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Closed-Form Derivation of Optimum Tuned Mass Damper Parameter Based on Modal Multiplicity Criteria

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Abstract

It is presently established that properly designed tuned mass dampers (TMD) systems can effectively control the vibrations induced due to the wind and/or the seismicity. The TMD is one of the most used passive control systems in high-rise structural systems in an era of recent developments. The optimum parameters for the damped system to date require a numerical search technique or shootout technique, which is the trial-error method, of-course add complexity in selecting parameters obtained at optimality conditions. The study of optimum parameters based on modal multiplicity criteria showcases the complete closed-form solution, independent to another optimum parameter. The parameters presented in this study solely depend on structural damping and mass ratio. The derived optimum parameters are compared with the existing parameters in this field. This study also investigates parametric uncertainty for structural configuration under the harmonic excitation. To confirm its robustness under random vibration, various earthquake time history loads are applied to study the effect of structural displacement and acceleration response. In the end, the energy concept is deployed to explain how effectively the optimum TMD parameters help to reduce the kinetic energy, damping energy, strain energy, and input energy.

Keywords: Tuned Mass Damper, Vibration Control, Optimum Parameters, Closed-Form Solution, Robustness, Parametric Uncertainty, and Energy Dissipation

1. Introduction

A very small mass connected between the structural system through an elastic spring and a dashpot is a very effective solution to absorb undesirable energy if tuned properly [1]. The introduction of TMD in the structural systems is quite often in application since Frahm has invented these mechanical devices and made US patent in 1909 [2]. In the recent past, The John Hancock Tower, Funade Bridge Tower, London Millennium Bridge, and CN Tower are the tall structures where TMD has been installed successfully to attenuate the wind-induced vibration [3].

Generally, the mass of TMD is tuned to its first mode of vibration to suppress the unwanted oscillatory response of the system [4]. A lot of studies have been made for tuning TMD with un-damped and damped systems, but the closed-form solution for the damped system to date is unavailable with zero tolerance [5]. The tuning of the device is generally referred to as calibrating or specifically optimizing in most of the practices commonly. The structural application to reduce seismic vibration was begun in 1985 when Villaverde [6] proposed the closed-form derivation for the optimum frequency and the optimum TMD damping through a curve fitting technique. This method of optimizing was equalizing modal frequencies and modal damping in the fundamental mode of vibration with minimal error numerically. Then, Sadek et al. [7] made the US Patent for optimum frequency and the optimum TMD damping for an un-damped and damped system in 1997, using curve fitting techniques. In this method of optimization, the parameters were searched numerically to have equal modal frequency and equal modal damping with minimum tolerances. That was the period when a lot of studies have been carried out to prove the viability of TMD for random excitation [8]. In all those investigations, either a harmonic or a white-noise excitation load has been triggered to the ground and then the response of the structural system is noted. The data were collected and analyzed for minimal response and through curve fitting technique the closed-form derivations were proposed. These all processes required input energy or input triggering force in terms of frequency so the all derivation proposed by all author has to be valid for specific excitation frequency ranges only [9,10]. That was the shortcoming of all proposed optimization process. The seismic vibration generally carries a large number of the harmonic frequencies, when it harmonized with any of the available processes. This fact adds a further complication for judging optimum parameters in case of such a complicated situation. In this circumstance, a large number of solutions are available, which one to choose or how to harmonize is challenging. Those parameters only get simplified, when derived parameters are not dependant on the excitation frequency of the input force [11].

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If such parameters can be derived through free vibration characteristics, then the optimized system parameters may have the least effect on the frequency of applied force. In this study, the hypothesis of modal multiplicity is proposed. The earlier study for the search of optimum parameters has noticed that the modal result may have multiplicity [12].

The specific objective of this study is to (1) derive the closed-form expression for the optimum frequency and the optimum damping of TMD, (2) examine comparatively these parameters under the harmonic excitation, (3) evaluate the reliability of these optimum parameters under parametric uncertainty, (4) compare the optimum parameters under earthquake excitation and (5) validate proposed model through energy concept.

2. Structural Model and Dynamic Equilibrium

The SDOF building model with TMD considered for the present study is shown in Fig. 1.

Under the free vibration, the dynamic equilibrium of the TMD system is shown as,

$$\begin{bmatrix} M & 0 \\ 0 & M_t \end{bmatrix}\begin{Bmatrix} \ddot{X} \\ \ddot{X}_t \end{Bmatrix}+\begin{bmatrix} C+C_t & -C_t \\ -C_t & C_t \end{bmatrix}\begin{Bmatrix} \dot{X} \\ \dot{X}_t \end{Bmatrix}+\begin{bmatrix} K+K_t & -K_t \\ -K_t & K_t \end{bmatrix}\begin{Bmatrix} X \\ X_t \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (1)$$

where $X \,\&\, X_t$ represent the displacement of the main mass and the TMD mass respectively, similarly $\dot{X} \,\&\, \dot{X}_t$ represent relative velocity and $\ddot{X} \,\&\, \ddot{X}_t$ represent relative accelerations of the main mass and the TMD mass respectively.

In equation (1), M & M_t, K & K_t, and C & C_t are respectively mass, stiffness, and damping of the main system & the TMD system. This can be represented into system parameters and can be rewritten as follow,

$$\begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{x}_t \end{Bmatrix}+\begin{bmatrix} 2\xi\omega_0+2\mu\xi_t\omega_0 f & -2\mu\xi_t\omega_0 f \\ -2\mu\xi_t\omega_0 f & 2\mu\xi_t\omega_0 f \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{x}_t \end{Bmatrix}$$
$$+\begin{bmatrix} \omega_0^2+\mu\omega_0^2 f^2 & -\mu\omega_0^2 f^2{}_t \\ -\mu\omega_0^2 f^2 & \mu\omega_0^2 f^2 \end{bmatrix}\begin{Bmatrix} x \\ x_t \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (2)$$

where μ signify mass ratio (M_t/M), ξ is structural damping ratio $C/2M\omega_0$, ξ_t is TMD damping ratio $C_t/2M_t\omega_0$, the fundamental frequency of main mass is $\omega_0=\sqrt{K/M}$, the natural frequency of TMD is $\omega_t=\sqrt{K_t/M_t}$ and f is frequency ratio ω_t/ω_0.

