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Dynamic Response Control of Stiffened Plate with Hole in Stiffener: A Novel Concept of Additional Open Branched Stiffeners

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Abstract

Modern day structures have found diverse applications of stiffened plates in civil, mechanical, aerospace, marine and offshore engineering. The stiffeners, mainly in the form of beams in these stiffened plates are often exposed to unfavourable environmental and service loads, inducing localized damage to the stiffeners. Sometimes, holes are deliberately made on the stiffeners for passing service pipes, cables etc. Both of the events cause localised loss of stiffness, finally affecting the global dynamic performances. One of the practised techniques to recover the lost stiffness and to safeguard the damaged site is to introduce enclosed prismatic stiffeners around the hole. However, this often fails to critically readjust the localised stiffness loss. A constructive alternate to alleviate such situation, is to provide open branched configurations of additional stiffeners within the near vicinity of the damage. The present research work deals with the demonstration of the suitability of the above solution through numerical modelling of dynamic behaviour of a rectangular isotropic stiffened plate with a central longitudinal stiffener using finite element software ANSYS. The damage, represented equivalently as a circular cut-out in the stiffener, is considered to be located arbitrarily. Free vibration characteristics viz. natural frequencies and mode shapes of the plate in undamaged and damaged conditions have been determined by solving eigenvalue problem employing Block Lanczos algorithm. Various configurations of open-branched stiffeners around the circular cut-out, placed at specified locations have been explored. The sensitivities of relevant geometric parameters, such as the distance of the branching from the damage site, orientations of the branching etc. indicate that it is possible to design a system of open branched stiffeners around a damaged site of a stiffener to keep the overall dynamic responses practically unchanged. Such idea of branched stiffener is novel and has potential practical applications towards damage mitigation of various engineering infrastructures.

Keywords: Stiffened plate, Free vibration, Natural frequencies and mode shapes.

1. Introduction

Stiffened plates and shells constitute various structures belonging to numerous modern day engineering fields such as aerospace, civil, naval architecture etc. They hold distinct advantages over their unstiffened counterparts in terms of global stiffness properties leading to their better static and dynamic responses. Because of their greater applications, the stiffened plates generally become more susceptible to damage as they often get exposed to structural/environmental loadings, leading to mostly localised losses of stiffness. Also there are several instances where, holes of various shape and size need to be made deliberately to satisfy various serviceability requirements. This may affect the global dynamic response of the main structure which the stiffened plate constitutes. To counter these damages one of the most common practices involve placing of regular enclosed configurations of additional stiffeners around the damage site. But when the damage occurs on the main stiffener, imposing enclosed stiffeners around the site often leads to considerable under-stiffening of the whole stiffened plate. Thus alignment of the stiffeners dictate the dynamic behaviour of the whole plate. Although most common and general practices are to employ regular stiffener distributions, however applications also extend towards arbitrary orientations of the stiffeners in designing of stiffened plates. Because of the current research work requires branching of additional stiffeners originated from another, it is sensible to explore previous research works on arbitrary orientations of the same as well. Kirk 1970 [1] has computed first symmetric and anti-symmetric modes analysing the free vibration responses of centrally stiffened isotropic plates via a Ritz formulation. Troitsky 1976 [2] has provided a complete assessment of the static bending, free vibration and in-plane buckling behaviour of isotropic stiffened plates. Aksu and Ali 1976 [3] have employed finite difference method to present free vibration response of a centrally stiffened plate. Olson and Hazell 1977 [4] have experimentally and theoretically studied the free vibration characteristics of integrally machined rib-stiffened aluminium plates of regular orientations along with

