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# Use of critical response spectrum for design of multi-story steel buildings under multi-component seismic excitation

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**Abstract**

During an earthquake, buildings are simultaneously excited by three-components of ground motion (two horizontal and one vertical) orientations of which are not known apriori. To take account for the uncertainty in the direction of incidence of earthquake waves, a structure is required to be designed such that it is safe for all directions of incidence. For this purpose, the combined effects of two horizontal components of motion are commonly determined using simplified methods such as the percentage rules (e.g. 100%+30%, 100%+40%), square root of the sum of squares (SRSS) and Complete Quadratic Combination (CQC-3) rules. The modern building codes recommend to estimate the orthogonal response quantities for the individual horizontal component of motion using response spectrum superposition methods with design spectrum in the principal direction of motion. In the present paper, a new method is proposed to evaluate the maximum response of multi-story buildings under the simultaneous action of two horizontal components of ground motion using the concept of critical response spectrum. The critical response spectrum is computed using the resultant response of a bidirectional single degree of freedom system at each time step under the simultaneous action of the two horizontal components of motion. For an illustration of the proposed method, steel building asymmetric in the plan is analysed using critical response spectra of the three different pairs of recorded ground motion and the results obtained are validated by comparison with the exact time-history solutions. The exact response is taken as the maximum of the responses estimated by applying the two horizontal time-histories of ground acceleration at different angles from 0 to 180 degree with respect to the structural x-axis. On the other hand, in the response spectrum method, two values of the desired response quantity are obtained by applying the critical response spectrum along the x- and y-directions of the structure and are combined using SRSS method. It has been found that the use of the critical spectrum provides a very convenient method for estimating the maximum response under multi-component excitation without the need for computation of critical incident angle of incidence.

*Keywords*: Multi-component seismic excitation, Time-History, Critical earthquake response, Critical response spectrum, Combination method, Asymmetric Building

## 1. Introduction

In seismic design practice, the translational components in the 3 orthogonal directions are considered and rotational components usually ignored. The critical orientation of the components, as well as the methods for combining their individual effects, have been of interest to the civil engineering profession. In traditional earthquake engineering, structures are designed to resist only horizontal translational components of acceleration. Occasionally, in the design of important structures with long spans, the vertical component of excitation is also considered. Penzien and Watabe [1] stated that the three components of an earthquake are uncorrelated along a set of axes termed as principal axes. The major principal axis is horizontal and directed toward the epicentre, the intermediate axis is horizontal and perpendicular to the orientation of the major component, and the minor principal axis is vertical. The critical response could be obtained when these components are applied along the structural axes.

Newmark [2] and Rosenblueth and Contreras [3] proposed the Percentage Rules to approximate the combined response as the sum of the 100% of the response resulting from one component and some percentage ($\lambda$) of the responses resulting from the other two-components. Newmark suggested to be 40% and Rosenblueth and Contreras suggested to be 30%. Smeby and Kiureghian [4] assumes the components of ground motion along the principal directions to be uncorrelated and they proposed an extension of. the CQC rule (Der Kiureghian [5]), known as the CQC3 rule to combine responses due to the three seismic. components. This rule is based on the elementary concepts of stationary random vibrations and accounts for correlation of the response.

Many studies have investigated to relative performances of these rules, but to the best of authors knowledge none of them have compared with the exact time-history results.

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Wilson et. al. [6] concluded that the percentage rules could underestimate the design forces in certain members compared to the SRSS. Menun and Der Kiureghian [7] compared the results of the CQC3 rule with those of the SRSS, the 30% ($\lambda$ = 0.3), and the 40% ($\lambda$= 0.4) rules, assuming CQC3 results to provide a benchmark. The studies by Fernandez-Davila et al. [8]; Zaghlool et al. [9] have pointed out that percent rules and the SRSS rules are inappropriate to take into account the effects of horizontal orthogonal components of ground motion. Salazar et al. [10] observed that both the 30% and the SRSS rules could underestimate the combined response. Salzar et al [11] studied the response of moment-resisting steel frames and observed that for the two horizontal components the SRSS rule is, in general, less conservative than the 30% rule. They showed that critical response could be obtained for an orientation different from that of the principal components.

Sadhu and Gupta [12] formulated the modal combination rule for the ordered peak response of multistoried buildings excited by the multi-component ground motions. Their formulation is based on the stationary random vibration theory and suitable approximations regarding the peak factors and non stationarity. They have shown that the proposed rule performs better than the CQC3 rule when the building is stiffer to the ground motion. However, the authors have also pointed out that this rule underestimates the response in some cases, especially when ground motion is stiffer to the ground. Secondly, this rule requires the response spectrums in principal directions, which may not be always available. Another drawback in using a principal component is that if a minor principal component does not coincide with the vertical direction, the critical response is underestimated.