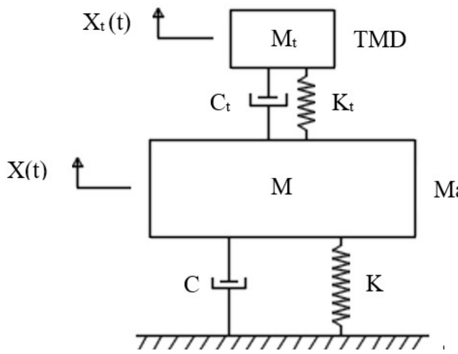

Fig. 1. SDOF system with TMD

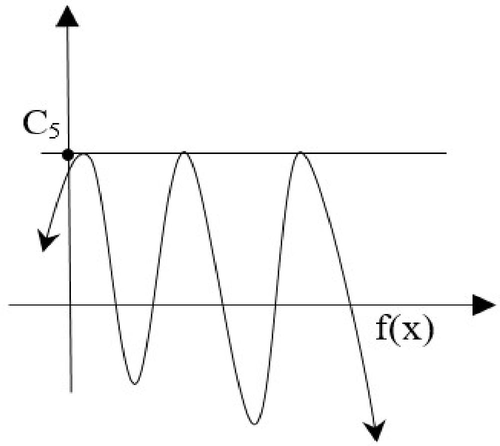

Fig. 2. Typical quartic plot

The characteristic polynomial can be present as follow,

$$\begin{vmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C} \end{vmatrix}=0 \quad (3)$$

where $\overline{M}, \overline{C} \,\&\, \overline{K}$ are respectively mass, damping, and the stiffness matrices and I is an identity matrix. By assuming that there exists a modal multiplicity criterion, the system equation can be express in term of Eigenvalue (λ).

It has to clear that the characteristic polynomial is the order of two-time the size of the mass matrix that means, this will be a quartic equation. The number of real roots of a quartic equation follow the Descartes' rules of the sign. Moreover, the close form roots may be found out by using Euler's Method, LaGrange's' Resolvent Techniques, Gerolamo Cardano Method, Converting to a depressed quartic, and then Ferrari's techniques of factoring into lower-order, etc. [20].

Here the solution has been derived using factoring techniques. To understand modal multiplicity criteria, let's assume, $f(x)=c_0+c_1\alpha+c_2\alpha^2+c_3\alpha^3+c_4\alpha^4=0$ is a quartic polynomial of α, where c_i is a constant positive coefficient of α and i=0 to 4. If this equation is simplifying as $f(x)=(\alpha-c_5)^3(\alpha-c_6)=0$, then it is said as root c_5 has 3 multiplicity and c_6 has unique multiplicity. This criterion provides three times the line intersects the root point. Moreover, if c_5 is a non-negative number that means that it has three real roots. Fig. 2 demonstrates graphically a quartic plot.

The typical behavior of a standard polynomial derived based on equation 3 has a pair of the real and complex conjugate in terms of modal parameters and as per the Descartes rule of the sign. Let's assume a modal parameter is represented in terms of $\alpha_1 \pm i\alpha_2$ and each has two multiplicities. Here α_1 and α_2 are the function of modal frequencies and modal damping. The root of such an equation expressed as,

$$(\lambda-\alpha_1-i\alpha_2)^2(\lambda-\alpha_1+i\alpha_2)^2=0 \quad (4)$$

$$\therefore\ \lambda^4-4\alpha_1\lambda^3+\mathbf{A}\lambda^2-\mathbf{B}\lambda+\mathbf{C}=0 \quad (5)$$

where

$$\mathbf{A} = \left(6\alpha_1^2 + 2\alpha_2^2\right); \mathbf{B} = \left(4\alpha_1^3 + 4\alpha_1\alpha_2^2\right); \mathbf{C} = (\alpha_1^2 + \alpha_2^2)^2$$

After comparing the coefficients of the characteristic polynomial derived from equation (3) and equation (5), it gives four equations under equilibrium, additionally. The α_1 and α_2 can be found out by simultaneously solving any of two equations as,

$$\alpha_1 = -\frac{1}{2}\left(\xi + f\xi_t + f\mu\xi_t\right)\omega_0 \tag{6}$$

$$\alpha_2 = -\frac{1}{2} i\sqrt{\xi^2 - 4f + f(1+\mu)\xi_t\left(2\xi + f(1+\mu)\xi_t\right)}\,\omega_0 \tag{7}$$

If once can follow the procedure briefly explained in the published article of Miranda and Thakur et al. [9,11], the quartic equation of f and ξ_t can found out as,

$$\left(1+f(-2+f+f\mu)\right)\left(1-\xi^2+f(-2+f+f\mu)(1+\mu-\xi^2)\right)=0 \tag{8}$$

$$\begin{aligned}&\xi^4 + 2\xi\left(\mu - 2\xi^2\right)\xi_t + (1+\mu)^2\xi_t^4 = \\ &\mu\xi^2 + \left(\mu + \mu^2 - 6\xi^2 - 2\mu\xi^2\right)\xi_t^2 + 4(1+\mu)\xi\xi_t^3\end{aligned} \tag{9}$$

The equation (8) and (9) are the quartic equation of frequency ratio and the TMD damping respectively. These equations can be solved using factoring techniques to obtain optimum frequency ratio (f^{opt}) and optimum TMD damping ratio (ξ_t^{opt}) in terms of structural damping (ξ) and mass ratio (μ) as,

$$f^{opt} = \frac{1-\xi^2}{1+\mu-\xi^2+\sqrt{\mu\xi^2\left(1+\mu-\xi^2\right)}} \tag{10}$$

$$\xi_t^{opt} = \frac{\xi - \sqrt{\mu\left(1+\mu-\xi^2\right)}}{1+\mu} \tag{11}$$