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experimental verifications. Mukherjee and Mukhopadhyay 1986 [5] have presented a comprehensive review on free vibration response of isotropic stiffened plates for both arbitrary and regular orientations of stiffeners. There came numerous research works concentrated on eccentric placements of straight stiffeners especially for isotropic rectangular plates since then. Mukherjee and Mukhopadhyay 1988 [6] have proposed a first order shear deformation formulation of isotropic arbitrarily oriented stiffened plates of arbitrary boundary using an isoparametric quadratic plate bending element. Koko and Olson 1992 [7] have analyzed free vibration of rectangular stiffened plate using super finite element needing only a single element each for plate and stiffening beam segments. Bedair and Troitsky 1997 [8] have employed sequential quadratic programing technique on strain energy functional represented by in-plane and out-of-plane displacement functions to study free vibration characteristics of concentric and eccentric simply-supported stiffened plates. Barik and Mukhopadhyay 2002 [9] have produced a new stiffened plate element to study free vibration response of arbitrary stiffened plates which can take care of arbitrary orientation of stiffeners on arbitrarily shaped plates. Peng et al. 2006 [10] have provided a mesh-free Galerkin formulation for vibration and stability analyses of arbitrarily oriented stiffened plate. Curvilinear stiffened plates have also been explored by several researchers which can be considered as an extension of the concept of arbitrarily oriented straight stiffeners. Tamijani et al. 2010 [11] have presented the Ritz formulation of curvilinear stiffened isotropic plates for analysing free vibration response with experimental validation. Phillips 2012 [12] explored practical usage of retrofitting with multiple regular enclosed stiffeners around the damage sites of a ship, through complex underwater repair. Minh et al. 2015 [13] explored static and free vibration analyses of eccentrically stiffened folded plates using a cell-based smoothed discrete shear gap method. Shi et al. 2015 [14] have employed quadratic isoparametric finite elements based on first order shear deformation theory in the formulation of dynamic and stability responses of stiffened plates with both concentric and eccentric curvilinear stiffeners. Cho et al. 2016[15] carried out free vibration analysis of stiffened panels with lumped mass and stiffness attachments using assumed mode method as an alternative of conventional FEM. Qin et al. 2017 [16] have presented an iso-geometric analysis based on non-uniform rational B-splines (NURBS) to study free vibration, linear and nonlinear static deformation and in-plane pre-stressed free vibration response behaviour of curvilinear stiffened plates. Liu et al. 2020 [17] have presented an experimental verification of free vibration analysis of curvilinear stiffened plates along with employing discrete shear triangle (DST) and Timoshenko beam elements to numerically model plate and curvilinear stiffeners respectively. The present research work can be considered as an extension of the concept of arbitrarily oriented stiffeners. Here undamped free vibration responses of a rectangular isotropic central longitudinally stiffened plate with and without a single hole, which is placed arbitrarily in the central longitudinal stiffener only and nowhere else on the plate planform, have been studied first. The insertion of this kind of a hole on the main central stiffener may be required according to various serviceability requirements viz. passing of ducts, cables etc. It affects the global dynamic response of the whole stiffened plate. In order to closely restore the dynamic response of the original stiffened plate, additional stiffeners around that hole, placed arbitrarily in the form of branching configurations are implied on the central stiffener only and nowhere else on the plate planform.

In this paper hole on the stiffener has been taken as a circular cut-out, the centre of which has been aligned arbitrarily along the longitudinal centreline of the main stiffener. Because of a hole on the central stiffener, corresponding loss of mass affects the bending stiffness crucially, which eventually is reflected on the significant reduction of fundamental frequencies and thus causing significant changes in overall global dynamic behaviour. Also because of mass-stiffness interplay, introduction of additional mass on the damaged stiffener in order to restore global dynamic response can prove to be altogether detrimental if proper adjustments are not made.

