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Seismic Response of Two-way Asymmetric building with Semi-Active Stiffness Damper Under Bi-Directional Excitations

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Abstract

This article present a seismic response of a two-way asymmetric building installed with a semi-active stiffness damper (SASD). The seismic response analysis of asymmetric structure is studied in terms of peak responses subjected to bi-directional excitations. The responses are obtained by numerically solving the governing equation of motion using state-space method. The effect of stiffness ratio and lyapunov constant on peak response which includes lateral, torsional displacement, and acceleration. The responses of asymmetric controlled and uncontrolled systems are simulated through MATLAB. To study the effectiveness of dampers, the controlled response of the asymmetric system is compared with the corresponding uncontrolled response. The parametric study indicated that the semi-active stiffness damper is quite effective in reducing various responses. Furthermore, on the optimum value of stiffness ratio, effectiveness of semi-active stiffness damper is reduced for lateral and torsional acceleration.

Keywords: Asymmetric building; Bi-directional excitation, Semi-active Stiffness Damper,Seismic performance, Optimum.

1. Introduction

Recently, several sizeable earthquakes have caused severe damage to civil structures all over the world. Conventionally, structures are designed to resist dynamic forces through a combination of strength, deformability, and energy absorption. The structural asymmetry is due to the disproportionate distribution of the mass and/or the rigidity of the different structural elements. Due to the several sizeable earthquakes, these kinds of asymmetric structures were damaged in the past. Asymmetry in a plan, buildings have often been thought of as being undesirable because of their vulnerability to earthquakes due to coupled lateral and torsional motions. The eccentricity that is generated by an uneven distribution of mass and/or rigidity should be avoided to minimize risk. However, due to challenging design and practical criteria, it is not possible to eliminate the eccentricity of the structure, so in such a situation, the use of energy dissipation devices is recommended to mitigate structural damage. Until a few years ago, the majority of research studies on the behaviour of the plan asymmetric buildings were subjected to using unidirectional earthquake motions. Previously, the efficacy of passive control, active control, and semi-active control of asymmetric systems under the uni-directional earthquake had been studied through numerous experiments. Jangid and Datta [1] investigated the dynamic response behavior of a torsionally coupled system with multiple tuned mass damper and found that the effectiveness of multiple tuned mass damper in controlling the lateral response of the torsionally coupled system decreases with the increase in the degree of asymmetry. Goel [2]investigated how supplemental damping affects the edge deformations in asymmetric-plan systems and found that maximum reduction in edge deformation obtain when the supplemental damping is distributed as far away from the center of supplemental damping. Date and Jangid [3] investigated the effectiveness of active control systems for one storey coupled structures and found that an active control system to be more effective for laterally flexible torsionally coupled structures in comparison to laterally stiff structure. Lin and Chopra [4]investigated the responses of one-way asymmetric-plan, one-storey systems with non-linear fluid viscous dampers attached in series to a linear brace. It was found that structural response is weakly affected by damper non-linearity and is increased by a small amount due to bracing flexibility. It was shown that nonlinear viscous damper is more effective in reducing responses that linear viscous damper with reduced damper forces. Mevada and Jangid [5] investigated the seismic response of torsionally coupled system with variable damper and found that the semi-active variable damper is quite functional in reducing lateral-torsional displacement. Kori and Jangid [6] investigated the application of semi-active variable stiffness damper for response control of seismically excited structure and found the optimum location of a damper. Mevada and Jangid [7] investigated the seismic response of single-storey,

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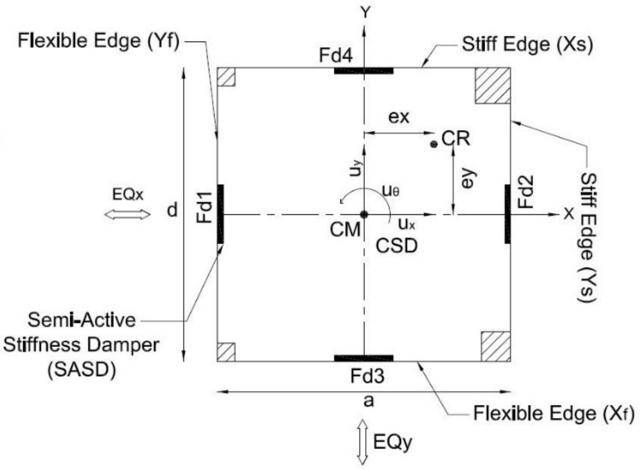
one-way asymmetric building with passive and semi-active variable stiffness dampers and found that resetting control laws is more effective than switching laws control.