In the present report, a new method is proposed to evaluate the maximum response of multi-story buildings under the simultaneous action of two horizontal components of ground motion using the concept of critical response spectrum. The critical response spectrum is computed using the resultant response of a single degree of freedom at each time step under the simultaneous action of the two horizontal components of motion. For illustration, a steel building, asymmetric in plan, is analysed using proposed critical response spectra of the nine different pairs of recorded ground motion and the results obtained are validated by comparison with the exact solutions.

The exact response is taken as the maximum of the responses estimated by applying the two horizontal time-histories of ground acceleration at different angles from 0 to 180 degree with respect to the structural axis. On the other hand, in the response spectrum method, two values of the desired response quantity are obtained by applying the critical response spectrum along the x- and y-directions of the structure and are combined using SRSS method. It will be shown that the proposed critical spectrum provides a very good estimate of maximum response without the need for computation of critical incident angle. The proposed method evaluates the critical response of multi-storied buildings subjected to bi-directional ground motions using the concept of the critical response spectrum, rather than the traditional principal components. The results obtained from the proposed method is compared with CQC3 rule, MAX+30%, MAX+40% and SUM method of combining response for multi-component seismic ground motion.

## 2. Theoretical background

### *2.1 Time history solution method*

The equation of motion for a linear, multi-degree of freedom system subjected to three components of ground motion excitation can be written as

$$M\ddot{U} + C\dot{U} + KU = -MI\ddot{X} \qquad (1)$$

where *M, C* and *K* are the mass, damping and stiffness matrices respectively, U is the vector of nodal displacements relative to the ground, $X = [X_1 X_2 X_3]^T$ is the vector of ground motion translational components with horizontal components $X_1(t)$ and $X_2(t)$ and verticle component $X_3(t)$, and $I=[I_1 I_2 I_3]^T$ is the influence matrix such that its k$^{th}$ column $I_k$ couples the degrees of freedom of the structure to the ground motion component $X_k(t)$. Usually, the response quantity of interest, *v(t),* is a linear combination of the nodal displacements and can be written in the generic form $v(t)=q^TU$, where q is the vector constants. For example, when the response of interest is the internal force in a member, then q is given in terms of the elements of the stiffness matrix of that member.

Transforming into normal co-ordinates, $U=\Phi Y$, the ith uncoupled modal equation is

$$\ddot{Y}_i + 2\zeta_i\omega_i\dot{Y}_i + \omega_i^2Y_i = -\Gamma_i^T\ddot{X} \qquad (2)$$

where $Y=[Y_1\ Y_2 ...\ Y_n]^T$ is the vector of normal coordinates, $\Phi=[\Phi_1\ \Phi_2\ ...\Phi_n]$ is the matrix of n modal vectors $\omega_i$ and $\zeta_i$ are the natural frequency and damping ratio of mode *i*, respectively, and *k* = 1,2,3, associated with the three input components.

The time-history solution for multi-component excitation is given by

$$U(t) = \sum_{k=x,y}\sum_{l=x,y}\sum_{i}\sum_{j}\psi_{ki}\psi_{lj}Y_i(t)Y_j(t) \qquad (3)$$

where, $\psi_{ki}= \Gamma_i^{(k)}q^T\phi_i$ is an effective participation factor associated with mode *i* and input component *k* and $Y_i(t)$is the solution of eq. (2) for the $i^{th}$ mode, which is like the equation of motion for a single degree of freedom oscillator.

The time history solution procedure given above is applicable when the ground motion is applied in the directions of the structural axis, however, to find the maximum response the ground motion is required to be applied in the all possible orientations in the horizontal plane. This process requires large number of computations thus practically for design purpose it is not adopted.

### *2.2 Response spectrum method*

To perform the seismic analysis and design of the structure for a particular location the actual time history record is required, however, it is not possible to have such records for every location. Further, the seismic analysis of the structures can not be carried out based on the peak values of ground acceleration as the response of the structure depends on the frequency content of the input ground motion and structures own dynamic properties. Therefore in most of the cases, the response spectrum method is used for seismic analysis. This method provides a computational advantage as the prediction of the response involves the calculation of the maximum values of the response quantities in each mode of vibration. These maximum values of the displacements and member forces in each mode are conveniently obtained from the smooth design spectra which are based on the average of several earthquake ground motions.