The objective of the current work is to retrofit the central stiffener only where a circular cut-out is introduced, with additional open branched stiffeners around the hole in order to bring the fundamental frequency of the stiffened plate closer to the undamaged one. While doing so, any notable changes affecting the global dynamic response in terms of modal behaviour has been kept away by suitably modifying the branching alignments. Thus, these newly added open branched stiffeners are needed to be aligned on the central stiffener following a critical evaluation of some important parameters defining their suitable alignments. Simultaneous comparison with the performances of the enclosed square and circular stiffener configurations are performed as well. It may again, be duly noted that both the cut-out and the additional branched and enclosed retrofitting stiffeners are placed on the central stiffener only and nowhere else on the plate. Although, there exists possibilities of employing non-parallel branching with non-prismatic stiffeners in restoration of dynamic behaviour, current research work has been restricted to parallel prismatic open branched stiffening distributions only. After an initial arbitrary selection of governing branching parameters, undamped free vibration analysis has been performed, followed by comparing the obtained frequencies and mode shapes with the undamaged state. Consequent sensitivity studies of the aforementioned parameters towards the modal responses are worked out. SOLID186, a 20-noded quadratic 3-dimensional brick element from ANSYS 18.2 element library has been selected to perform the numerical experimentation in 'Workbench Mechanical' environment of ANSYS 18.2. Block Lanczos algorithm for extracting eigen values and corresponding eigen vectors and has been used to determine the undamped frequencies and mode shapes.

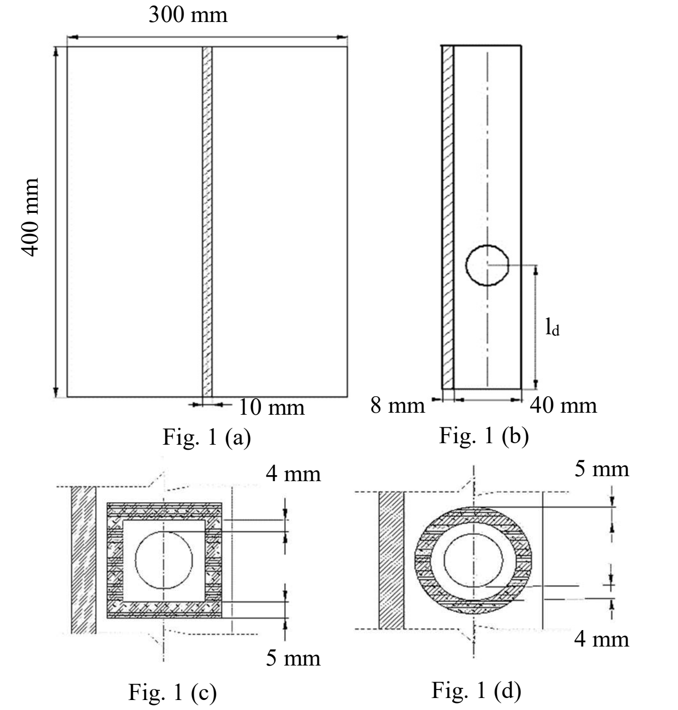

Fig. 1. (a) Schematic top view of undamaged stiffened plate; (b) Schematic side view of damaged stiffened plate; (c) Square enclosed stiffener; (d) Circular enclosed stiffener.

2. Numerically Simulated Examples

Three numerical problems based on a rectangular single stiffened plate of dimension 300 mm x 400 mm have been solved numerically, schematic plan of which is shown in Fig. 1 (a). Material used for all problems is structural steel having Young's modulus E = 200 Gpa, Poisson's ratio **ν** = 0.3 and mass density **ρ** = 7850 kg/m^3. Main longitudinal stiffener cross section is taken as 40 mm x 10 mm; whereas all other branched and enclosed stiffener cross-sections are 5 mm x 5 mm. Thickness of the plate is 8 mm. The diameter of the cut-out has been taken as half of the total depth of the central stiffener i.e. 20 mm and the centre of which has been placed along various locations on the longitudinal central line of the main stiffener. The distance of the centre of the cut-out from the transverse free end of the main stiffener has been referred as 'l_d' in Fig. 1 (b).

