

Numerical Simulation of Bunker Buster Slab under Projectile Impact

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Abstract

In the present investigation, 3-D finite element modelling is carried out to predict the penetration depth and residual velocity of a projectile impacting a target. For the purpose of numerical simulation, target is a concrete slab of planar dimension 5 m × 4 m with an unconfined compressive strength of 50 MPa. This target is considered to be part of protective structure like bunker buster slab. The explicit algorithm of ABAQUS/Explicit® is employed in the simulation of projectile impact. The projectile is modelled as a rigid body and the concrete slab as a deformable body using Lagrangian formulation. The energy dissipation, penetration depth and residual velocities of the projectile impacting at different velocities are compared with the empirical equation as available by simulating a real impact scenario numerically. Investigation is carried out over a wide range of impacting velocities under sub-hydrodynamic range.

Keywords: Rigid Projectile, Concrete, Penetration Depth, Residual Velocity, Energy Dissipation, Impact, Buster Slab

1. Introduction

Concrete has been used as a prime material to resist against impact by projectiles and penetrating missile throughout the world. This impact of projectile is a highly non-linear problem wherein local failure of the structure/component occurs due to high loading rate and localized temperature, which have significant effect on both tensile and compressive properties of the concrete. Atchley and Furr. [1] studied energy absorption and strength of plain concrete subjected to static and dynamic loading and formulated that both compressive and tensile strength increases with increase in rate of loading and then came to a constant value. Special structures like bunkers, nuclear containments, and underground tunnels need to be designed for worst possible loading conditions like penetrating missiles and explosion or combination of both. In this study, authors are concerned with the penetration of a rigid projectile into concrete slab and to analyse for penetration depth, residual velocity after perforation and the kinetic energy dissipation therein. If the kinetic energy imparted by the projectile exceeds the strain energy or internal energy of the target, the target undergoes local as well as global failure. It is important to note that due to rarity of such loading conditions, there are no specific guidelines for design of structures in many parts of the world and hence, need detailed intervention to arrive at indigenous technologies for self-reliance. Various experimental studies, in the past, had been conducted to analyse impact and penetration depth of projectiles and on this basis of many

empirical formulas were introduced. Kennedy [2] presented a review of procedures for analysis and design of concrete structures to resist missile impact wherein he reported comparison and collection of formulae to predict the penetration depth of missile impact in concrete. Forrestal et al. [3] reported dimensionless empirical equation for penetration depth of ogive-nose projectiles into concrete targets. The equation depends on dimensionless constant which depends only on unconfined compressive strength of concrete (f'_c) and is independent of dimension of projectile and its striking velocity. The limitation of this method is that penetration depth computation is accurate only for striking velocity lower than 800 m/s. This puts a limit on the use of this equation for striking velocities higher than 800 m/s. With the introduction of powerful numerical tools like finite element method and advancement in computational techniques, it has become slightly easier to analyse impact problems under varying conditions and compare it with the existing experimental as well as available empirical formulae. Zaidi et al. [4] studied analytical models and methods for predicting penetration, and perforation of concrete and reinforced concrete against hard projectile. Goel [5] gave numerical study of multi-layered structure against ballistic impact by an ogive-nose projectile and inferred that multi-layered armour is more effective in energy dissipation as compared to a single layered armour system. Smith and Cusatis [6] presented numerical analysis of projectile impact on regular strength concrete (RSC),

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high-strength concrete (HSC), and engineered cementitious composites (ECC) using the Lattice Discrete Particle Model (LDPM) and various related parameters were investigated. Xue et al. [7] studied the Corundum-Rubble Concrete (CRC) under projectile Impact experimentally and analytically. They proposed modified Taylor model for prediction of the penetration depth of CRC targets.

