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# Dynamic Instability Characteristics of Spherical Panels Subjected to linearly varying In-plane Loading

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**Abstract**

The present paper reports the parametric instability phenomenon of spherical shell panels under linearly varying harmonic load employing finite element technique. An isoparametric curved element with eight nodes is considered in the formulation.. The sanders' theory is used to model the spherical panel incorporating the properties of transverse shear and rotary inertia. A computer programme is created based on the formulation to carry out all essential calculations.The correctness of the formulation is established by comparing with results available in the literature.The impact of different parameters such as load factors, shallowness ratio on the instability regions of spherical panels with three support conditions such as SSSS (all edges simply supported), CCCC (all sides clamped) and CFCF (two opposite edges clamped and other two edges free) are studied. The study demonstrates that the parametric instability behavior of the spherical shell panels is significantly influenced by its shallowness parameter, boundary conditions and type of loads. These results will help the designers for deciding the dimensions of the spherical panels exposed to linearly varying periodic load at the time of the preliminary design.

*Keywords*: Dynamic instability, Spherical panel, Finite Element Method, linearly varying harmonic Load.

## 1. Introduction

Spherical panels are extensively utilised as an important structural components in civil, mechanical, aerospace and marine engineering structures. In civil engineering spherical panels are mostly used in construction of roof for a large column free area. People today like architecturally attractive constructions, and spherical curved panels are frequently employed for aesthetic point of view. Spherical panels are more structurally rigid than flat panels because of their geometry.These structures are commonly exposed to a wide range of static and dynamic in-plane stresses in practical applications. When dynamic load acts on a structure, transverse vibration develops in it. The combination of various non-uniform stress along with natural frequency caused by transverse vibration can induce parametric resonance and structural instability. Hence a thorough understanding of the stability criteria of spherical panels under various forms of non-uniform harmonic load and support conditions is highly required for a genuine design of a strong, dependable, and functional structure.

Several researchers have investigated the structural instability due to sudden dynamic load and buckling phenomenon of flat and curved panels exposed to various edge load. Bolotin [1] reported the dynamic stability theory by solving problems of simple resonance and presented the dynamic instability regions using Fourier analysis. Leissa et al. [2] solved the free vibration problem of cantilevered shallow shell using Ritz method of displacement functions. Leissa and Ayoub [3] presented numerical results for static stability and free vibration of a simply supported plate under concentrated forces using Ritz and finite element techniques. Deolasi and Datta [4-5] studied the instability of rectangular plates with partial in-plane edge load both compression/tension and damping effects on simple combination resonance of plates with in-plane patch load employing FEM. Prabhakara and Dutta [6] reported the buckling load and studied the dynamic instability of plates with a cut-out, centrally placed and exposed to tensile and compressive in-plane edge. Matsunaga [7] analyzed the buckling aspects and vibration of shallow simply supported shells using a higher order shallow shell theory of two-dimensional type. Sahu and Datta [8-11] reported the dynamic instability of isotropic, laminated curved panels and studied the effects of cut-outs under partial compressive periodic load employing Bolotin's approach. Ravi kumar et al. [12-13] reported the results for the parametric instability and buckling under tensile load of shell structure and also reported the influence of circular cutout on parametric instability exposed to partial tensile load using FEM. Ravi kumar et al. [14] also reported the parametric instability behavior of laminated flat panel subjected to partial non-uniform follower load. Srivastava et al. [15] used FEM and reported the effects of varied parameters with respect to instability phenomenon of flat stiffened panel with periodic partial compressive edge load. For the first time, Kang and Leissa and Kang

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[16] reported the numerical solutions on buckling of plate under linearly varying load employing method of frobenius. Leissa and Kang, Wang et al. [17-18] presented the exact solutions for critical buckling load and free vibration of an SS-C-SS-C flat panel exposed to linearly varying stresses using Frobenious and differential quadrature method. The stability phenomenon of laminated curved panel under periodic patch load was investigated by Ravi kumar et al. [19]. Udar and Datta [20-22] reported the phenomenon of simple and combination resonance with damping and the impact of circular cutouts on dynamic instability phenomenon of laminated panels under partial and point load using FEM. Girish and Ramachandra [23] studied the non-linear static response and free vibration characteristics of simply supported laminated cylindrical panel using Galerkin method. The dynamic instability behavior for composite hypar and conoidal shells with uni-axial uniform load was reported by Pradyumna and Bandyopadhyay [24] using FEM. Ovesy and Fazilati [25] studied the influence of parabolic load on the DIRs of cylindrical laminated shell adopting finite strip method. Dey and Ramachandra [26] considered Donnels theory and investigated the parametric instability behavior of simply supported circular cylindrical composite panel under partial periodic load. Dey and Ramachandra [27] investigated the behavior of dynamic stability and post buckling of composite laminated cylindrical shell panel under partial edge compressive load using Galerkin's method. Kumar et al. [28] studied the stability phenomenon of a composite conical panel exposed to axial harmonic and static compressive load employing differential quadrature and Bolotin's approach.