In numerical examples tackled, simply supported boundary conditions have been imposed on the plate boundary. Additional open branched and enclosed stiffeners have been applied around the cut-out and their performance in retrieving the modal response have been studied. In the first example centre of the cut-out has been placed at at one quarter length from transverse free end following Fig. 1 (a). The second and the third problems tackle the cases where the cut-out is located at an arbitrary length and the central point measured from the transverse free end of the main stiffener respectively. Also in all the three numerical examples, placement of the enclosed stiffeners with respect to the centre of the cut-out around the cut-out has been kept unaltered, schematic view of which has been presented in Fig. 1 (c) and Fig. 1 (d). Since the objective is to study the effectiveness of the alignments of the open branched configurations, depth of all additional branched and enclosed stiffeners have been taken as 5 mm for all the problems tackled.

2.1 Example 1. Single Stiffened Plate with Cut-out Placed at a Quarter Length of the Main Stiffener

In this numerical example, hole represented by the circular cut-out in the stiffener has the centre, placed at a distance 'l_d' = 100 mm as in Fig. 1(b). The simply supported centrally stiffened plate has been analysed first for both undamaged and damaged conditions for undamped free vibration response. Next, additional enclosed stiffeners in the form of square and circular configurations have been imposed around the cut-out and free vibration response of these configurations have been performed and compared with damaged and undamaged ones.

Next the damaged central stiffener is retrofitted with newly introduced open branching configurations denominated as OBS (Open Branched Stiffeners). Various parameters governing size, shape and alignment of a representative configuration containing two OBSs, have been explored in Fig. 2. Each of the parallel stiffeners, orthogonal to the longitudinal centreline of the main stiffener has been referred as 'stem' and additional parallel stiffeners attached to the stems at different angular orientations are ascribed as 'branches'. The angles are specified as branch angles (**α**, **β**) which also play a key role to define the branch lengths as all the branches are extended to the longitudinal free end of the main stiffener. The stems of OBS-A and OBS-B are of equal length (l_s) measured from the longitudinal free end of the main stiffener, are placed at distances of d_1 and d_2 respectively from the centre of the cut-out as shown in Fig. 2. Also the parallel branching is performed in such a way that it divides the stems in equal segments afterwards. For the present case, stem lengths are taken as 25 mm while values of 'd_1' and 'd_2' are kept equal as 15 mm. Also the branches originating from each stem are aligned at angles 84° and 75° respectively for OBS-A and OBS-B. The subsequent results of undamped free vibration are presented in Table-1.

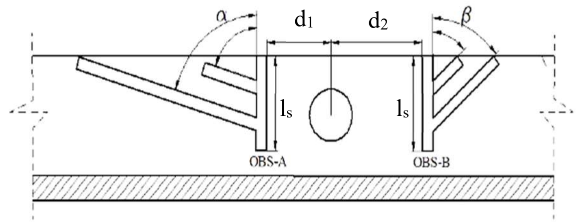

Fig. 2. Schematic side view of OBS configurations with governing parameters.

Table-1. Comparison of natural frequencies (Hz) of enclosed and OBS configurations in Example.1.

Mode No.	Undamaged	Damaged	Circular	Square	Branched
1	611.47	602.93	603.29	604.05	610.34
2	948.86	948.72	948.77	948.38	944.16
3	1346.7	1347.7	1346.7	1346.6	1347
4	1478.7	1468.6	1469.3	1470.7	1470.5
5	1879.9	1880.7	1878.2	1877.9	1872.1
6	1960.1	1957.3	1959.9	1959.1	1959.5

It can be observed from Table-1 while comparing the modal frequencies of damaged and undamaged configurations, the most significant alteration in their overall dynamic performances happens due to the significant changes in fundamental frequencies. Both the additional enclosed stiffeners fail to efficiently address the problem which can be seen in Table-1, leaving the structure potentially under-stiffened. But the OBS distribution obtained from simple variation of the aforementioned parameters yields considerably better outcome than its enclosed counterparts.