Erzar et al. [8] studied ultra-high-performance fibre-reinforced concrete (UHPFRC) under rigid projectile penetration. Zhang et al. [9] conducted high velocity penetration experiments using three different projectiles for varying velocities (from 1000 and 1360 m/s). Penetration depth and trajectory stability were compared and analyzed. Rajput et al. [10] numerically simulated a ballistic impact of ogive nose projectile on thin concrete slab and found comparable results with concrete slab. Hanifehzadeh and Gencturk [11] studied reinforced concrete (RC) and steel-concrete-steel sandwich (SCSS) panels under high-speed projectile impact. Two different modeling methods i.e., smoothed particle hydrodynamics (SPH) and conventional finite element (FE) analysis along with element erosion was applied to study the penetration phenomenon.

Abdel-Kader and Fouda [12] experimentally studied the response of plain concrete subjected to the impact of hard projectiles. The main aim of this study was to understand the influence of compressive strength on concrete. Xu et al. [13] employed finite element procedure to study the normal impact of a projectile on a plain concrete slab. In their investigation, they compared the numerical results with the experimental results considering modes of local damage and penetration depth. Li et al. [14] investigated ultra-high performance fibre reinforced concrete under bullet impact and provided the various design parameters for engineering applications.

Similarly, many authors reported several investigations on the projectile impact by considering various target materials. However, authors focus, in the present study is impact of projectile on concrete target which is part of bunker buster slab. Hence, in this manuscript, authors tried to predict the penetration depth of a rigid projectile impacting a concrete slab at different velocities using FE simulation. Based on this analysis, efforts are made to arrive at minimum slab thickness required to safe guard the structure and residual velocity of the perforating projectile. Further, the deviation from the path of motion of projectile is noted and efforts have been made to monitor the energy dissipation and conservation of energy in this highly non-linear phenomenon.

2. Material Properties

The material behaviour of concrete slab under projectile impact is modelled using concrete damage plasticity model (CDP model) available in ABAQUS/Explicit® [15]. This is a continuum plasticity-based damage model and it assumes two failure mechanisms for concrete i.e. cracking of concrete in tension and crushing of concrete under compression [16]. The compressive behaviour of concrete is formulated through stress-strain curve by use of stress and corresponding inelastic strain. The model was first put forward by Lubliner et al. [17] and they formulated that the failure surface can be analysed with

respect to two hardening variables ε_t^{-pl} and ε_c^{pl} for tensile and compressive equivalent plastic strains, respectively. The material properties used in the present study are reported in Table 1. Further, projectile is considered as rigid body and is assigned the relevant properties to act as rigid material during simulation.

3. Finite Element (FE) Modeling

The numerical modelling is done using ABAQUS/Explicit® [15] and model utilizes encastred boundary conditions on concrete slab along with rigid body constraints to projectile. Herein, concrete is a rectangular target and it is a part of a large protective slab. This is considered for analysing the effect of localized failure condition due to penetration along with scabbing and spalling phenomenon, if any. The dimensions of concrete slab considered are 5 m × 4 m as shown in Fig. 1. Fig. 1(a) shows the CAD model of slab which is developed using deformable solid feature available in ABAQUS/Explicit® [15].

