is neglected and eq. (6) is used to find story stiffness.

$$[K] = \begin{bmatrix} 521634.57 & -229089.00 & 0 \\ -229089.00 & 422450.67 & -193361.67 \\ 0 & -193361.67 & 193361.67 \end{bmatrix} \text{N/m}$$

Table 9: Comparison of natural frequencies between SAP and alternate formulation with correction factor for approach 3

Formulae with correction factor with approach 3		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	16.571	1	11.646	42.28
2	42.853	4	37.476	14.35
3	59.604	9	64.954	-8.24

Approach 4: For Story 1, ($1/\Sigma K_{gb}$) is neglected and since we have to find response in y-direction, the stiffness of beams parallel to x-axis is neglected and eq. (6) is used. ($K_{ga} = 2K_y$ and $K_{bg} = 2K_y$)

$$[K] = \begin{bmatrix} 234389.85 & -90575.43 & 0 \\ -90575.43 & 177869.78 & -87294.35 \\ 0 & -87294.35 & 87294.35 \end{bmatrix} \text{N/m}$$

4.2. Condensation methods

The reduction of stiffness and mass matrix is carried out using static condensation and SEREP methods using MATLAB 2016R.

Static Condensation

The global stiffness and global mass matrix are extracted by using SAP software and then it is condensed by using MATLAB 2016R. The row and column number of stiffness and mass matrix are corresponding to the node number given in Fig. 3. For condensation, master DOF must be identified correctly otherwise it will lead to error. Eq. (7), (8), (9) and (10) are used for static condensation.

Table 10: Comparison of natural frequencies between SAP and alternate formulation with correction factor for approach4

Formulae with correction factor with approach 4		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	11.035	1	11.646	-5.25
2	29.147	4	37.475	-22.22
3	38.979	9	64.9544	-39.99

If we choose master DOF as Uy of node 4,8 and 13 and condense the matrix by retaining them, following are reduced stiffness and mass matrix

$$[K] = \begin{bmatrix} 328423.8219 & -172966.9567 & 28761.39185 \\ -172966.9567 & 267501.8542 & -126990.6246 \\ 28761.39185 & -126990.6246 & 102741.243 \end{bmatrix} \text{N/m}$$

$$[M] = \begin{bmatrix} 2.695971484 & -0.167872928 & -0.479261448 \\ -0.167872928 & 4.922756903 & -2.1298959 \\ -0.479261448 & -2.1298959 & 5.462961755 \end{bmatrix} \text{Kg}$$

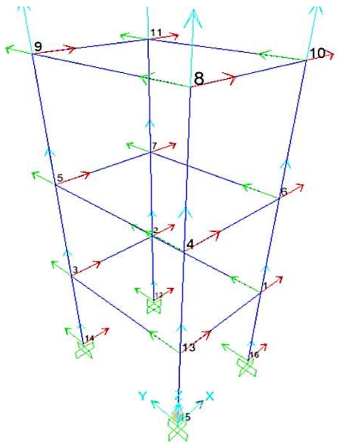

Fig 3: Frame structure with node numbers

[K] =

51725214	-3854767	-40340.7	-5.1E+07	7435722	-1.7E+07	17442217	-6591483	3499228	638692.3	-162527	-3989167
-3854767	15425152	-1009169	3670243	-1.5E+07	1584146	-4688110	10258253	-4571814	-2282117	1648449	6118289
-40340.7	-1009169	680689	146590.7	343539.8	538933.2	63177.58	-316.724	-33539.5	158358.5	-91663.9	-151054
-5.1E+07	3670243	146590.7	51666368	-7265568	17019809	-1.8E+07	6168931	-3964320	-567443	326280	4355103
7435722	-1.5E+07	343539.8	-7265568	51825010	-2047718	4287285	-4.8E+07	6170172	2192955	-1612524	-7551658
-1.7E+07	1584146	538933.2	17019809	-2047718	55550878	-5.5E+07	1883020	-3685056	-454980	493345.3	3401091
17442217	-4688110	63177.58	-1.8E+07	4287285	-5.5E+07	56303054	-3192028	4730536	860738	-1037755	-4742711
-6591483	10258253	-316.724	6168931	-4.8E+07	1883020	-3192028	44914570	-5131805	-1547836	1191218	5813766
3499228	-4571814	-33539.5	-3964320	6170172	-3685056	4730536	-5131805	44679439	778446.9	-2409324	-4.3E+07
638692.3	-2282117	158358.5	-567443	2192955	-454980	860738	-1547836	778446.9	462028.2	-376881	-973774
-162527	1648449	-91663.9	326280	-1612524	493345.3	-1037755	1191218	-2409324	-376881	1129301	1889939
-3989167	6118289	-151054	4355103	-7551658	3401091	-4742711	5813766	-4.3E+07	-973774	1889939	42857400