3. Characteristic behavior and optimum parameters

To study the behavior of optimum frequency ratio (f^{opt}) and optimum TMD damping ratio (ξ_t^{opt}), a unit change in mass ratio is applied for specific structural damping (ξ). The characteristic behavior of the optimum frequency (f^{opt}) and the optimum TMD damping (ξ_t^{opt}) is plotted using equations (10) & (11). The behavior in this section is studied for two damped cases $\xi = 0.03$ & 0.05 (Fig. 3). The plot shows that the optimum frequency ratio is lower at the higher structural damping ratio. While the optimum TMD damping (ξ_t^{opt}) is increasing with increasing mass ratio but for higher structural damping, it required a lower value. The present study required the highest damping ratio which is the same as Sadek et al. [7]. Moreover, the present study has a lesser optimum TMD frequency requirement in comparison with other studies [7, 13, 14]. It is also interesting to note that the parametric behavior of the present study closely matches with Sadek et al. [7].

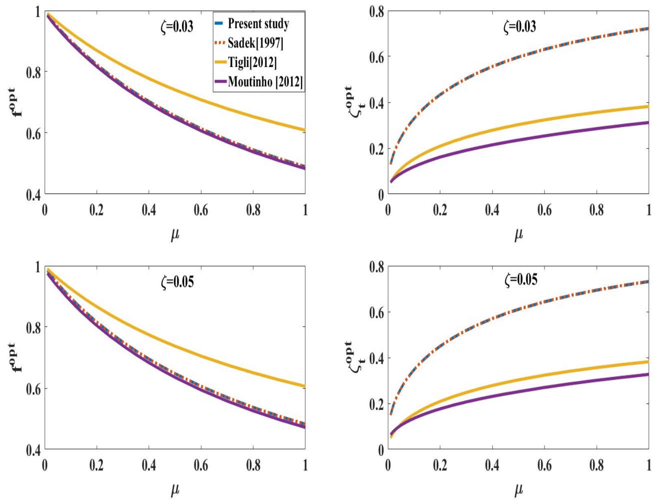

Fig. 3. Effect of mass ratio on optimum parameters

Table-1. Effect on optimum parameters (ξ=0.05)

	Sadek [1997]		Present Study	
μ	f^{opt}	ξ_t^{opt}	f^{opt}	ξ_t^{opt}
0.0001	0.9994	0.0600	0.9994	0.0600
0.0002	0.9991	0.0641	0.9991	0.0641
0.001	0.9974	0.0816	0.9974	0.0815
0.0025	0.9950	0.0998	0.9950	0.0998
0.004	0.9929	0.1129	0.9929	0.1128
0.01	0.9852	0.1490	0.9852	0.1489
0.02	0.9735	0.1890	0.9735	0.1889
0.04	0.9521	0.2442	0.9521	0.2440
0.06	0.9322	0.2851	0.9322	0.2848
0.08	0.9133	0.3185	0.9133	0.3181
0.1	0.8954	0.3470	0.8954	0.3466
0.5	0.6474	0.6107	0.6474	0.6102
1	0.4823	0.7321	0.4823	0.7317
1.5	0.3845	0.7946	0.3845	0.7942
2	0.3279	0.8332	0.3197	0.8328

The earlier study on TMD shows that system needs relatively less optimum frequency at a higher mass ratio [6]. The optimum frequency (f^{opt}) is decreasing with increasing mass ratio, for the same mass ratio at higher structural damping, it has a higher optimum frequency ratio.

Now it becomes essential to discuss all optimum parameters for damped structure. Moutinho [13] has given parameters based on numerical search technique and tabled them in published articles for damped structure for $\xi = 0.01; 0.03$ & 0.05 cases. For the rest of the cases, it is required to follow the numerical search technique given in that published article. However, Sadek et al. [7] has proposed optimum parameters for damped structures as,

$$f^{opt} = \left(1/1+\mu\right)\left(1-\beta\sqrt{1/1+\mu}\right) \tag{12}$$

$$\xi_t^{opt} = \beta/(1+\mu) + \sqrt{\mu/(1+\mu)} \tag{13}$$

The comparison of optimum parameters is showcased in Table-1. The more rational method is to apply a white noise excitation to the structure as a random process, and the response quantity of displacement, velocity, and accelerations of the structure is collected in terms of the frequency of applied forces. This quantity can be minimized through the maxima-minima principle. These optimum parameters, one can derive through classical optimization technique. The response function of displacement is derived under a white noise excitation. This function is optimized for TMD damping & frequency ratio. The simplified expression of these parameters is given by Tigli [14] as,

$$f^{opt} = \frac{\beta\xi(2\xi^2(1+\mu)-\mu)+\xi\sqrt{2(1+\mu)(\xi^2(1+\mu)(2+\mu)+\beta^2(2\xi^2(1+\mu)(\xi^2-2)+\mu))}}{2\xi^2(1+\mu)^2+\beta^2(\mu-4\xi^2(1+\mu))}$$

$$\& \ \xi_t^{opt} = \sqrt{(3\mu^2+4\mu)/(8+8\mu)} \qquad (14)$$

Tsai et al. [15] used a numerical search technique to attenuate peak displacement amplitude for two different harmonic excitations, their outcomes are expressed as,

$$f^{opt} = \frac{\sqrt{1+0.5\mu}}{1+\mu} + \sqrt{\mu}\left(0.288-0.661\sqrt{\mu}+1.12\mu\right)\xi - \sqrt{\mu}\left(2.298-6.739\sqrt{\mu}+8.316\mu\right)\xi^2 + \frac{1}{\sqrt{1-2\xi^2}} - 1 \qquad (15)$$

$$\xi_t^{opt} = \frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{\frac{\mu}{1+\mu}} + 0.151\xi + 0.238\mu\xi - 0.187\xi^2 \qquad (16)$$

The work by Thakur et al. [8] has proposed parameters as shown in equation (17) are dependent parameters. Hence it required either the iterative process or need to solve numerically. The solution strategy of the quartic equations is also given in the published article. This work is basically inspiration behind what was given by Villaverde [6], earlier 2012. This justifies the necessity of the present study for closed-form derivation in independent parameters.