In this paper, seismic response of a one-storey, two-way asymmetric system installed with semi-active stiffness damper under bi-directional seismic excitation is investigated. The objective of the study includes the performance of the system by comparing various parameters like lateral and torsional displacement and acceleration for both uncontrolled and controlled system.

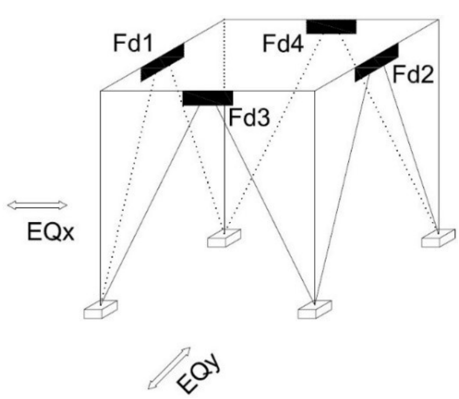
2. Structural Model

The model considered is idealized as a one-storey building which consists of a rigid floor supported by the structural element. The model is assumed to remain in a linear elastic state. Further, it is assumed that the floor slab of the structure is considered as flexural rigid and columns are axially rigid. Figure 1 shows the arrangement of semi-active stiffness damper in the considered structural model.

The mass of floor is assumed to be uniformly distributed and hence the center of mass (CM) coincides with the geometrical center of the floor. The columns are of different sizes used to produce centre of rigidity (CR) at distance ex from the (CM) in X-direction and ey from (CM) in Y-direction. The system is asymmetric about both X and Y-direction which makes $(3-DOF)$ system. These three degrees of freedom are lateral displacements in X-direction (u_x) and Y-direction (u_y) torsional displacement (u_θ) as



(a) Plan view of two-way asymmetric system.



(b) Arrangement of dampers in system

Fig. 1. Plan and isometric view of two-way asymmetric building showing arrangement of a damper

represented in Figure 1(a). Semi-Active Stiffness Dampers $(SASD)$ are provided at an equal distance from CM about both axis as represented in Figure 1(a) to achieve supplemental damping eccentricity (e_{sd}) zero[8].

3. Solution of Equations of Motion

The governing equations of motion fora considered structure is expressed in the matrix form as

$$M\ddot{u} + C\dot{u} + Ku = -M\Gamma\ddot{u}_g + \Lambda F \quad (1)$$

Where $[M]$,$[C]$, and $[K]$are the mass, damping, and stiffness matrix of the system, respectively; $u = \{u_x u_y u_\theta\}^T$is the displacement vector; $\dot{u} = \{\dot{u}_x\dot{u}_y\dot{u}_\theta\}^T$is velocity vector;$\ddot{u} = \{\ddot{u}_x\ddot{u}_y\ddot{u}_\theta\}^T$is the acceleration vector; Γ is the influence coefficient vector;$\ddot{u}_g = \{\ddot{u}_{gx}\ddot{u}_{gy}\ 0\}^T$ground acceleration vector;$\ddot{u}_{gx}$ is ground acceleration in X-direction and $\ddot{u}_{gy}$ is the ground acceleration in Y-direction; Λ is the damper location matrix which depends on the placement of dampers; $F = \{F_{dx}F_{dy}F_{d\theta}\}^T$is the vector of control forces; $F_{dx},F_{dy,}$ and $F_{d\theta}$ are resultant control forces of damper along x-, y- and θ- direction, respectively.