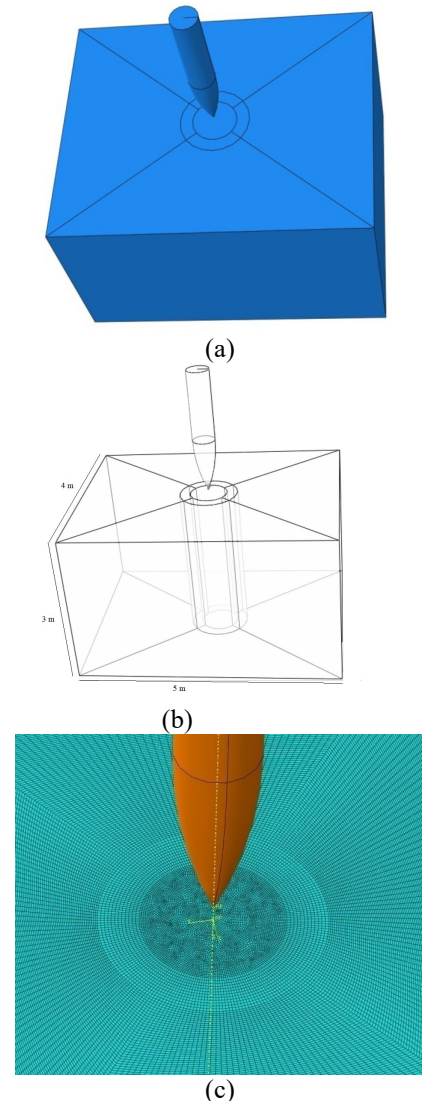


Fig. 1: (a) CAD model of slab, (b) Wire frame of whole concrete slab of 5 m × 4 m dimensions, (c) Meshing of different impact zone and top view of the concrete slab

Table-1: Material properties of concrete used in present FE simulation [18]

Dilation Angle	Eccentricity	f_{bo}/f_{co}	K	Viscosity Parameter
380	1	1.12	1	0.666
Compressive Behavior				
Main option			Sub-option	
Yield stress (MPa)	Inelastic Strain	Rate	Damage Parameter	Inelastic Strain
13	0	1.5	0	0
20	0.0007	1.5	0	7.473x10-5
24	0.001	1.5	0	9.85x10-5
37.5	0.002	1.5	0	0.0001541
22.5	0.0034	1.5	0	0.0007615
16	0.05	1.5	0.195402	0.0025576
			0.596382	0.0056754
			0.894865	0.0117331
Tension Behavior				
Main option			Sub-option	
Yield stress (MPa)	Inelastic Strain	Rate	Damage Parameter	Inelastic Strain
3.5	0	1.5	0	0
1.75	0.00015	1.5	0	3.333x10-5
0.8	0.00035	1.5	0.406411	0.0001604
0.25	0.0006	1.5	0.69638	0.0002798
Young's Modulus(N/m ²)	Poisson's Ratio		Density (kg/m ³)	
3.00x10 ¹⁰	0.2		2400	
Fracture energy (N/m)	Concrete compressive strength		Concrete tensile strength	
100	53		2.1	

Further, this model is divided into three different zones of impact on the basis of direct projectile strike, localized failure and to account for wave propagation. Fig. 1(b) shows wire frame of whole concrete slab of 5 m × 4 m dimensions. Further, mesh density is varied across these three zones and it can be observed from Fig. 1(c) that mesh intensity decreases as we move out from centre or zone of direct impact to the outer periphery (zone three). The concrete slab is modelled as deformable solid and meshed with hex-dominated continuum element (C3D8R) using linear reduced integration [15]. At the zone of direct impact, mesh size is 3mm wherein, in zone two or transition zone, a size of 5 mm is used. The outermost zone is meshed with 8 mm C3D8R as shown in Fig. 1(c). This gradual decrease in mesh size along the zones allows in more accurate result along with low cost computation or CPU time. These mesh sizes are decided based on the mesh convergence study. Moreover, for error free simulation, mesh size should be checked for proper aspect ratio especially in the impact zone to attenuate the danger of excessive distortion compared to wave speed and hence, adaptive meshing may also be adopted in this zone, if required. The first zone of impact has a radius of 0.4 m and the second zone has radius of 0.6

m. The boundary conditions of slab are fully clamped boundary conditions with projectile as rigid body constraints.

FE modelling is done with surface-to-surface contacts between the projectile and the slab to account for the exact behaviour of the target on impact by a high velocity projectile. The deformation of the projectile is neglected in the model and hence, the projectile is modelled as an analytical rigid body which moves through space with respect to reference point on which the translational mass of 907 kg is assigned. The projectile has varying cross section along the length with a diameter of 0.457 m at the neck to 0.43 m diameter at the bottom. Penalty algorithm is used for contact between target and projectile with coefficient of friction being 0.1. Coefficient of friction is considered to be low since the velocity of projectile is considerably high.