While obtaining the final OBS configuration with two parallel branches coming out from each stem, only a few trials have been performed and their modal performances are recorded. The Y-Z plane view of the trials and corresponding frequencies are presented in Fig. 3 and Table-2 respectively.

Table-2. Natural frequencies (Hz) obtained from different configuration trials for obtaining final OBS

Mode No.	Trial-I	Trial-II	Trial-III	Trial-IV	Trial-V
1	602.73	608.1	610.72	614.81	610.34
2	948.06	946.24	944.62	942.91	944.16
3	1347.5	1347.5	1348.2	1345.9	1347.
4	1469.1	1475.9	1476.1	1477.9	1470.5
5	1877.4	1874.1	1869.8	1870.5	1872.1
6	1958.7	1961.1	1961.5	1959.4	1959.5

Fig. 3 (a). Trial-I

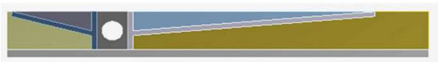

Fig. 3 (b). Trial-II

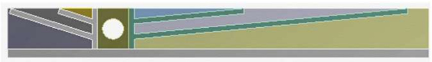

Fig. 3 (c). Trial-III

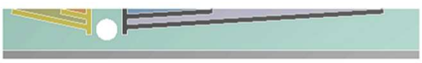

Fig. 3 (d). Trial-IV

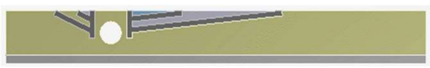

Fig. 3 (e). Trial-V

Fig. 3. (a-e). Configuration trials for obtaining final OBS

Table-3. Variations of governing variables of Example 1 regarding final configuration and sensitivity study

	d_1 (in mm)	d_2 (in mm)	l_s (in mm)	α	β
Sensitivity Study	15-150	15-60	25-40	60°-86.5°	60°-79°
Final Configuration	15	15	25	84°	75°
Optimum Range	15-65	15-30	25-30	81°-85°	72°-76°

As seen from Table-2, initial trials based on single OBS are not able to provide satisfactory output even with increased branch lengths, which has prompted the introduction of OBS configurations with dual parallel branches providing better control over the restoration of overall dynamic performance of the stiffened plate. A parametric study of the governing variables has been conducted and the variations of the same are listed in Table-3.

From the present analysis it can be concluded that unlike enclosed additional stiffeners, OBS are not needed to be placed at close proximity to the damage location. This can prevent causing impairments while retrofitting in practical scenarios. Also increased branch length governed by branch angles '**a**' and '**b**', can lead to over-stiffening of the structure if proper adjustments are not made. Lastly, reduced stem-length 'l_s' imparts better response than the higher or full stretch to base of the main stiffener because of the increased eccentricity of overall OBS with respect to the plate mid-plane. In order to validate the concept, further studies have been conducted as described below.

2.2 Example 2. Single Stiffened Plate with Arbitrarily Placed Cut-out on the Main Stiffener

In this example circular cut-out has been placed at an arbitrary location ('l_d' = 27 mm with respect to Fig. 1) prompting suspension of installation of the OBS-B. Parametric details of the final configuration of OBS-A is produced in Table-4 and resulting retrofitted structure has been presented in Fig. 4 along with its enclosed counterparts.

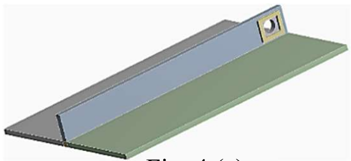

Fig. 4 (a).

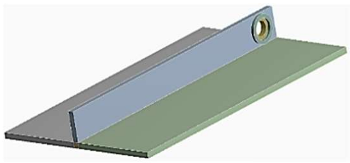

Fig. 4

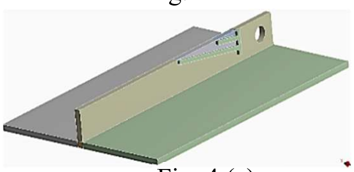

Fig. 4 (c).