The mass matrix $[M]$can be expressed as,[2]

$$M= \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & mr^2 \end{bmatrix} \quad (2)$$

Where $m$represent the lumped mass of the floor; and r is the mass-radius of gyration about the vertical axis through CM. Mass-radius of gyration is given by;$r = \sqrt{\frac{a^2+d^2}{12}}$; where a and d are the plan dimension of the building.

The stiffness matrix$[K]$ can be expressed as,[9]

$$\mathrm{K} = \begin{bmatrix} k_{xx} & k_{yx} & k_{\theta x} \\ k_{xy} & k_{yy} & k_{\theta y} \\ k_{x\theta} & k_{y\theta} & k_{\theta\theta} \end{bmatrix} \quad (3)$$

$k_{xx} = \sum_i k_{xi}$, $k_{yy} = \sum_i k_{yi}$ where k_{xx} and k_{yy} are the total lateral stiffness in X- and Y- direction respectively.

$$K_{x\theta} = K_{\theta x} = \sum_i (y_i \times K_{xi})$$
$$K_{y\theta} = K_{\theta y} = \sum_i (x_i \times K_{yi}) \quad (4)$$

$k_{yx} = k_{xy} = 0$ denotes that u_x and u_y are uncoupled degrees of freedom.

$$k_{\theta\theta} = \sum_i k_{xi}y_i^2 + k_{yi}x_i^2 \quad (5)$$

$k_{\theta\theta}$ is the torsional stiffness of the system about a vertical axis at CM. Where $k_{xi}$and k_{yi} are the lateral stiffness of i^{th} column in X-direction and Y-direction respectively; x_i and y_i are the distance of i^{th} column from theCM respectively. Theclassical damping of the system is developed using Rayleigh damping matrix ,$[C]$ can be expressed as [9]

$$C = \alpha M + \beta K \quad (6)$$

Where α and β are the coefficients expressed in equation (7) below.

$$\alpha = \xi \frac{2\omega_i \omega_j}{\omega_i + \omega_j}$$
$$\beta = \xi \frac{2}{\omega_i + \omega_j} \quad (7)$$

Where ω_i and $\omega_j$depends on the damping ratio of two vibration modes. For the study, 5% damping is considered for two vibration modes. The state-space method is used to solve the governing equations of motion. The solution of equation of motion can be written as, [10]

$$\dot{z} = Az + HF - E\ddot{u}_g \quad (8)$$

Where $z = \{u\dot{u}\}^T$ is a state vector. A, H and E are the system matrix, distribution matrix of damper force, and distribution matrix of earthquake excitation respectively. These matrices are expressed as,

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}k & -M^{-1}C \end{bmatrix};$$
$$H = \begin{bmatrix} 0 \\ M^{-1}\Lambda \end{bmatrix} \text{ and } E = \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} \quad (9)$$

In which I is the identity matrix.

The equation (8) is discretized in the time domain and the earthquake excitation forces and damper control forces are assumed to be constant within any time interval, the solution may be written in an incremental form[11].

$$z_{k+1} = A_k z_k + H_k F_k - E_k \ddot{u}_{gk} \quad (10)$$

Where k denotes the time step; and $A_k = e^{A\Delta t}$ represent the discrete-time step system matrix. whereΔt is a time interval. The constant coefficient matrices H_k and E_k can be written as

$$H_k = A^{-1}(A_k - I)H \text{ and } E_k = A^{-1}(A_k - I)E \quad (11)$$

4. Modelling of Dampers

Recently, Yang et al. have developed a semi-active stiffness damper which consist of a hydraulic damper with a cylindrical chamber and piston arrangement. The cylindrical chamber is filled with fluid. The control valve with the bypass pipe connects two chambers of the hydraulic damper. Figure 2 shows the schematic diagram of semi-active stiffness damper[12]. Semi-active stiffness damper can be worked in two different ways. When the control valve is closed semi-active stiffness damper worked as a bracing member. In this stage, stiffness is generated due to the bulk modulus of the fluid in the cylinderk_f. On the other hand, semi-active stiffness damper worked as hydraulic damper when the control valve is open. When the control valve is open damper provides only less amount of damping without any stiffness. In these stage, damper is worked as a passive component in the system. The stiffness of the bracing is denoted byk_b, then the effective stiffness of the entire device $k_h$is given by