$$f^{opt} = (\xi_t - \xi)/(\xi_t + \mu\xi - \xi) \ \&$$

$$\xi_t^{opt} = \xi(f-1)/(\mu f + f - 1) \qquad (17)$$

Tuned mass damper devices are frequency sensitive devices so tuning to its optimum frequency is of utmost importance. Moreover, at optimum tuning frequency, TMD damping has multiple roots are in existence. The effective TMD damping is always lesser than optimum damping. Fig. 4 shows the effect of TMD damping on the modal frequencies and the modal damping (ω_0=10rad/sec, $\mu = 0.02$, $\xi = 0.02$). This has been plotted using the roots of characteristic polynomial derived from equation 3. The considered modal parameters in terms of $\alpha_1 \pm i\,\alpha_2$ as discussed earlier herein can also be expressed as, $\omega_j\xi_j \pm i\omega_j\sqrt{1-\xi_j^2}$, where j=1 & 2. The roots of the characteristic equation in terms of $\omega_1 \& \omega_2$ and $\xi_1 \& \xi_2$ against TMD damping indicate that as TMD damping is increasing the first mode frequency is increasing and 2nd mode frequency is decreasing. Similarly, the modal damping of both modes increasing as TMD damping is increasing. At particular TMD damping, the modal damping in the first mode decreasing and 2nd modal dramatically increasing. At this specific point, the TMD damping is called effective TMD damping. The TMD damping corresponding to this point is much of important under the

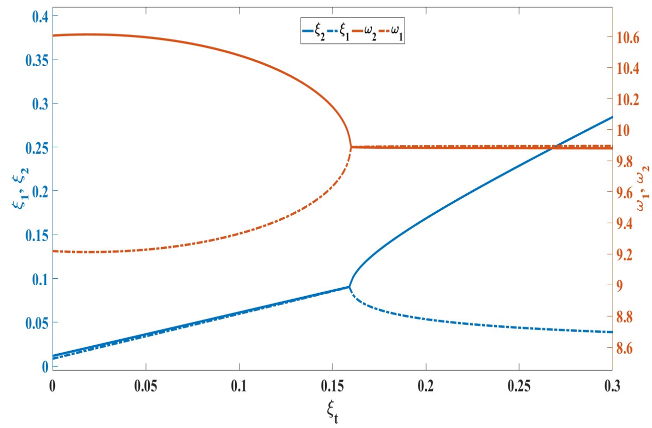

Fig. 4. Effect of TMD damping on modal frequency (ω_1, ω_2) and modal damping (ξ_1, ξ_2)

Table-2. Comparison of effective modal damping ratio

μ	Thakur 2019	Villaverde 1985	Present Study
0.0001	0.0251	0.025	0.0250
0.0002	0.027	0.027	0.0271
0.001	0.0358	0.0358	0.0358
0.0025	0.045	0.0449	0.0450
0.004	0.0516	0.0515	0.0516
0.01	0.07	0.0696	0.0699
0.02	0.09	0.0898	0.0906
0.04	0.1195	0.1176	0.1198
0.06	0.1421	0.1383	0.1422
0.08	0.16	0.1553	0.1610
0.1	0.1777	0.1698	0.1776
0.5	0.372	0.3053	0.3719
1	0.517	0.3685	0.5177
1.5	0.63	0.4012	0.6298
2	0.724	0.4215	0.7245

harmonic excitations. In the present study, the effective damping numerically calculated with a tolerance of 0.00001. Here, the effective TMD damping is 0.159. Table-2 shows a comparative study of effective TMD damping, the earlier study used the optimum TMD to estimate effective damping as an average of structural damping and optimum TMD damping [6].

4. Harmonic excitation

The harmonic excitation on the proposed model gives an idea about the effect of parameters on the condition of optimality. Here the harmonic force $10\sin(\omega t)$ is applied where the frequency of force is taken 10 rad/sec, the same as a fundamental frequency. The structural damping is taken as 3% and mass ratio as 2%. The effect of acceleration response of main mass and TMD mass with an application of sinusoidal load is shown in Fig. 5. The peak acceleration response of the main mass reduces 263%, approximately.

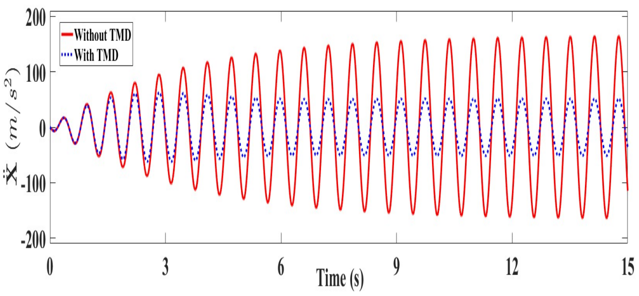

Fig. 5. Effect on Displacement FRF under the harmonic time history

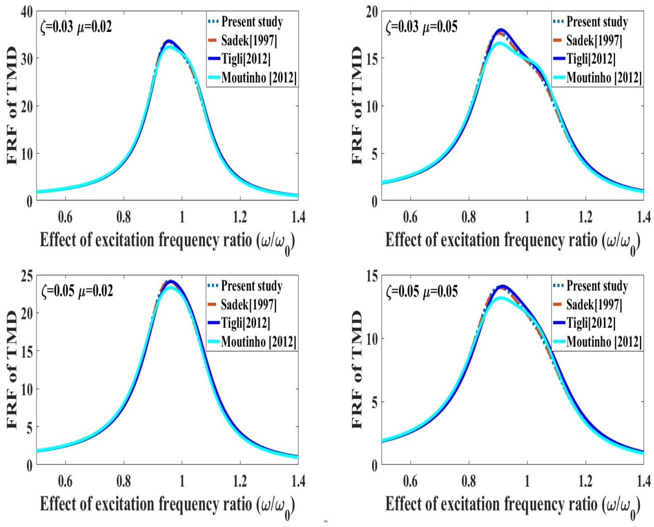

Fig. 6. Effect on the peak displacement FRF of TMD against excitation frequency ratio

Furthermore, the proposed model studies under the harmonic excitation when ground excitation frequency ω, act with the assumed functional load as $\ddot{x}_g = a_0 e^{-i\omega t}$. The fundamental frequency of the model is (ω_0) 10 rad/sec. The peak displacement response of TMD mass for μ=0.02 &0.05 and ξ = 0.03 & 0.05 against excitation frequency ratio presented in Fig. 6. It is substantial that Moutinho [13] model shows the least peak displacement response of the TMD mass for all cases. The proposed model has been compared with Sadek et al. [7], Moutinho [13], and Tigli [14].