Fig. 4 (a). Square enclosed retrofitted plate; (b). Circular enclosed retrofitted plate; (c). OBS retrofitted plate.

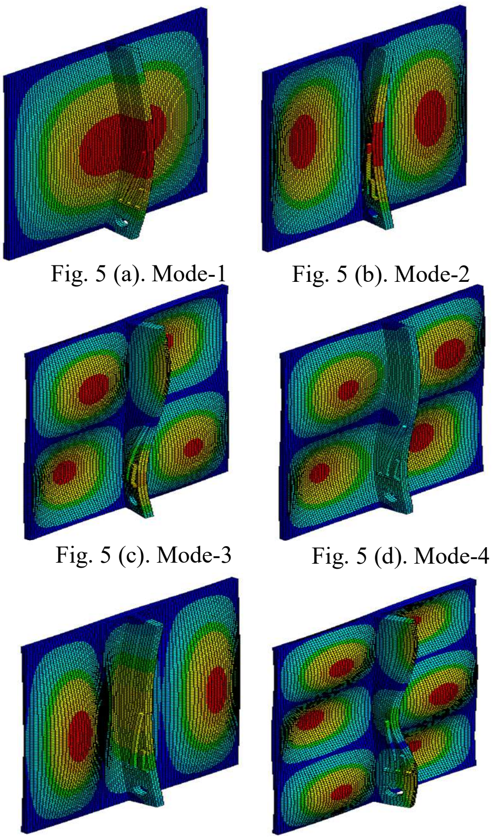

Fig. 5 (a). Mode-1 Fig. 5 (b). Mode-2

Fig. 5 (c). Mode-3 Fig. 5 (d). Mode-4

Fig. 5 (e). Mode-5 Fig. 5 (f). Mode-6

Fig. 5. (a-f). First six mode shapes of OBS retrofitted stiffened plate.

Table 4. Governing parameters of OBS in Example 2

d_1 (in mm)	l_s (in mm)	a
45	30	81.5°

Modal frequencies corresponding to OBS and enclosed configurations are listed in Table-5. The mode-shapes obtained after incorporating OBS-A have been presented in Fig. 5.

Table-5. Comparison of natural frequencies (Hz) of enclosed and OBS configurations in Example.2.

Mode No.	Undamaged	Damaged	Square	Circular	Branched
1	611.47	606.54	606.88	606.87	611.14
2	948.86	947.78	948.66	948.79	945.22
3	1346.7	1345.6	1346.4	1346.5	1345.3
4	1478.7	1464.3	1465.4	1465.3	1467.3
5	1879.9	1878.9	1878.6	1878.6	1874.6
6	1960.1	1960.4	1959.8	1959.7	1962.2

It can be observed from current analysis that retrofitting with a single OBS configuration placed away from the hole which is situated even at an arbitrary location can produce better restoration of original dynamic response compared to enclosed regular stiffeners imparted at closer proximity to the cut-out.

2.3 Example 3. Single Stiffened Plate with Centrally Placed Cut-out on the Main Stiffener

In this example main stiffener has been introduced to a centrally placed circular cut-out whose centre is at 'l_d' = 200 mm according to Fig. 1 (b). Modal frequencies of the damaged plate, presented in Table-7 show the difference on global dynamic response with the undamaged plate is most among the problems encountered till now. To overcome the lost stiffness additional square and circular enclosed stiffeners are imposed around the damage location following the placement specifications mentioned earlier and corresponding modal frequencies are presented in Table-7. It can clearly be observed that the effects of enclosed retrofitting in countering the changes in global dynamic response is fairly nominal. Next, additional OBS configurations i.e. OBS-A and OBS-B, are presented following the specifications mentioned in Table-6 and modal frequencies corresponding to the same are presented in Table-7. The retrofitted stiffened plate with OBS have been presented in Fig. 6 (a). Although overall dynamic response gets improved significantly by retrofitting the damaged stiffener with OBS configurations compared to the performance of enclosed ones as observed from Table-7, it fails to overcome the effects of lost stiffness by a notably good margin.