Fig 4: Reduced Stiffness Matrix

[M] =

1.178049	0.127972	0.003987	0.22481	0.105115	-0.18134	-0.0215	-0.17374	0.057138	0.105985	-0.11596	-0.00533
0.127972	1.489241	0.010684	0.307317	0.266514	0.186163	-0.26635	-0.36665	0.113206	0.140929	-0.10409	-0.03799
0.003987	0.010684	0.011871	0.004351	0.014757	0.005867	-0.01107	-0.01024	0.00416	0.003136	-0.00347	-0.00642
0.22481	0.307317	0.004351	1.217865	-0.02388	0.031545	-0.20864	-0.05946	0.012538	0.135211	-0.04166	0.029178
0.105115	0.266514	0.014757	-0.02388	1.457955	0.14566	0.085753	-0.08953	0.181799	0.019019	-0.01054	-0.18534
-0.18134	0.186163	0.005867	0.031545	0.14566	1.157632	0.20605	0.06826	-0.0698	-0.03806	-0.02533	-0.03278
-0.0215	-0.26635	-0.01107	-0.20864	0.085753	0.20605	1.311369	0.117991	-0.11717	0.10557	0.135851	-0.14333
-0.17374	-0.36665	-0.01024	-0.05946	-0.08953	0.06826	0.117991	1.420106	-0.07823	-0.17214	-0.13194	0.280123
0.057138	0.113206	0.00416	0.012538	0.181799	-0.0698	-0.11717	-0.07823	1.155077	-0.02023	-0.03558	0.395698
0.105985	0.140929	0.003136	0.135211	0.019019	-0.03806	0.10557	-0.17214	-0.02023	0.522305	0.023818	-0.02853
-0.11596	-0.10409	-0.00347	-0.04166	-0.01054	-0.02533	0.135851	-0.13194	-0.03558	0.023818	1.63165	-0.51221
-0.00533	-0.03799	-0.00642	0.029178	-0.18534	-0.03278	-0.14333	0.280123	0.395698	-0.02853	-0.51221	1.206605

Fig. 6: Calculated Frequency of first 12 modes by SEREP

Table 11: Comparison of natural frequencies between SAP and Static Condensation

Master DOF: U_y of node 4,8 and 13		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	12.62536164	1	11.64631	8.406539
2	28.68304671	4	37.47586	-23.46260
3	59.94195491	9	64.95375	-7.715944

If we choose master DOF as U_y of node 3,5 and 9 and condense the matrix by retaining them, following are reduced stiffness and mass matrix:

$$[K] = \begin{bmatrix} 328423.8 & 28761.39 & -172967 \\ 28761.39 & 102741.2 & -126991 \\ -172967 & -126991 & 267501.9 \end{bmatrix} \text{N/m}$$

$$[M] = \begin{bmatrix} 2.923294 & 0.060262 & 0.219389 \\ 0.060262 & 2.721817 & 0.341657 \\ 0.219389 & 0.341657 & 3.144767 \end{bmatrix} \text{Kg}$$

Mode	Frequency(in Hz)
1	11.6574734
2	37.5366531
3	49.37823005
4	61.06431296
5	64.97477972
6	78.19336963
7	1111.591992
8	1111.997747
9	1112.926908
10	1113.46933
11	1267.334748
12	1267.800025

Table 12: Comparison of natural frequencies between SAP and Static Condensation

Master DOF: Uy of node 3,5 and 9		SAP		Error %
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	12.1446841212499	1	11.64631	4.27
2	40.1845035462814	4	37.47586	7.22
3	68.7103992502788	9	64.95375	5.78

By SEREP SEREP is also performed by using the MATLAB 2016R. Here, the matrix is condensed from 72x72 to 12x12 matrix. Translational DOF in *y*-direction is released for all the nodes (master DOF). Other DOF are slave DOF. Since, there are 12 master DOF the reduced matrix will be 12x12.

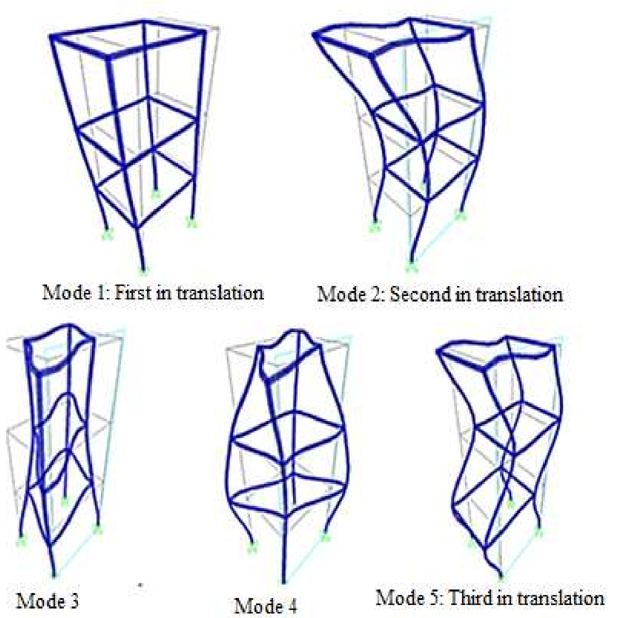

Fig. 7: First five mode of reduced system (by SEREP) by considering translation in *y*-direction.