It is evident that; the present study shows the least peak displacement response of the main mass. The reduction of peak displacement response for present study in comparison with without TMD is respectively 56%, 36 %, 86% and 55% for {β=0.03, μ=0.02}, {β=0.03, μ=0.05}, {β=0.05, μ=0.02}, and {β=0.05, μ=0.05} cases. The average reduction of peak displacement response is 58.5%. Moreover, the peak displacement response of the present study is par with the authors of the study (Table-3).

Moreover, it is seen from Fig. 7 that by the proposed hypothesis the dispersion of two peaks of the main mass is nearly equal and the peak response closely matches with Sadek et al. [7]. The higher mass ratio provides better damping, hence has a lesser peak displacement response [6,16].

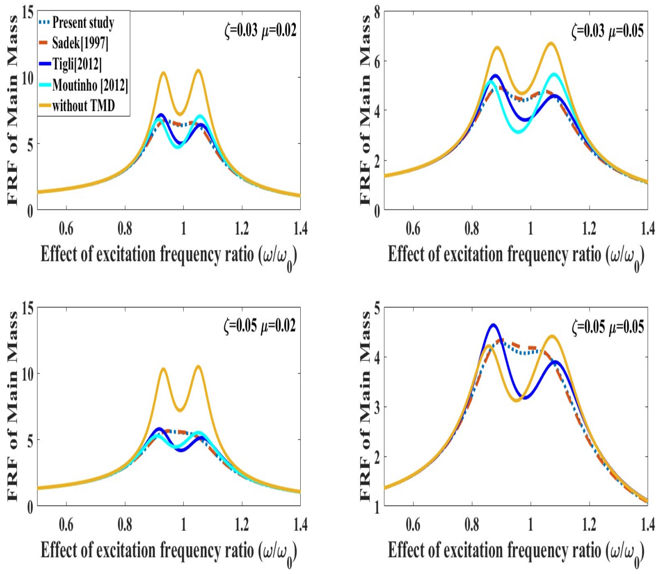

Fig. 7. Effect on Peak Displacement FRF of the main mass against excitation frequency ratio

Table-3. Peak displacement response for the harmonic excitation

		β=0.03		β=0.05	
	mm	μ=0.02	μ=0.05	μ=0.02	μ=0.05
Present Study	X	6.7364	4.9107	5.6292	4.2911
	X_t	33.5635	17.7078	24.1699	14.0398
Sadek [1997]	X	6.7566	4.9168	5.6608	4.3356
	X_t	33.4960	17.6340	24.1376	13.9938
Tigli [2012]	X	7.1591	5.3910	5.8046	4.6336
	X_t	33.6019	17.9784	24.1025	14.0913
Moutinho [2012]	X	7.0603	5.4441	5.5270	4.4083
	X_t	32.3384	16.5686	23.3058	13.1758
W/O TMD	X	10.4950	6.6877	10.4950	6.6877

The optimization processes that are based on forced vibration, generally have different objective functions for displacement, velocity, and acceleration process. Through this process when the minimization of displacement response parameters are used this does not work for acceleration and velocity response or vice-versa. To comprehend the present study work and its relevance, study peak acceleration response is inevitable. Fig. 8 shows the comparative behavior of the peak acceleration response. In that Fig., $\ddot{X}$ represent the peak acceleration response of the main mass. Table-4 shows that the response shown for the present study fairly matches with Sadek et al [7]. Additionally, in comparison with and without TMD cases, the maximum reduction in peak acceleration response is 30.35 % for $\xi = 0.05; \mu = 0.02$ and an average reduction is 17.38%.

Table-4. Comparative study of peak acceleration response (m/sec^2)

ξ	μ	Sadek [1997]	Tigli [2012]	Moutinho [2012]	Present Study	W/O TMD
0.03	0.02	8.1606	8.3778	9.0436	8.1653	9.7951
0.03	0.05	6.4949	6.6206	7.5811	6.5166	6.8678
0.05	0.02	6.8296	7.0087	7.2818	6.8218	9.7951
0.05	0.05	5.6474	5.8635	6.3296	5.6718	6.8678

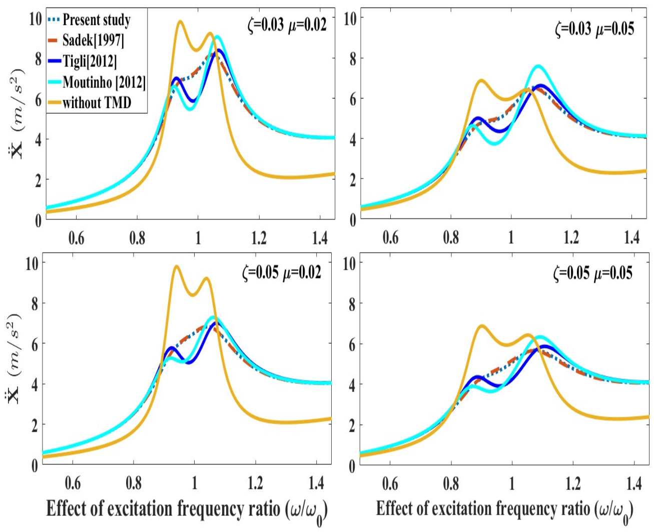

Fig. 8. Effect on peak acceleration response of main mass against excitation frequency ratio

5. Effect of uncertainty on parameters

The applied physics problems are quite intricate and need to simplify for application purposes. Hence, the procedure carries some inborn error. The design process requires rectifying uncertainty, misjudgment, and/or any such errors. To check their consistency, this section illustrates the parametric uncertainty. The proposed hypothesis aims to establish the closed-form derivation based on modal multiplicity criteria. The work in the following segment is studied for $\pm 5\%$ frequency uncertainty. Furthermore, $\pm 10\%$ the uncertainty of mass ratio and structural damping ratio is showcased independently and interactively.