$$k_h = \frac{k_f k_b}{(k_f + k_b)} \quad (12)$$

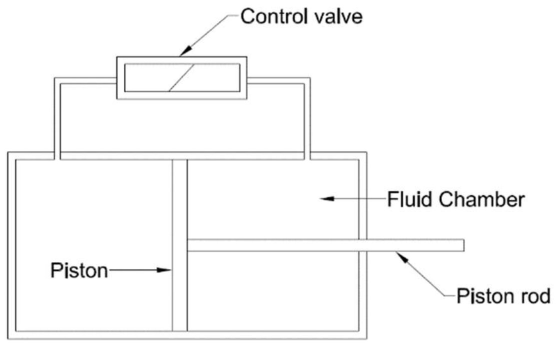


Fig. 2. Schematic Diagram of Semi-Active Stiffness Damper

For the semi-active control operation, two different control modes can be used, namely, resetting and switching. Based on the literature review it was observed that the resetting control law is quite effective in suppressing the structural response than a switching control law. So for the present study resetting semi-active stiffness damper (RSASD) controlled is used.

The damper forces of i^{th}semi-active stiffness damper considering resetting control law can be obtained as

$$F_{di} = k_{hi}(u_i - u_{ri}) \quad (13)$$

Where u_{ri} is resetting position of i^{th}damper; When the RSASD is reset (valve is pulsed to open and close), $u_{ri} = u_i$.At that instant, the applied damper force is zero. Yang et al. develop a resetting control law considering the Lyapunov function V as follows[13].

$$V = 0.5u^T K_u + 0.5\dot{u}^T M \dot{u}^T + \alpha_L u^T M \dot{u} \quad (14)$$

Where α_L is a Lyapunov constant such that the Lyapunov function is positive definite as follows

$$\begin{bmatrix} K & \alpha_L M \\ \alpha_L M & M \end{bmatrix} > 0 \quad (15)$$

Based on this, by minimizing $\dot{V}$, derived the resetting control law as follows [12]

$$u_{ri} = u_i \text{ when } \dot{u}_i + \alpha_L u_i = 0 \quad (16)$$

The effective damper stiffness plays a significant role in developing the control system for the semi-active stiffness damper. The affective stiffness ratio k_r is given in equation (17) below[7].

$$k_r = \frac{k_{hi}}{k_{si}} \quad (17)$$

Where k_{si} is the storey stiffness.

5. Numerical Study

The seismic response of linearly elastic, idealized single-storey, two-way asymmetric building installed with SASD is investigated by a numerical study using MATLAB. From the numerical simulation, various response quantities are found out at $\boldsymbol{CM}$ of the structure. These response quantities are lateral displacement in X and Y direction, torsional displacements of floor mass ($\boldsymbol{u_x}, \boldsymbol{u_y}$ and$\boldsymbol{u_\theta}$), lateral acceleration in X and Y direction, torsional accelerations of floor mass ($\ddot{\boldsymbol{u}}_{\boldsymbol{x}}$, $\ddot{\boldsymbol{u}}_{\boldsymbol{y}}and\ddot{\boldsymbol{u}}_{\boldsymbol{\theta}}$), base shear in X and Y direction ($\boldsymbol{Vb_x}$, $\boldsymbol{Vb_y}$), base torque $\boldsymbol{Vb_\theta}$, and drift of the structure. The responses of the system are found out using following parametric variations: stiffness ratio $\boldsymbol{k_r}$and constant $\boldsymbol{\alpha_L}$for SASD. The various response quantities are listed above are obtained by performing time history analysis considering three different ground motion. Various details of considered ground motions are listed in table 1 below. The recorded time histories of the ground motion of the considered earthquakes are shown in Figure 3 [8].In this study, the aspect ratio of the plan dimension is maintained as a unityto achieve supplemental damping eccentricity ($\boldsymbol{e_{sd}}$) zero.In order to study the effectiveness of the control system, the answers are expressed in terms of the $\boldsymbol{R_e}$ defined as follow [14]. It is a ratio of the peak responses of the controlled asymmetric system to the corresponding uncontrolled system.