The aforementioned literature evaluation reveals that substantial study has been done on parametric instability of flat and shell panels made up of isotropic and composite material under various types of loads, including partial patch load, parabolic and linearly varying in-plane load. However limited research work has been conducted on parametric instability phenomenon of plates exposed to linearly varying loading. On the contrary, to the best of the author's knowledge, no comprehensive work on the dynamic instability behaviour of doubly curved shell panels exposed to linearly varying loads has been found in the literature.

The present study is focussed on a parametric instability analysis of spherical shell exposed to linearly varying harmonic load showing the impact of different parameters including type of loading, shallowness parameter for three boundary conditions (SSSS, CCCC, CFCF) are demonstrated using FEM.. According to the FEM formulation and taking into account the in-plane edge load of the curved panel, a computer code has been developed.

## Theoretical concept and formulation

A spherical curved shell panel with dimension ($a$ x $b$), thickness '$h$' with radius of curvature $R_x$ and $R_y$ subjected to non-uniform harmonic load, as depicted in Fig.1 is considered as the basic problem. The loading function of linearly variable load is formulated as $N_x = -N_0\left(1-\gamma\frac{y}{b}\right)$

Where, $N_x$ is the loading function along x axis. $N_0$ is the compressive load / unit length at $y = 0$. The in-plane load is applied at the edges $x = 0$ and $x = a$ and different load pattern

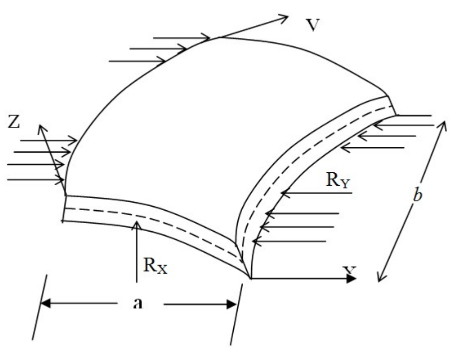

**Fig. 1** A typical Spherical panel with harmonic load

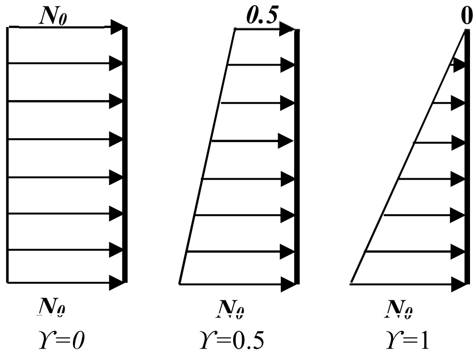

**Fig. 2** Types of edge load along the edges $x = 0$, $a$

such as uniform load ($\gamma = 0$), trapezoidal load ($\gamma = 0.5$) and triangular load ($\gamma = 1$) are obtained by considering different load factor '$\gamma$' as shown in Fig 2. The material is assumed as homogeneous, isotropic and linearly elastic.

### 2.1 Formulation

The equation for dynamic stability of structures is presented as:

$$[M]\{\ddot{q}\} + \left[[K_b] - P\left[K_g\right]\right]\{q\} = \{0\} \quad (1)$$

[M] stands for overall mass and [K] stands for elastic stiffness matrices respectively. $[K_g]$ represents the geometric stiffness matrix produced by the in-plane load $P$. The displacement and acceleration vectors are denoted by the symbols $q$ and $\ddot{q}$.