Static Condensation after SEREP:

Since we have chosen 12 master DOF, SEREP condense stiffness matrix from 72x72 to 12x12. This 12x12 matrix can be further reduced to 3x3 by using static condensation. In 12x12 reduced matrix there will 12 DOF (1 to 12). The master DOF chosen for static condensation are 1,5 and 9 from the 12 DOF.

$$[K] = \begin{bmatrix} 346838 & 12480.7 & -157757 \\ 12480.7 & 100293.7 & -111600 \\ -157757 & -111600 & 247704.6 \end{bmatrix} \text{N/m}$$

$$[M] = \begin{bmatrix} 3.185637 & 0.022244 & 0.30291 \\ 0.022244 & 3.222087 & 0.882224 \\ 0.30291 & 0.882224 & 3.938611 \end{bmatrix} \text{Kg}$$

Table 13: Comparison of natural frequencies between SAP and Static Condensation after SEREP.

By Static Condensation after SEREP		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	11.82273756	1	11.64631	1.51
2	40.24801342	4	37.47586	7.39
3	64.77673263	9	64.95375	0.27

Two Stage SEREP (SEREP followed by SEREP):

Since we have chosen 12 master DOF, SEREP condense stiffness matrix from 72x72 to 12x12. This 12x12 matrix can be further reduced to 3x3 by using again SEREP. The master DOF chosen for SEREP are 1,5 and 9 from the 12 DOF. And, modes chosen are 1,2 and 5 in 12*12 matrix. Reduced stiffness and mass matrix are given by:

$$[K] = \begin{bmatrix} 558764.9321 & 62651.10622 & -314133.037 \\ 62651.10622 & 159809.0405 & -212594.4111 \\ -314133.037 & -212594.4111 & 451087.1 \end{bmatrix} \text{N/m}$$

$$[M] = \begin{bmatrix} 5.348403964 & 4.24E-07 & -7.84E-07 \\ 4.24E-07 & 4.114160724 & -7.57E-06 \\ -7.84E-07 & -7.84E-07 & 5.348418 \end{bmatrix} \text{Kg}$$

Table 14: Comparison of natural frequencies between SAP and SEREP followed by two stage SEREP

SEREP followed by SEREP		SAP		Error%
Mode	Frequency (Hz)	Mode	Frequency (Hz)	
1	11.657	1	11.64631	0.09
2	37.536	4	37.47586	0.16
3	64.974	9	64.95375	0.03

The frequency corresponds to the 1,4 and 9th mode in 3D frame model.

Selection of master and slave DOF: Master and slave DOF should be selected in such a way that the resulting reduced system not lose the properties of the actual system. Master DOF should be chosen so that it is possibly reflect the actual system. For example, the system we considered for analysis has the 72DOF: 36 translational, 36 rotational. We can neglect the 36 rotational DOF and consider them slave DOF as we considered the displacement in *y*-direction. Neglecting them will not change significantly the property of the system. In same manner we can consider the translational DOF in *x* and *z* direction as slave DOF. We considered the 12 translational DOF in *y*-direction as master DOF as it reflects the actual system behaviour in *y*-direction.

5. Conclusions:

Condensation is required to reduce computational effort and to reduce the degree of freedom. It helps in analyzing huge structures with ease. Condensation eliminate unwanted degrees of freedom, which has no impact on the response of structure. Approximate stiffness formulae do not give satisfactory result. The error is very high by using these formulas as there are various assumptions related to each formula and most formulas are formulated keeping 2D structure in focus. But in the present study methodology for condensing 3D frame structure is proposed and validated. Static Condensation is not worked well with the space frame model. It gives value more the actual. It is only accurate for stiffness reduction. Also, choosing master DOF becomes very critical issue in it.

For frame structure, direct condensation from (72x72) to (3x3) is not yielding the accurate result. So, it is alternatively done by first reducing it to the (12x12) by using SEREP, then further reduce it to (3x3) using static condensation and again SEREP. Two stage SEREP shows very close result with very less error. When SEREP is used for multi degrees of freedom, the choice of active degrees of freedom and identification of modes required is very critical. SEREP produces the natural frequencies identical to the original system. A care should be taken to clearly distinguish rotational and translational modes as per requirement.

Disclosures

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