4. Results and Discussions

The concrete slab is subjected to impact by a rigid projectile at velocities of 20, 70, 120, 180 and 353 m/s. The concrete slab, considered herein, has an unconfined compressive strength of 53 MPa and a tensile strength of 2.1 MPa. In this analysis, striking velocity time history results shows the impact results and ballistic resistance of concrete in terms of velocity of projectile exiting out of the concrete slab at the back, which is termed as the residual velocity. The ballistic limit of the given concrete slab may be inferred higher if the velocity reduction on impact is higher.

4.1 Comparison of Penetration Depth with the Existing Formulae

Due to the complexities of the impact load and evaluation of its damage on concrete structures, its design criteria, that have been developed so far, are based mainly on experimental data and available empirical formulae. There exist many empirical formulae proposed by various researchers and research institution that can predict the penetration depth against different striking velocity (V_s) on concrete target of varying grades. Although there exist several formulae, authors compared their results with two formulas i.e. Petry's and Chelapati, Kennedy and Wall formula known as (CKW)-BRL formula due to their acceptability amongst researchers by enlarge [19].

Petry's formula (Eq. 1), for penetration depth, is derived from high velocity test conducted on infinitely thick concrete walls [20]. The response to the projectile impact is governed by the dimension and velocity of the projectile, the reinforcement available and the thickness of the concrete wall which governs the mode of failure.

$$X_p = K_m A V' R \quad (1)$$

Here, K_m is material property constant ($= 4.76 \times 10^{-3} \text{ ft}^3/\text{lb} = 2.97 \times 10^{-4} \text{ m}^3/\text{kg}$); A is sectional weight of concrete per unit cross-sectional area of contact (F/L^2); R is thickness ratio and velocity factor, V' is given by,

$$V' = \log_{10} \left[1 + \left(\frac{V_0^2}{V^{*2}} \right) \right] \quad (2)$$

Here, V^{*2} is reference velocity ($= 215000 \text{ ft}^2/\text{s}^2 = 19973 \text{ m}_2/\text{s}^2$).

Using Chelapati, Kennedy and Wall (CKW-BRL) formula, penetration depth can be computed as [19],

Table 2: Penetration Depth as per CKW-BRL, Petry's Formula and Present FE Simulation

Striking Velocity (m/s)	Penetration Depth (m)		
	CKW-BRL Formula	Petry's Formula	FE Simulation
20	0.05	0.01	0.09
70	0.25	0.12	0.32
120	0.50	0.30	0.73
180	0.83	0.53	Perforation
353	2.04	1.10	Perforation

$$X_p = \left(\frac{6Wd^{0.2}}{d^2} \right) \left(\frac{V_0}{1000} \right)^{1.33} \quad (3)$$

Here, W is the weight of missile and d is the diameter of the missile. In the present study, the author has calculated the penetration depth of rigid projectiles on concrete slab with average compressive strength of 53 MPa and tried to correlate it with the penetration depth obtained in numerical simulation. Table 2 shows the penetration depth computed using these two formulas for different striking velocities of the projectile. It can be observed from this table that there exists some variation amongst these two empirical formulas. Further, based on present numerical simulation, penetration depth at projectile striking velocities of 20, 70, 120 and 353 m/s are computed and the same are reported in Table 2. From this table it can be observed that at low velocities i.e. 20, 70 and 120 m/s, numerical simulation results are comparable with the results obtained using empirical formulae. However, at velocity of 180 and 353 m/s complete perforation is observed in present numerical simulation. Fig. 2 shows comparison of perforation depth with the striking velocities considered in the present investigation. It can be observed from this figure that at lower velocities difference between empirical and numerical simulation is small and it increases with the increase in striking velocities. At higher velocities, empirical formulas failed to predict the perforation which is very well captured by the numerical simulation. Thus, it can be said that although empirical formulae provide easy assessment, however, detailed FE analysis helps in to understand the behaviour of such complex phenomena. In the present investigation, CDP material model for concrete is used for analysis but if detailed parameters are available other advanced material model can be employed to arrive at better assessment of penetration depth. This requires detailed material characterization considering the effect of strain rate, which is beyond the scope of present investigation. Nevertheless, CDP model is able to predict the reliable behaviour of the concrete under such situation with detailed availability of other material models.