$P_{(t)}$ stands for the applied harmonic load and comprises of static portion of harmonic load ($P_S$) and dynamic portion ($P_d$)and is given by

$$P(t) = P_S + P_d \cos\Omega t \quad (2)$$

$$P_S = \alpha P_{cr},\ P_d = \beta P_{cr} \quad (3)$$

$\alpha$ and $\beta$ stands for the static and dynamic load factor respectively. The static critical buckling load is denoted by $P_{cr}$.

The following equation is obtained by substituting equations (2), (3), and (4) into Eq. (1).

$$\left[[K_b] - \alpha P_{cr}\left[K_g\right] \pm \tfrac{1}{2}\beta P_{cr}\left[K_g\right] - \tfrac{\Omega^2}{4}[M]\right]\{q\} = 0 \quad (4)$$

Equation (4) is an eigenvalue problem for known values of α, β and $P_{cr}$.

The element matrices for the finite element formulation are found out using the standard procedure as follows:

$$[K_g]_e = \int_{-1}^{1}\int_{-1}^{1}[G]^T[S][G]|J|d\xi d\eta \quad (5)$$

$$[K]_e = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\,|J|d\xi d\eta \quad (6)$$

$$[M]_e = \int_{-1}^{1}\int_{-1}^{1}[N]^T[P]\,[N]|J|d\xi d\eta \quad (7)$$

Where, $[K]_e$ $[K_g]_e$, $[M]_e$ denotes the element elastic stiffness matrix, geometric stiffness matrix and mass matrix respectively. $[B]$ is the strain matrix for the element, $[G]$ is strain matrix for geometric stiffness, $[S]$ denoted as stress matrix, $[N]$ represents the shape function, $[J]$ represents the Jacobian matrix, and $\xi$ and $\eta$ denoted as local natural co-ordinates.

The load vector under linearly variable edge load ($N_x$) at every node of the element can be established as illustrated below

$$\{p_e\} = \int_{-1}^{+1}[N]^T N_x|J|d\eta \quad (8)$$

Here, the load vector is denoted as $\{p\}_e$, $[N]$ is the shape function and $|J|$ is the Jacobean matrix.

In this analysis, an eight node isoparametric curved shell element with five degrees of freedom ($u$, $v$, $w$, $\theta_x$ and $\theta_y$) per node, is considered. In order to solve the eigenvalue problem of equation (4), The FORTRAN programming language is used to create a FEM code. The global $[K]$ and $[K_g]$ and $[M]$ matrices are obtained by assembling corresponding element matrices and stored in sky-line pattern. The equation (5) is solved using the sub-space iteration approach.

## 3. RESULTS AND DISCUSSION

In this analysis, the non-dimensional form of the excitation frequency, $(\Omega) = \bar{\Omega}a^2\sqrt{\rho h/D}$ are obtained and dynamic instability regions (DIRs) are plotted for a spherical (SPH) shell panel with load factors ($\gamma$ = 0, 0.5 and 1.0). Shallowness ratio ($R_y/b$ =5, 10, 15) for three cases of support conditions (SSSS, CCCC and CFCF). In the current analysis, $a/b$ =1, $b/h$ =100, and $v$ = 0.3 are taken into consideration. Here the buckling load is expressed in non-dimensional form ($\lambda$) as [$\lambda=N_xb^2/D$] Where D = $Eh^3/12\ (1-v^2)$, E = $2\mathrm{x}10^{11}$ N/m$^2$ and $\bar{\Omega}$ denoted as excitation frequency in rad/s. $\alpha$ is considered as 0.2 and $\beta$ is taken from 0 to 1 with 0.2 as increment. The detailed support conditions is explained below.

S: $v = w = \theta_y = 0$ (at $x$ = 0, $a$) and $u = w = \theta_x = 0$ (at $y$ = 0, $b$)
C: $u = v = w = \theta_x = \theta_y = 0$ (at $x$ = 0, $a$ and $y$ = 0, $b$)
F: $u = v = w = \theta_x = \theta_y \neq 0$ (at $x$ = 0, $a$ and $y$ = 0, $b$)

### 3.1 Study of convergence and FEM code validation

In case of Finite element solution, the discretization of structure is pivotal for accurate convergence of results. So a convergence study is conducted to find out the **Ω** value for uniformly loaded simply supported curved panel and the results are presented in Table 1.The Ω value are derived for various mesh sizes using the present FEM code and compared with the results given by Matsunga [7]. It can be ascertained that with mesh size refinement, i.e. from 4x4 to 10x10, the value of Ω decreases and converges at mesh size of 10x10. The value of Ω computed from FEM code agree well with the results reported by Matsunga [7]. Thus, the current study uses a mesh size of 10x10, and all subsequent results are computed using this mesh size.