$$R_e = \frac{Peak\ response\ of\ controlled\ asymmetric\ system}{Peak\ response\ of corresponding\ uncontrolled\ system} \quad (18)$$

The $\boldsymbol{R_e}$ less than one value suggests the control system is successful in minimizing responses and on the other hand,the value of $\boldsymbol{R_e}$ more than one implies that the control system is not successful in reducing the response.

For the analysis various physical quantities are taken as; length and width of the structure are 6mand, storey height of 6m. To make the structure two-way asymmetric three different sizes of the columns are used. Two columns are of dimension 0.3m × 0.3m, one column is of 0.35m × 0.35m and another one column is of 0.4m × 0.4m is considered.The arrangement of the different columns is shown in Figure 1 above. Total lumped mass (m) of the system is 4.009×10^4 Kg. The transverse frequency of the system is 12.11 rad/sec, torsional frequency of the system is 19.7914 rad/sec. Normalize stiffness eccentricity in X and Y direction respectively $\bar{e}_x = 0.2148$, $\bar{e}_y = 0.0932$. Normalize supplemental damping eccentricity in X and Y direction respectively $\bar{e}_{sdx} = 0$, $\bar{e}_{sdy} = 0$. [8]

Table-1Information of earthquake motions considered for the study[8].

Earthquake	Recording Station	Time (sec)	PGA (g) EQ_x	PGA (g) EQ_x
Imperial Valley, 19th May 1940	El Centro	40	0.31	0.22
Loma Prieta, 18th October 1989	Los Gatos Presentation Center	25	0.97	0.59
Northridge, 17th January 1994	Sylmar Converter Station	40	0.89	0.61

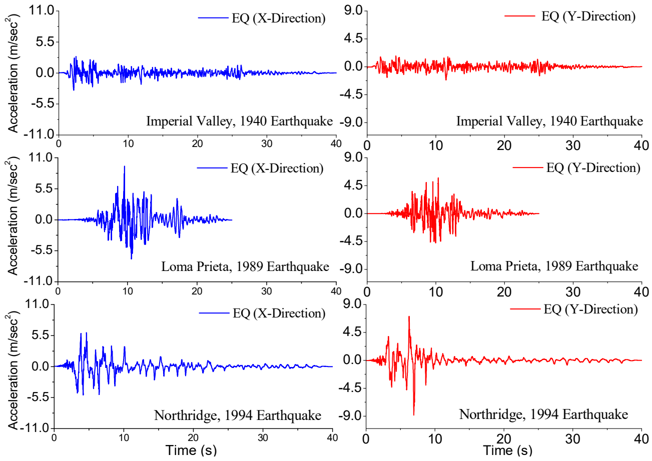


Fig.3 Recorded Time Histories of Earthquakes[8].