4.2 Impact Velocity to Residual Velocity of Projectile

Another important parameter is residual velocity of the projectile after its perforation through the target. This residual velocity will decide whether this layer has worked for its intended function or not. For this study, a concrete

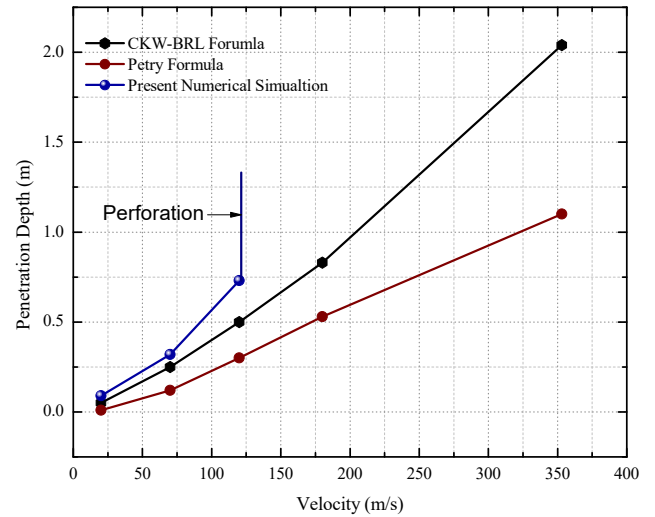


Fig. 2: Comparison between results of numerical simulation and empirical formulae

Table-3: Striking Velocity and Residual Velocity for the Slab Considered in the Present Investigation

Impact Velocity (m/s)	Residual Velocity (m/s)
20	0
70	0
120	0
180	77.6
353	293.4

slab with thickness 3 m has been considered. It is impacted with a rigid projectile at different velocities to study the energy dissipation and residual velocity. CDP model is employed to model the slab as discussed in previous section. Table 3 shows the comparison of striking velocity with the residual velocity of the projectile for the slab considered in the present study. It can be observed from this table that, at striking velocities of 20, 70 and 120 m/s, velocity of the projectile exponentially decays to zero on impact and internal energy of the slab is sufficient enough to arrest the projectile without complete perforation. Further, during simulation it is observed that at 20 and 70 m/s velocities, a rebound velocity is observed on the projectile and the reason for such behaviour may be attributed to that fact at lower velocities the impact may not be considered as a 'plastic' impact. The energy imparted on target due to impact is partially absorbed by the ejected concrete on front forming a spalling crater and the residual energy is not sufficient to create cracking of concrete hence the momentum transferred on to the target get reflected of the end surface which imparts a reverse momentum on to the projectile. Further, at higher velocities i.e., 180 and 353 m/s, it is observed that projectile completely perforate the target with sufficient residual velocity. The kinetic energy of the projectile was non-zero in this scenario. At higher velocities, ballistic limit of concrete slab is well below the striking velocity of the projectile which led to a complete perforation with a residual velocity. Fig. 3 shows the variation of residual velocity with the time for various striking velocities considered in the present investigation.

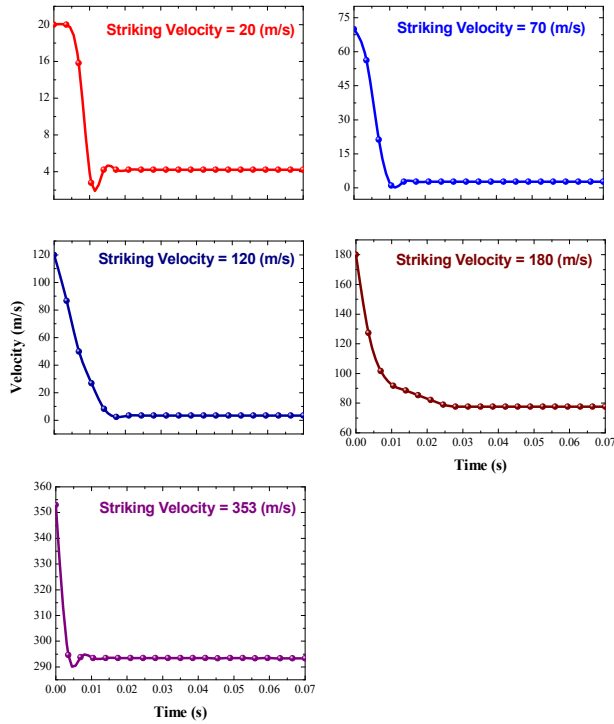


Fig. 3: Variation of residual velocity with the time for various striking velocities considered in this study.