**Table 1.** Convergence study of $\Omega$ for a SSSS doubly curved panel with uniform load ($a/b$ = 1, $a/h$ = 10, $v$ = 0.3, Excitation frequency,$(\Omega) = \bar{\Omega}h\sqrt{\rho/G}$.

| Mesh division | $\Omega$ | | |
|---|---|---|---|
| | CYL ($a/R_x$=0, $b/R_x$=0.2) | SPH ($a/R_x$=$b/R_x$=0.2) | HYP ($a/R_x$=-0.2, $b/R_x$=0.2) |
| 4X4 | 0.094353 | 0.09830 | 0.09274 |
| 8X8 | 0.094261 | 0.098219 | 0.092647 |
| 10X10 | 0.094258 | 0.09822 | 0.092644 |
| Sahu &Datta [8] | 0.09426 | 0.09822 | 0.09264 |
| Matsunga [7] | 0.09436 | 0.09826 | 0.09276 |

For validation of the present FEM formulation and computer code, the dimensionless critical buckling load for SSSS plate under linearly varying load are computed as per the developed computer programme code and compared the results obtained by (Kang and Leissa [16] and Timoshenko [29], From Table 2 It is noticed that the results obtained using the current code are consistent with those found in the past results. Secondly the DIRs for a plate with partial edge load (compression) at one end given by Deolasi and Datta [4] are considered from the available literature and compared with the results from the current FEM code and are depicted in Fig. 3. It is evident that the DIRs obtained using the current code agrees well with past results. As a result, the current FEM code for dynamic stability of spherical panel (SPH) with linearly varying load has been validated.

Thereafter, the study of dynamic instability analysis of spherical panels under load factor (γ = 0, 0.5, 1) considering various parameters is carried out and demonstrated in the subsequent sub-sections.

**Table 2.** Comparison of $\lambda=N_xb^2/D$ of SSSS flat panel with various load factor $\gamma$ and $a/b$.

| $\gamma$ | Method | $a/b$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0.4 | 0.5 | 0.6 | 0.8 | 0.9 | 1.0 | 1.5 |
| 1 | Timoshenko [29] | 149.00 | - | 96.00 | 80.00 | - | 77.00 | 83.00 |
| | Kang and Leissa [16] | 149.50 | 114.70 | 96.16 | 80.26 | 77.66 | 77.10 | 82.59 |
| | Present results | 151.10 | 115.98 | 97.18 | 80.89 | 78.14 | 77.43 | 82.89 |
| 4/5 | Timoshenko [29] | 131.00 | - | 82.00 | 68.00 | - | 65.00 | 70.00 |
| | Kang and Leissa[16] | 113.40 | 99.30 | 82.44 | 68.09 | 65.70 | 65.09 | 70.20 |
| | Present results | 132.30 | 100.03 | 83.02 | 68.44 | 65.95 | 65.26 | 70.36 |
| 2/3 | Timoshenko [29] | 107.00 | - | 70.00 | 59.00 | - | 57.00 | 60.00 |
| | Kang and Leissa [16] | 120.80 | 90.69 | 74.98 | 61.08 | 59.45 | 58.86 | 63.64 |
| | Present results | 121.43 | 91.19 | 75.40 | 61.93 | 59.64 | 58.98 | 63.76 |

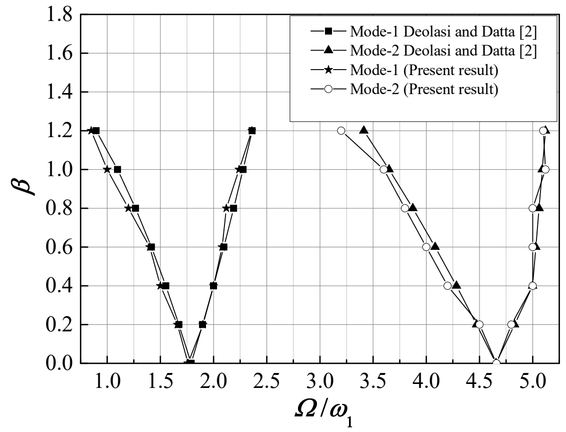


**Fig.3.** Comparison of DIR as obtained by present computer code and available in literature for a plate with partial compressive in-plane load [$a/b$ = 1, $\alpha$ = 0.2, $c/a$ = 0.8]

**3.2 Dynamic instability Studies**

The (DIRs) for spherical curved panels (SPH) subjected to load cases (γ = 0, 0.5 and 1.0), shallowness ratio ($R_y/b$ =5, 10,15) for three support conditions such as SSSS, CCCC, CFCF support are depicted in Fig 4-6, respectively.