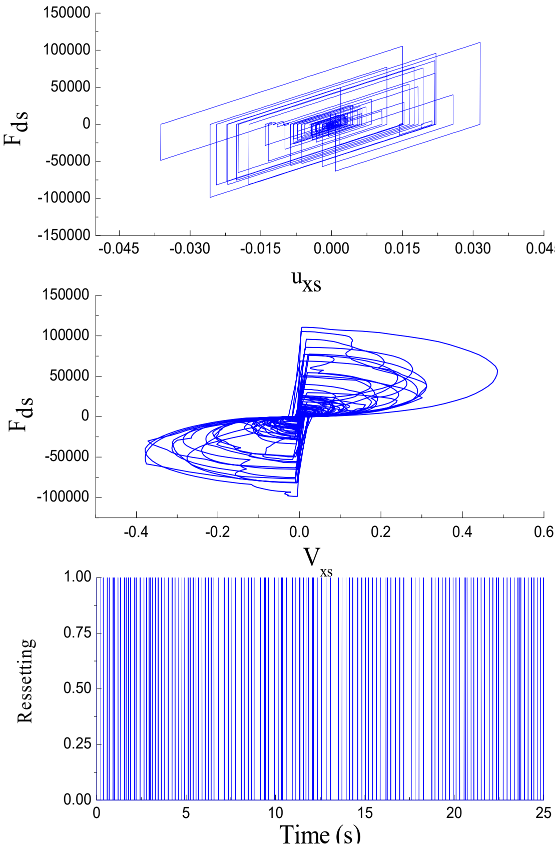


Fig. 4.Damper force-displacement and velocity loops and Control Actions (Resetting) of SASD for Loma Prieta, 1989

Figure 4 shows the damper force-displacement, velocity relationship (hysteretic loop), and resetting control action of semi-active stiffness damper(SASD) under Lama Prieta, 1989 Earthquake. To examine the efficiency of SASD, the stiffness ratio k_r as described in equation (17) is varied from 0.05 to 0.4 and constant α_L of resetting control law is varied from 0 to 5.

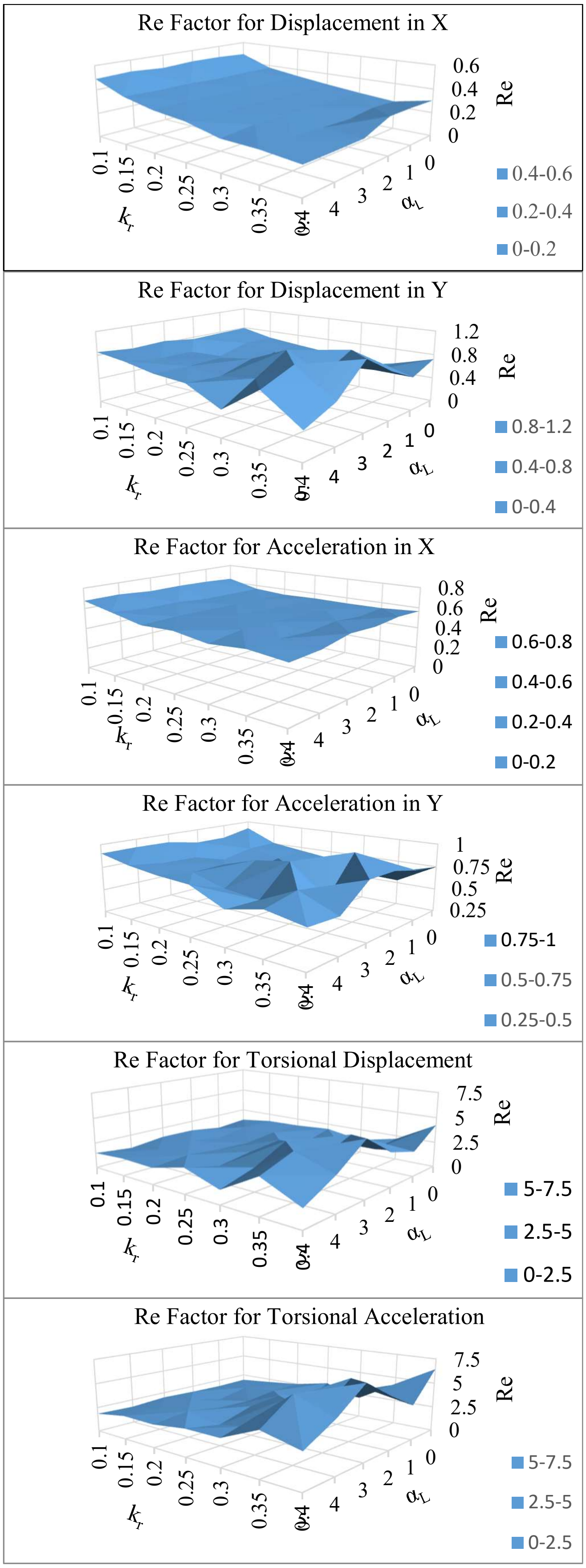


Figure:5 Re factor for SASD for various displacement and acceleration for Imperial Valley,1940 Earthquake