5.1 Frequency uncertainty

The derived parameters are compared with + 5% and - 5% of frequency change with the earlier study. Fig. (9) & (10) illustrated the peak displacement response of TMD and the main mass respectively. It is evident that the proposed model has the least effect on frequency uncertainty (Table-5) and has consistently least displacement response for TMD and main mass.

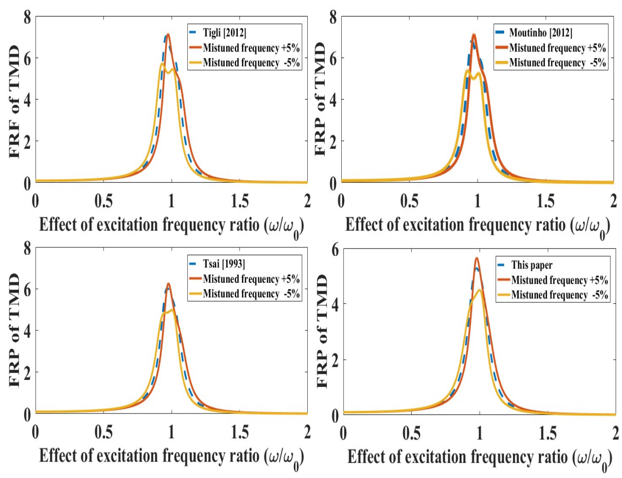

Fig. 9. Effect of frequency uncertainty on peak displacement FRF of TMD mass

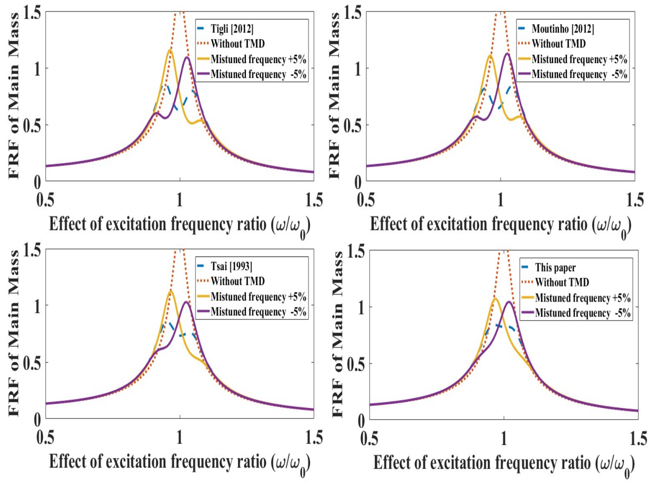

Fig. 10. Effect of frequency uncertainty on peak displacement FRF of Main Mass

Table-5. Effect on displacement response of frequency uncertainty

		β=0.03, μ=0.01			
mm		Tigli [2012]	Mountinho [2012]	Tsai [1993]	Present study
Author	X	8.6616	8.3914	8.5741	8.3876
Result	X_t	70.4471	68.0016	60.3431	52.8424
Mistuned	X	11.5804	11.0989	11.2365	10.6928
Freq. +5%	X_t	71.4203	71.2187	62.6280	56.5702
Mistuned	X	10.9349	11.2602	10.2970	10.4182
Freq. -5%	X_t	57.2399	53.9043	49.8522	45.0304
W/O TMD	X	16.6741	16.6741	16.6741	16.6741

Table-6. Effect of mass and structural damping uncertainty on displacement response

ξ	μ				
	10%	5%	0%	-5%	-10%
10%	5.69	3.85	2.10	0.43	-1.16
5%	4.53	2.74	1.03	-0.59	-2.15
0%	3.41	1.67	0.00	-1.59	-3.10
-5%	2.33	0.63	-1.00	-2.55	-4.02
-10%	1.29	-0.38	-1.96	-3.47	-4.91

5.2 Mass ratio and structural damping ratio uncertainty

Table-6 demonstrates the mass and damping ratio uncertainty in the prediction of displacement response. It is seen that if the mass is uncertain with that case has the highest positive error of 3.41 % and a negative error of 3.10%. While in case of miscalculation of structural damping has a maximum positive error of 2.10% and negative error of 1.96%. Furthermore, if uncertainty is considered in both cases $\pm 10\%$, then the maximum positive error is 5.69% and the maximum negative error is -4.91%. This justifies the consistency of the proposed derivation, please note that maximum deviation is applied as $\pm 10\%$.

6. Earthquake excitations

To have a perception about the present study to mitigate the peak displacement response in the case of earthquake excitation, the three seismic time histories have been chosen to demonstrate the behavior of the proposed model. The essential properties of recorded seismic time-histories are shown in Table-7.

It is comprehended from Table-8 that the peak displacement response of TMD for the present study model has the least magnitude. The average displacement response is 27.31% lesser than the average magnitude of both authors.