A missile impact is considered to be a highly nonlinear plastic phenomenon in which conservation of energy is followed. For this study, authors assumed that the missile to be rigid such that the strain energy of the missile during impact is approximated to zero. If the missile is deformable a considerable amount of kinetic energy that is required to penetrate the target is lost as strain energy. From Fig. 3, it may be inferred that a fully perforating projectile have a residual kinetic energy while the arrested projectile with velocity equal to zero has kinetic energy tending to zero while the total energy remains constant. Further, from Fig. 3, it also can be inferred that at striking velocity 20 m/s the kinetic energy of the missile isn't enough to impart a higher degree of local damage, which is indicated by a bounce off velocity of 4.5 m/s attained by the projectile in the opposite direction to the motion after impacting the slab.

Further, at 70 m/s the velocity came to zero and a bounce off velocity of 2.5 m/s is observed. It indicates chances of spalling as penetration velocity is not enough to have a cylindrical hole in the slab. When the striking velocity of projectile is 120 m/s, the residual velocity can be noted as zero along with no notable rebound velocity. From this behaviour, it can be inferred that the missile on striking the concrete, will stick on to the concrete slab forming a cylindrical hole of diameter slightly greater than the missile diameter. Further, by going through the striking velocity time history of 180 m/s, the velocity is reduced to 67 m/s at time, $t = 0.03$ seconds (Fig. 3). This final velocity is the residual velocity of the projectile. Similarly, velocity time history of the projectile at striking velocity of 353 m/s, results in a residual velocity of 292 m/s at time, $t = 0.005$ seconds. At striking velocities of 180 and 353 m/s, projectile completely perforated the concrete slab forming a cylindrical hole of diameter slightly greater than the diameter of the projectile (Fig. 3 and Fig. 4).

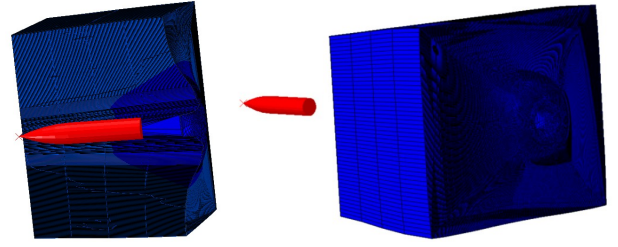


Fig. 4: Penetration and perforation of projectile on impact of concrete slab of thickness 3 m

4.1. Energy Equilibrium Studies

The internal energy of the target should overpower the kinetic energy of the projectile for it to be restricted from complete perforation of the target. Fig. 5 shows energy time history plot wherein, kinetic energy, internal energy and the total energy has been plotted against time for velocities 20, 70, 120, 180, 353 m/s. From Fig. 5, at striking velocity of 20 m/s, it can be inferred that the kinetic energy reduces from 181.4 kJ to 5.87 kJ where it has a rebound velocity of 4.5 m/s and attains a kinetic energy of 8 kJ which is gradually decaying to zero. Further, from this figure, it can be inferred that the kinetic energy is exponentially decaying on impact and is not enough to overpower the internal energy and strain energy of the target and it can be observed that the kinetic energy falls below the strain energy. From Fig. 5, for striking velocity of 70 m/s, kinetic energy got reduced from 2.22 MJ to 3.36 kJ at time, $t = 0.036$ s and it is exponentially decaying to zero. Similar to earlier case, strain energy and internal energy overpowers the kinetic energy of the projectile, and hence the ballistic limit of the concrete slab is not exceeded. At 120 m/s striking velocity, kinetic energy is reduced from 6.5 MJ to 5.3 kJ and all the above said velocity the projectile was unable to perforate the concrete