The various responses are obtained fora considered system with Ty = 0.5188 sec under considered earthquake and variations of k_r and α_L are shown in Figure 5.The response ratio, R_e are obtain from displacement counterpart u_x, u_y andu_θ, acceleration counterpart $\ddot{u}_x$, $\ddot{u}_y$and$\ddot{u}_\theta$ due to bi-directional excitation and its variation against k_r and α_L for Imperial Valley,1940 Earthquake shown in Figure 5.It can be observed from figure 5 that with the increase in the value of k_r, the ratio R_e for the displacement responses decrease continuously. This means the effectiveness of the control system ismore in reducing displacement with higher values of k_r. On the other hand, $R_e$for variousaccelerations decreases initially with an increase in $k_r$and then increases with further increase in k_r. The response reduction for torsional displacement and torsional acceleration is not effective using SASD. The ratio R_e for the torsional displacement and torsional acceleration is more than one for any sets of parametric variations.

From these parametric variations, optimum values of α_L is 2.5 and stiffness ratio k_r is 0.3 for lateral displacement and acceleration. On the other hand optimum values of α_L and stiffness ratio k_r are different for torsional displacement and acceleration. Optimum values of α_L and k_r for torsional displacement and acceleration are 0.05 and 0.5 respectively.Figure 6 shows the time histories for Loma Prieta, 1989 Earthquake using separate optimum parameters for both lateral displacement lateral acceleration and torsional displacement and torsional acceleration. It can be observed that considerable response reduction in displacement and acceleration. But damper is less effective in reducing responses for rotational displacement, rotational acceleration.

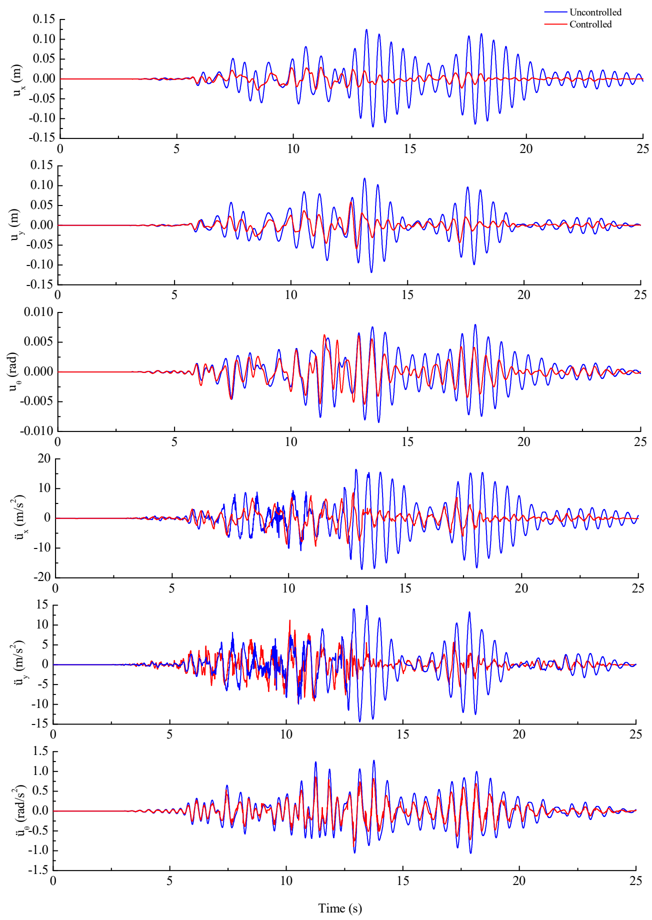


Fig. 6. Time Histories for various uncontrolled and controlled displacement and acceleration using SASD under Loma Prieta, 1989 Earthquake

Table 2 Response quantities for SASD control systems under considered earthquakes