**3.2.1 Effect of ($R_x/a$, $R_y/b$ = 5, 10, 15) on DIRs of SPH panel (load case $\gamma$=0, 0.5, 1)**

It is seen from Fig.4-6 that the DIRs starts at a higher $\Omega$ value with small value of $R_y/b$. The DIRs starts shifting to the lower frequency zone with the increase of $R_y/b$ value. It has been noted that for all support conditions taken into consideration here, the $\Omega$ value rises as the shallowness ratio decreases.

The width of DIRs is wider for small value of shallowness ratio, but as $R_x/a$, $R_y/b$ value increases the width of DIRs decreases and maintains a constant value with further increase of shallowness ratio. Further it is revealed that for all the values of shallowness ratio considered here the dynamic instability occurs at higher $\Omega$ value in case of CCCC support and follows the order as CCCC > SSSS > CFCF. This occurs due to the edge restraint and as the restraint at the edges decreases the DIR tend to shift to lower frequency zone.

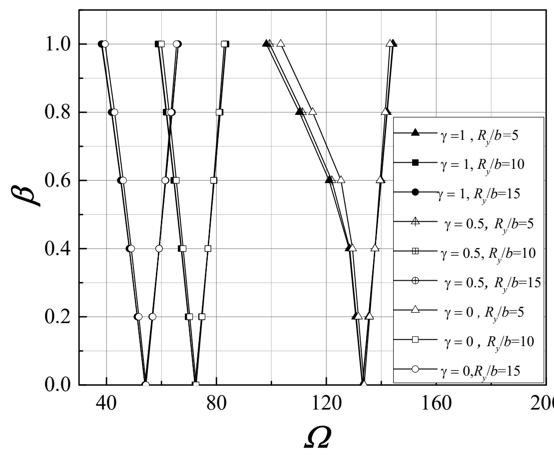


**Fig.4** DIRs showing the effect of $\gamma$ and ($Ry/b$) for SSSS spherical panel

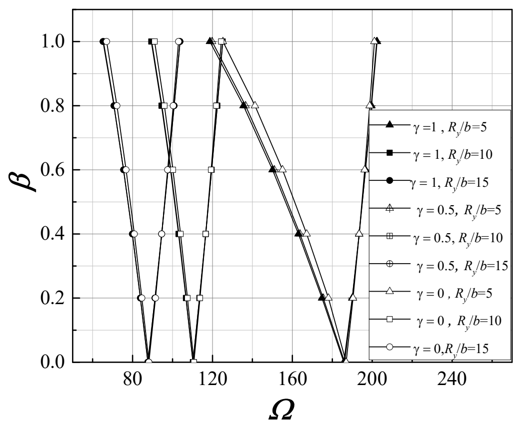


**Fig.5** DIRs showing the effect of $\gamma$ and ($Ry/b$) for CCCC spherical panel

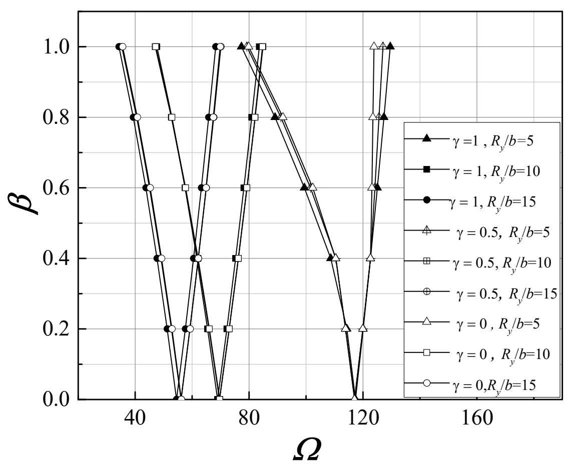


**Fig.6** DIRs showing the effect of $\gamma$ and ($Ry/b$) for CFCF spherical panel

**3.2.2 Effect of support conditions on DIRs for SPH panel ($\gamma$ = 0, 0.5, 1)**

It is evident from (Figs 4-6), that the beginning of DIRs starts at higher value of $\Omega$ for CCCC support and follows the order as CCCC > SSSS > CFCF for all load factors and shallowness ratio considered here. This is caused by the inclusion of simple support and free support at the edges, as well as the edge restraint in the case of clamped support. Also, it is noticed that the width of DIRs takes the order as CCCC > SSSS > CFCF.