Table-7. Time history essentials

Earthquake Name	Date	Mag-nitude	Station Name	Record /Component	PGA (m/s^2)
Kobe, Japan	16/01/1995	6.9	Abeno	RSN1100 /90	2.29
Kocaeli, Turkey	17/08/1999	7.51	Afyon Bay	RSN1146 /90	0.146
Northridge USA	17/01/1994	6.93	Camarilo	RSN958/ UP	0.476

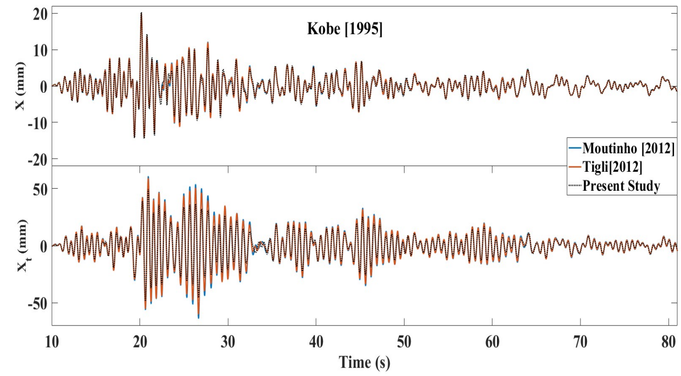

Fig. 11. Comparative displacement response (Kobe 1995)

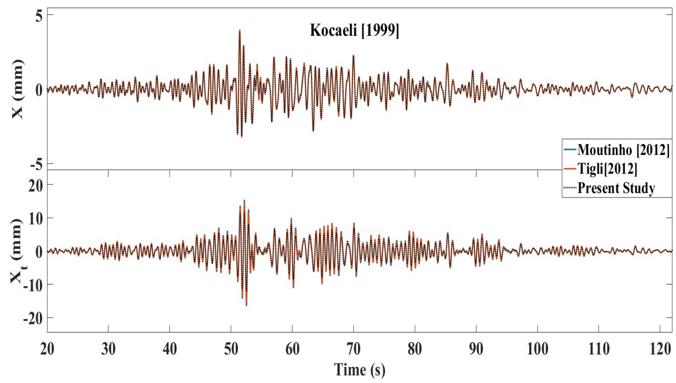

Fig. 12. Comparative displacement response (Kocaeli 1999)

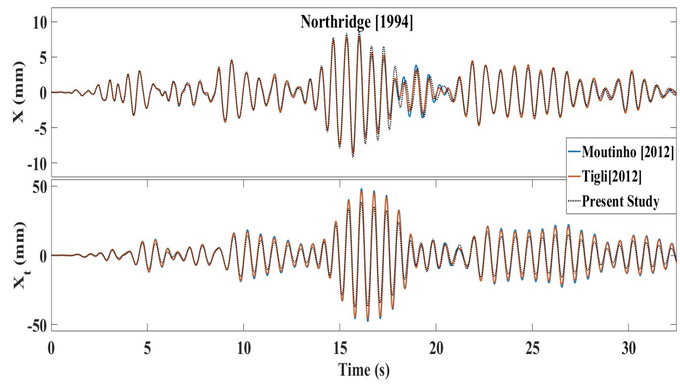

Fig. 13. Comparative displacement response (Northridge 1994)

Table-8. Peak displacement response under seismic excitation

Name of Earthquake	(mm)	$\mu=0.02$, $\omega_0=10$ rad/s, $\xi=0.03$		
		Moutinho [2012]	**Tigli [2012]**	**Present Study**
Kobe, Japan [1995]	X	20.0205	19.9784	20.2713
	X_t	63.7766	60.0740	49.2829
Northridge [1994]	X	8.3100	8.5000	9.1000
	X_t	48.4400	46.7700	38.3000
Kocaeli, Turkey [1999]	X	3.9993	4.0055	3.8899
	X_t	16.5395	15.5980	12.1740

While the average displacement response of the main mass is around 1.46% higher than the average displacement response of all authors.

The author of this paper has studied various time-histories, out of those three time-histories have been chosen. The chosen time history shows the minimum response of the main mass for a specific model only and each author has the minimum response for the specific time-history. The results are shown in Table-8, under the Kobe, Kocaeli, and Northridge earthquake excitation respectively. Tigli [14], Present study and Moutinho [13], show the least peak displacement response of the main mass (Fig. (11), (12), (13)).

7. Energy Concept

During the event of random vibration like an earthquake, the Input Energy (IE) caused by the ground motion gets converted into various forms depending upon types of mechanism. The energy-dissipating devices mainly dissipate energy through Kinetic Energy (KE), Strain Energy (SE), and Damping Energy (DE). The optimized TMD parameters provide the maximum energy reduction during such activity. The maximum reduction of energy will give an actual idea of its effectiveness. From the earlier study on dynamic energies show expressions as,

$$\mathrm{KE} = \frac{1}{2}\dot{Y}_t^T \bar{M} \dot{Y}_t \tag{18}$$

$$\mathrm{DE} = \int_0^t \dot{X}^T \bar{C} \dot{X} dt \tag{19}$$

$$\mathrm{SE} = \frac{1}{2} X_t^T \bar{K} X_t \tag{20}$$

$$\mathrm{IE} = \int_0^t \ddot{Y}^T \bar{C} \dot{X}_g dt \tag{21}$$

The expression from 18 to 21 is the basic equations for energy study on TMD [17]. Where $\bar{M}$, $\bar{C}$ & $\bar{K}$ are Mass Matrix, Damping Matrix, and Stiffness Matrix respectively. X & Y represent deformation response of the structure and absolute deformation respectively at time t, that means $Y(t) = X(t) + X_g(t)$. The X_g represent input ground displacement. The $\dot{Y}$, $\dot{X}$ and $\dot{X}_g$ represent absolute velocity, velocity response, and ground velocity motion. The $\ddot{Y}$, $\ddot{X}$ and $\ddot{X}_g$ represent absolute acceleration, acceleration response, and ground acceleration. It is to be clear from the above expression that KE and DE are independent of time. That mean Kinetic Energy and Strain Energy are instantaneous value only.

For the understanding purpose in this study, one harmonic time history ($10\sin(10t)$) and an earthquake time history (Kobe-1995) have been taken. The single degree of freedom (SDOF) system is compared with the TMD system. For the harmonic loading, the properties were chosen as M=100 Kg, μ=0.02, ξ=0.03, and ω_0=5 rad/sec. To study the energy reduction, the earthquake time history of Kobe (1995) has been taken, for M=10000 kg, μ=0.02, ξ=0.03, and ω_0=5 rad/sec. Fig. 14 shows a comparison of energy response with and without TMD under Kobe [1995] time-history. The TMD parameters here are taken as optimized parameters. There is a significant reduction of Kinetic Energy, Strain Energy, Damping Energy as well as Input Energy.