Responses	Control System	Imperial Valley, 1940	Loma Prieta, 1989	Northridge, 1994	Average percentage reduction
u_x (m)	UC	0.0562	0.1248	0.0079	-
	SASD	0.0148 (73.71)	0.0239 (80.85)	0.0031 (60.70)	**71.75 %**
u_y (m)	UC	0.0477	0.1194	0.0087	-
	SASD	0.0280 (41.22)	0.0590 (50.54)	0.0027 (68.51)	**53.42 %**
u_θ (rad)	UC	0.0036	0.0085	0.0006	-
	SASD	0.0027 (23.62)	0.0063 (26.25)	0.0004 (22.93)	**24.26 %**
$\ddot{u}_x$ (m/sec^2)	UC	8.6549	18.9160	1.1976	-
	SASD	4.6328 (46.47)	9.3865 (50.37)	0.8134 (32.07)	**42.97 %**
$\ddot{u}_y$ (m/sec^2)	UC	9.5052	16.7689	1.2409	-
	SASD	5.9003 (37.92)	12.2707 (26.82)	0.8761 (29.39)	**31.38 %**
$\ddot{u}_\theta$ (rad/sec^2)	UC	0.6966	1.2853	0.0925	-
	SASD	0.563 (19.17)	1.007 (21.57)	0.0690 (25.40)	**22.04 %**
V_{bx} (kN)	UC	3.47×10^5	7.59×10^5	4.80×10^4	-
	SASD	1.86×10^5 (46.47)	3.76×10^5 (50.37)	3.26×10^4 (32.07)	**42.97 %**
V_{by} (kN)	UC	3.81×10^5	6.72×10^5	4.98×10^4	-
	SASD	2.37×10^5 (37.92)	4.92×10^5 (26.82)	3.51×10^4 (29.39)	**31.38 %**

Table 2 shows the comparative response quantities for the uncontrolled (UC) system and $SASD$controlled responses under considered earthquake. From Table 2 it can be observed that semi-active stiffness damper is very efficient for reducing lateral displacement and acceleration responses in both X and Y-direction. On the other hand, semi-active stiffness damper is less effective for torsional displacement and acceleration. The behaviour of the structure for torsional displacement and acceleration is different due to the optimum values of the stiffness ratio k_r and constant α_L which makes the system very stiff and acts as bracing in the structure.

Table 3. Storey Drift

Earthquake	Drift as per IS: 1893 (Part 1):2002	Structure Without Damper (Uncontrolled)		Controlled SASD	
Imperial Valley, 1940	0.024 m	0.562 m	Unsafe	0.014 m	Safe
Loma Prieta,1989	0.024 m	0.124 m	Unsafe	0.023 m	Safe
Northridge, 1994	0.024 m	0.077 m	Unsafe	0.003 m	Safe

Table 3 shows the storey drift of the structure under the uncontrolled and controlled system. As per IS 1893(Part 1): 2016, storey drift in any storey due to the minimum design lateral forces, with a partial load factor of 1.0, shall not exceed 0.004 times the storey height. It can be observed from the table that the permissible limit of the drift of the considered structure is 0.024 m. when the structure is uncontrolled, the drift of the structure is exceeding the permissible limit. But when the structure is installed with the semi-active stiffness damper the drift of the structure is reduced and it is under the permissible limit.

6. Conclusions

The seismic response of linearly elastic, idealized single-storey, two-way asymmetric building installed with SASD is investigated. The responses are evaluated with parametric variations to study the effectiveness of damper. In order to investigate the effectiveness of SASD, the stiffness ratio k_r is varied from 0.1 to 0.4 and constant α_L of resetting control law is varied from 0 to 5. From the results of the present study, the following conclusions can be drawn:

1. The effectiveness of the semi-active stiffness damper is increased with the increase with the stiffness ratio.
2. The semi-active stiffness damper is very effective in reducing lateral displacement and acceleration. On the other hand, SASD is not effective for reducing torsional displacement and acceleration.
3. The significant reduction in the drift of the structure as per IS: 1893 (Part 1):2016when the structure is installed with semi-active stiffness damper.
4. There exists an optimum value of stiffness ratio and Lyapunov constant

Disclosures

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