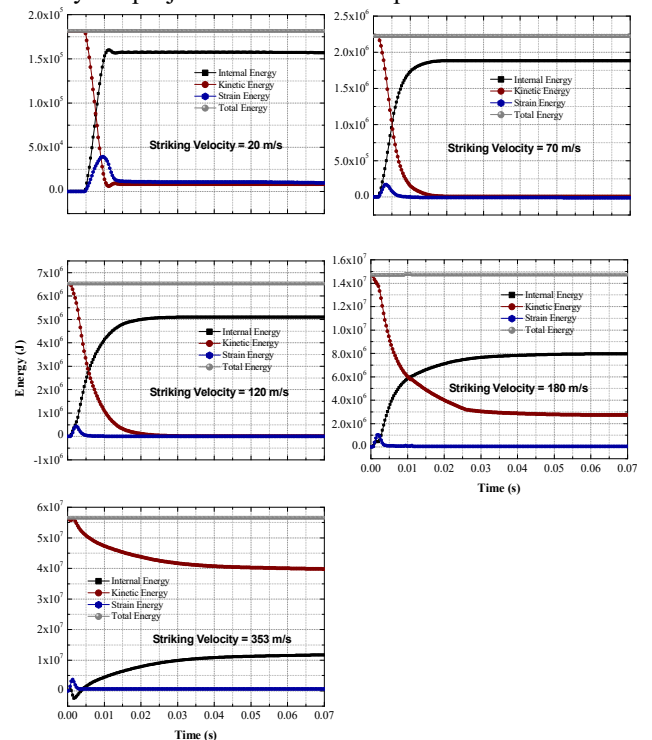


Fig. 5: Variation of energy with time for various projectile striking velocities

slab as can be inferred from Fig. 5. The kinetic energy of the projectile in all the three cases exponentially falls below the internal energy of the target due to dissipation of energy due absorption by the penetration of concrete wall. At velocity of 180 m/s and 353 m/s, it can be inferred from the figure that the kinetic energy dissipation is not strong enough to withstand a complete perforation. The strain energy of the target is well below the residual kinetic energy of the projectile. The kinetic energy of the target fell from 14.69 MJ to 2.73 MJ for 180 m/s and from 56.51 MJ to 39.78 MJ for 353 m/s, which can be observed from Fig. 5. Further, it can be said that the velocity of projectile is more than the ballistic limit of the concrete slab. The projectile at this velocity clearly perforates the target.

5. Summary and Conclusions

Hard or rigid projectile impact results in both local and dynamic global response of the concrete slab. If the kinetic energy of the incident projectile is more than the strain energy capacity of the slab, the target will be perforated and the global response will be in the form of flexural or shear failure. In this study, authors had analysed the impact of rigid projectile at velocities ranging from 20 m/s to 353 m/s. FE simulation is done using ABAQUS/Explicit® [15] and the results showed that at low velocities 20 m/s and 70 m/s, the projectile will strike the concrete slab and bounce off without creating much local damage which can be observed from the velocity time history and it shows a rebound velocity for the projectile. But at a higher velocity of 120 m/s the missile will stick to the concrete slab forming a cylindrical hole of diameter slightly greater than the missile diameter rather than rebounding off the slab. As the velocity of projectile increases beyond 160 m/s i.e. at 180 m/s and 353 m/s, projectile completely perforated the concrete slab with a residual velocity of 67 m/s and 292 m/s.

It was also noted that as the projectile perforated through the concrete slab it deviated from the path in the direction perpendicular to the motion of the projectile (z-axis). Further, it is observed that the internal energy of the target overpowers the kinetic energy of the projectile till velocity 120 m/s and at higher velocity of 180 m/s the internal strain energy of the target is not sufficient enough to dissipate the momentum which causes a complete local failure mechanism causing perforation of projectile with a residual velocity.

Disclosures

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