**3.2.3 Effect of type of loading ($\gamma$) on DIRs for SPH panel**

It is evident from (Figs 4-6), that the effect of $\gamma$ is insignificant as the occurrence of instability begins at the same value of $\Omega$ for each load case with all shallowness ratio considered here. But the width of DIRs is the highest in case of $\gamma$ = 1 followed by $\gamma$ = 0.5 and 0 for all support conditions with $Ry/b$ = 5 and this effect diminishes as the $Ry/b$ increases.

**CONCLUSION**

The stability phenomenon of spherical panels, with linearly variable harmonic load and three support conditions (SSSS, CCCC and CFCF) can be summarized as follows:

1. For all load factors ($\gamma$) and boundary conditions taken into account in this study, the dynamic instability starts at higher value of $\Omega$ for lower shallowness parameter, i. e. $Ry/b = 5$, followed by $Ry/b = 10$ and $Ry/b = 15$.
2. It is seen that, for all load factors ($\gamma$) with $Ry/b = 5$, the instability starts at higher value of $\Omega$ with CCCC support and takes the order as CCCC > SSSS > CFCF. Further the width of DIRs takes the order as CCCC > CFCF > SSSS.
3. The effect of load factors ($\gamma$) on the initiation DIRs is insignificant for all support condition and shallowness ratios considered. But the width of DIRs for all support conditions with $Ry/b = 5$ follows the order as $\gamma=1$ > $\gamma=0.5$ > $\gamma = 0$.

The aforementioned parametric study demonstrates that the various parameters strongly influence the dynamic instability phenomenon of spherical shell panels under harmonic load. Therefore, the designers must exercise caution while creating a structural system that will be reliable and well functional under harmonic load. The results as obtained can also serve as a pivotal point for future studies. Moreover, these results will help the designers for deciding the dimensions of the spherical panels at the first stage of structural design.