Table-9. Maximum energy dissipation

	Harmonic Time History			**1995 Kobe earthquake**		
Energy (KJ)	**w/o TMD**	**w/ TMD**	**% drop**	**w/o TMD**	**w/ TMD**	**% drop**
KE	23.74	4.12	82.66	1.45	0.54	62.63
DE	20.38	2.02	90.07	2.47	0.18	92.81
SE	23.16	2.05	91.16	1.20	0.17	85.46
IE	47.92	14.29	70.18	2.57	0.75	70.75
KE+DE +SE	47.88	5.29	88.96	2.56	0.62	75.88

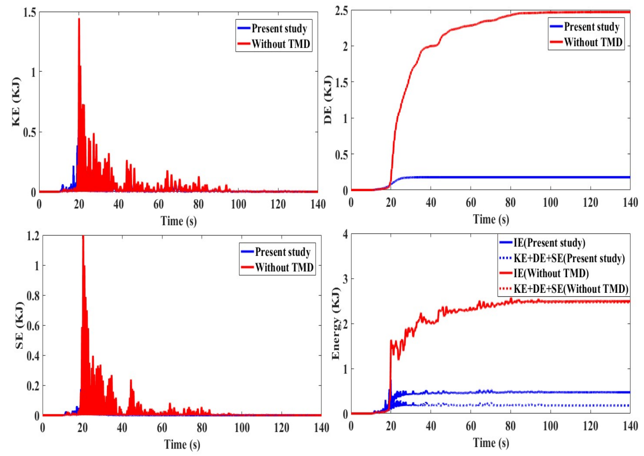

Fig. 14. Energy time history (Kobe 1995)

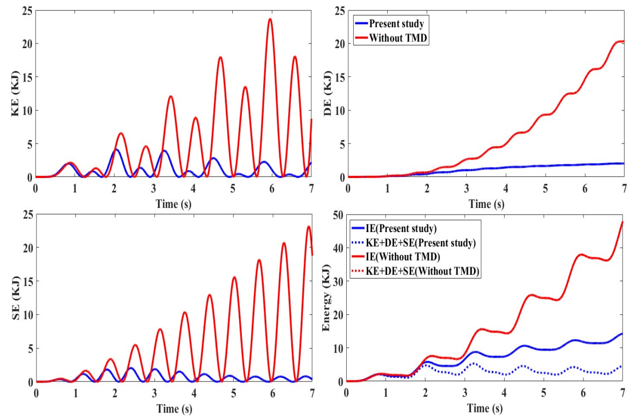

Fig. 15. Energy time history (Harmonic load)

The energy response of the present study under the harmonic excitation has a clear indication of a significant drop in energy (Fig. 15).

It is evident from Table-9 that the maximum reduction of kinetic energy, damping energy, and strain energy for the harmonic loading is 82%, 90%, and 91% respectively. The reduction of kinetic energy, damping energy, and strain energy are 62%, 93%, and 85% respectively when Kobe (1995) ground acceleration has been imposed on the present study model. Moreover, the present study reduces input energy about 70% in both cases, a harmonic and an earthquake (Table-9). The results validate the energy conservation principle, the sum of KE, DE, and SE is equal to IE for a case of without TMD. The energy is 48 kJ and 2.5 kJ for the case of a harmonic and an earthquake excitation respectively. This result also validates the authenticity of the present study.

8. Discussion

The proposed hypothesis aims to derive the closed-form parameters which are only dependent on structural properties (section 1, 2). The derived expression should be a ready reckoner and handy. The derived closed-form parameters match with the earlier study (section 3). The earlier study shows there must be unique parameters for the displacement response and acceleration response, moreover, those are sensitive to the frequency of applied forces. The proposed parameters control displacement and acceleration response optimally by using the same parametric expression (section 4). They must have the least effect on parametric uncertainty like frequency deviation, mass and/or structural damping ratio deviation (section 5). Also, they must produce the least peak displacement of the TMD mass under random excitation. The classical optimization process only aims to minimize the displacement response of the main mass. During this process sometimes the displacement of TMD mass also increases uncontrollably, in the present study there aren't any such issues [19].

In this study, the rigorous procedure was carried out to obtain closed-form parameters. The procedure was idealized based on the earlier study carried out by the researcher. Villaverde [6] has first noticed there exist modal multiplicity criteria at optimality conditions. Later these things were confirmed by other authors [12], [13], and [8]. Finally, the energy concept is presented to show how efficiently the optimum parameters reduce the resonating amplitude and, how faster it attenuates the transient response to improve the overall dynamic performance of the TMD system (Fig. 14, 15).

9. Conclusions

The various conclusions drawn from the present study are

[1] The hypothesis for modal multiplicity criteria for closed-form optimum parameters proposed and derived the optimum tuning frequency & the optimum damping of the TMD.

[2] The derived optimum parameters are only dependent on structural properties (μ and ξ) and independent to excitation frequency of input force.

[3] The derived parameters produced a relatively lesser optimum tuning frequency ratio, but higher optimum TMD damping ratio. This distinctive behavior makes TMD less frequency sensitive devices.

[4] The derived parameters are unique for the displacement response and the acceleration response.

[5] The present study has the least peak displacement response of the main mass under the harmonic excitation.

[6] It is concluded that the frequency uncertainty has the least impact on the prediction of peak displacement response of TMD and main mass.

[7] The miscalculation of parameters, like mass ratio, damping ratio and both have minimal effect on displacement response calculation.

[8] Under earthquake excitation, the derived parameters give the maximum reduction in the peak displacement response of the structure.

[9] Significant reduction of Input Energy (IE), Kinetic Energy (KE), Damping Energy (DE), and Strain Energy (SE) has been observed for the harmonic and the seismic excitation, for proposed optimum parameters. Additionally, it attains a steady-state response rapidly.

Disclosures

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