Table-6. Governing parameters of OBS in Example. 3

d_1 (in mm)	d_2 (in mm)	l_s (in mm)	α	β
15	15	25	84°	84°

Table-7. Comparison of natural frequencies (Hz) of enclosed, OBS, and modified OBS configurations in Example.3.

Mode No.	Damaged	Square	Circular	OBS	Modified OBS
1	597.59	599.59	598.01	605.83	611.11
2	950.32	948.55	948.8	944.4	943.91
3	1342	1345.1	1346.4	1340.9	1346.6
4	1469.1	1469.8	1470.1	1480.7	1482.8
5	1882.1	1876.8	1877.2	1870.4	1877.1
6	1959	1960.5	1960.4	1961.1	1966.9

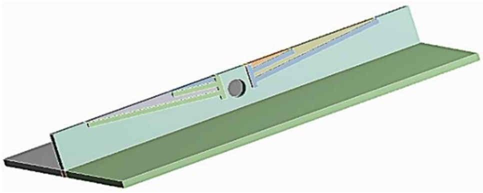

Fig. 6 (a).

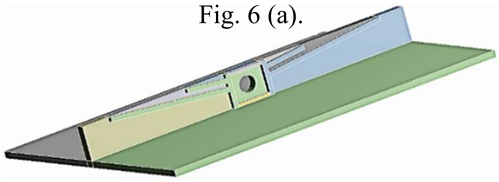

Fig. 6 (b).

Fig. 6 (a). Retrofitted plate with general OBS; (b). Retrofitted plate with the modified OBS.

Lastly, another configuration has been proposed where the stems of the OBS configurations are extended to the base of the main stiffener (i.e. ‘l_s’ = 40mm) without altering any other branching specifications mentioned in Table-6. The stems of OBS-A and OBS-B are then connected to each other with two parallel stiffeners of a depth 3.5 mm, symmetrically placed from the centre of the cut-out while reaching the longitudinal free-end and base of the main stiffeners as shown in Fig. 6. This branching configuration is referred as ‘Modified OBS’ and Modal frequencies corresponding to the same are presented in Table-6. This configuration, effectively representing an enclosed one with additional branching, successfully cater for the local stiffness loss towards closely restoring the global dynamic response of the undamaged plate. Thus, in this critical example also, positive effects of additional OBS retrofitting in damage mitigation is established. Thus, in the numerical experimentation it has been successfully established that placing open branched stiffener around the cut-out, both of which are on the central stiffener only, can improve the overall dynamic response of the stiffened plate structure. In immediate future, effects of non-parallel and non-prismatic configurations in branching can be explored. Also, the effects of various boundary conditions on OBS retrofitting and installation of curvilinear branches as OBS can be considered as immediate extension of this study.

3. Conclusions

In the current research work, numerical experimentations have been carried to study dynamic behaviour of longitudinally single stiffened plate with and without a hole on that main stiffener. The hole is represented by a circular cut-out centre of which lie arbitrarily along the centre line of the main stiffener. Next, additional open branched and enclosed stiffeners are added around the cut-out, exclusively on the central stiffener. While the stiffener is damaged this way, it severely understiffens the whole structure. Retrofitting the main stiffener with open branching configurations tackle this problem better than the enclosed ones. Moreover, these additional branched stiffeners are not necessarily needed to be put at close proximity to the damage site to restore the dynamic performance to an intimate extent of the undamaged state. Thus unlike the enclosed stiffeners, this provides the practising engineers additional relief by not affecting the damage site further during the retrofitting process in real life. This concept of retrofitting can efficiently be applied to various civil, aerospace, naval and off-shore industrial infrastructures.

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