## Disclosures

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**REFERENCES**


1. V.V.Bolotin, *The Dynamic Stability of Elastic Systems* (San Francisco: Holden-Day,1964).
2. A. W. Leissa, J. K. Lee and A. J. Wang, Vibrations of cantilevered doubly curved shallow shells,*Int. J. Solids Struct.*,19(1983) 411-424.
3. A. W. Leissa and E. F. Ayub**,** Vibration and buckling of simply supported rectangular plate subjected to a pair of in-plane concentrated forces*, J. Soundvib.*127 (1988) 155-171.
4. P.J.Deolasi and P.K.Datta, Parametric instability behaviour of a plate subjected to localized in-plane compressive or tensile periodic edge loading, *Comp. Struct.,* 54 (1) (1995) 73-82.
5. P.J.Deolasi and P.K.Datta, Simple and combination resonances of rectangular plates subjected to non-uniform edge loading with damping, Engg.Struct, 19(12)(1997) 1011-1017
6. D.L.Prabhakara and P.K.Datta,Vibration, buckling and parametric instability behaviour of plates with internal opening subjected to in-plane compressive or tensile periodic edge loadings, *Thin-Walled Struct.,* 27 (4) (1997) 287-310.
7. H. Matsunaga,Vibration and Stability of thick simply supported shallow shells subjected to in-plane stresses, *J. Soundvib.,* 225 (1999) 41-60.
8. S.K.Sahu and P.K. Datta, Parametric instability of doubly curved panel subjected to non-uniform harmonic loading, *J. Sound vib.,* 240 (1) (2001) 117-129.
9. S.K.Sahu and P.K. Datta, Dynamic stability of curved panel with cut-outs subjected to non-uniform harmonic loading, *J. Sound vib.,* 251 (1) (2002) 683-96.
10. S.K.Sahu and P.K. Datta, Parametric resonance characteristics of laminated composite doubly curved shells subjected to non-uniform loading, J. Reinf. Plast. Compos.20 (18), 1556-1576
11. S.K.Sahu and P.k datta Dynamic stability of laminated composite curved panels with cutouts.ASCE journal of engineering mechanics.
12. Ravi Kumar, L., Datta, P.K., Prabhakara, D.L., 2002. Tension buckling and vibration behaviour of curved panels subjected to non-uniform in-plane edge loading. Int. J. Struct. Stab. Dyn. 2, 409–423. DOI: 10.1142/S0219455402000579
13. L.Ravi Kumar, P.K.Datta,D.L. Prabhakara, Tension, buckling and parametric instability characteristics of Doubly curved panels with circular cutout subjected to non-uniform tensile edge loadings,*Thin-Walled Struct.*, 42(2004) 947-962.
14. Ravi Kumar, L., Datta, P.K., Prabhakara, D.L., 2003. Dynamic instability characteristics of laminated composite plates subjected to partial follower edge load with damping, j. mech sci 45(2003)1429-1448
15. A.K.L Srivastava, P.K. Datta, A.H. Sheikh, Dynamic Buckling & vibration of stiffened plates subjected to partial edge loading, *Int. J. Mech. Sci.,* 45(2003) 73-93.
16. J. H. Kang, and A. W. Liessa, Exact solutions for the buckling of rectangular plates having linearly varying in-plane loading on two opposite simply supported edges, *Int. J. Solids Struct.,*42(2005) 4220-4238.
17. Leissa, A.W., Kang, J.-H., 2002. Exact solutions for vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses. Int. J. Mech. Sci. 44, 1925–1945. DOI: 10.1016/s0020-7403(02)00069-3
18. X. Wang, L. Gan and Y. Wang,Vibration and buckling of a SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses**,** *J. Soundvib.,*298(2006) 420-431.
19. Ravi Kumar, L., Datta, P. K., & Prabhakara, D. L. (2005). Vibration and stability behavior of laminated composite curved panels with cutout under partial in-plane loads. *International Journal of Structural Stability and Dynamics*, *5*(01), 75-94.
20. Ratnakar.S.udar, P.k datta Dynamic stability of parametrically excited laminated composite curved panels under non-uniform edge loading with damping, *composite structures*. (2007)

21. Ratnakar.S. udar, P.k datta, Parametric combination resonance instability characteristics of laminated composite curved panels with circular cutout subjected to non-uniform loading with damping, *International journal of Mechanical sciences* (2007)

22. Udar, R. S., & Datta, P. K. (2008). Combination resonance instability of curved panels with cutout subjected to non-uniform loading with damping. *Journal of engineering mechanics*, *134*(7), 555-566.

23. Girish, J., & Ramachandra, L. S. (2008). Stability and vibration behavior of composite cylindrical shell panels under axial compression and secondary loads. *Journal of applied mechanics*, *75*(4).

24. S. Pradyumna, J.N .Bandyopadhyay, Dynamic stability behaviour of laminated hypar and conoidal shells using a higher-order shear deformation theory, *Thin walled structures* (2011)

25. H.R Ovesy, J. Fazilati, Parametric instability analysis of laminated composite curved shells subjected to non-uniform in-plane load, *Composite Structures* (2014)

26. Tanish Dey, L.S Ramachandra, Static and Dynamic instability analysis of composite cylindrical shells panels subjected to partial edge loading. *International Jornal of Non-linear mechanics* (2014)

27. Dey, T., & Ramachandra, L. S. (2015). Dynamic stability of simply supported composite cylindrical shells under partial axial loading. *Journal of Sound and Vibration*, *353*, 272-291.

28. Kumar, R., Ramachandra, L. S., & Banerjee, B. (2019). Dynamic instability characteristic of damped laminated composite conical shell panel under periodic axial compression. *Mechanics of Advanced Materials and Structures*, 1-16.

29. S. P. Timoshenko, The stability of the webs of plate girders, *Eng'g,* 138 (1934), 207-209.

30. S. P. Timoshenko and J. M. Gere**,** *Theory of elastic stability* (McGraw Hill,New York, 2$^{nd}$ ed., 1963).

31. Cook RD (2007) Concepts and applications of finite element analysis. John Wiley & Sons.