The maximum modal displacement response is found from the response spectrum is

$$Y_{i,max} = |Y_i(t)|_{max} = \Gamma_i \, S_d(\zeta_i, \omega_i) \tag{4}$$

where, $\Gamma_i$ is the modal participation factor for $i^{th}$ mode and $S_d(\zeta_i, \omega_i)$ is the spectral displacement for the $i^{th}$ mode. The final maximum response $Y_{max}$ is obtained by combining the response in each mode of vibration using the modal combination rules. The commonly used method for combining the maximum final response is given by Complete Quadratic Combination (CQC) method [5] given by

$$Y_{max} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij} \, Y_i \, Y_j} \tag{5}$$

where, $Y_i$ and $Y_j$ are the maximum responses in the ith and jth modes respectively and $\rho_{ij}$ is the modal correlation coefficient [5] given by

$$\rho_{ij} = \frac{8\left(\zeta_i \zeta_j\right)^{1/2}(\zeta_i + \beta\zeta_j)\beta^{3/2}}{(1-\beta^2)^2 + 4\zeta_i\zeta_j\beta\left(1+\beta^2\right) + 4({\zeta_i}^2 + {\zeta_j}^2)\beta^2} \tag{6}$$

The response due to multi-component seismic ground motion are determined using simplified methods such as the percentage rules (e.g. Max+30%, Max+40%), square root of the sum of squares (SRSS) and Complete Quadratic Combination (CQC-3) rules the details of these methods are given below

*2.2.1 CQC-3 Rule*

This method is proposed by Smeby and Der Kiureghian [4] and later by Menun and Der Kiureghian [7] for finding the critical response under the simultaneous action of ground motion. This method requires principal components of ground motion. The incident angle-dependent response is given by equation (7)

$$R_\theta = \left[\sum_{k=1,2,3}\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\psi_{ki}\psi_{kj}S_{ki}S_{kj} - \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\left[\psi_{1i}\psi_{1j} - \psi_{2i}\psi_{2j}\right][S_{1i}S_{1j} - S_{2i}S_{2j}]\sin^2\theta + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\psi_{1i}\psi_{2j}[S_{1i}S_{1j} - S_{2i}S_{2j}]\sin\theta\cos\theta\right]^{1/2} \tag{7}$$

where, $\psi_{ki}$ is the effective participation factor in the $k^{th}$ structural axis, $\rho_{ij}$ is the modal correlation coefficient [5], $S_{1i}$ and $S_{2i}$ are the major and minor principal components of the ground motion [1]. $\theta$ is the angle between the incidence angle of principal components w.r.t the structural axis. The critical angle for the response can be obtained by differentiating equation (7) w.r.t. $\theta$. The critical angle is given by equation (8).

$$\theta_{cr} = \frac{1}{2}\tan^{-1}\left(\frac{2\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\psi_{1i}\psi_{2j}[S_{1i}S_{1j} - S_{2i}S_{2j}]}{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\left[\psi_{1i}\psi_{1j} - \psi_{2i}\psi_{2j}\right][S_{1i}S_{1j} - S_{2i}S_{2j}]}\right) \tag{8}$$

Equation (8) has two roots between 0° to 180° corresponding to those values of $\theta$ gives the maximum and minimum values of the response.

*2.2.2 SRSS Rule*

In this method of combining response due to multi-component seismic groundmotion is given by eq (9)

$$R_{crit} = [\{R^x\}^2 + \{R^y\}^2]^{1/2} \tag{9}$$

where, $R^x$ is the contribution of the response when the critical response spectrum is applied in $x^{th}$ structural axis and $R^y$ is the contribution of the response when the critical response spectrum is applied in $y^{th}$ structural axis. The response quantities $R^x$ and $R^y$ are computed as given in eq (10)

$$\{R^x\} = \left\{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\psi_{xi}\psi_{xj}S_{x_i}S_{x_j}\right\}^{\frac{1}{2}} \qquad \{R^y\} = \left\{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\psi_{yi}\psi_{yj}S_{y_i}S_{y_j}\right\}^{\frac{1}{2}} \tag{10}$$

where, $\psi_{xi}$ , $\psi_{yi}$ are the effective modal participation factor in the $x^{th}$ and $y^{th}$ structural axis, $\rho_{ij}$ is the modal correlation coefficient [5], $S_{x_i}$ and $S_{y_i}$ are the $i^{th}$ modal response spectral acceleration values of the $x^{th}$ and $y^{th}$ components

*2.2.3 Max+30% Rule*

Rosenblueth and Contreras [3] proposed the Percentage Rules to approximate the combined response as the sum of the 100% of the response resulting from one component and 30 percentage of the responses resulting from the other two components. This method is recommended by ATC-3(1978), IS-1893:2016 and many other codes. The response is given by equation (11)

$$R = max\{R^x + 0.3R^y \; or \; 0.3R^x + R^y\} \tag{11}$$

*2.2.4 Max+40% Rule*

Newmark [2] proposed combined response as the sum of the 100% of the response resulting from one component and 40 percentage of the responses resulting from the other two components. This method is recommended by ASCE (1986), ATC-32, for seismic analysis of nuclear structures. The response is given by equation (12)

$$R = max\{R^x + 0.4R^y \; or \; 0.4R^x + R^y\} \tag{12}$$

*2.2.5 SUM Method*

In this method, it is assumed that the contribution of the response due to multi-component seismic ground motion occurs simultaneously hence, the response is combined as absolute sum of the response contribution as given in equation (13)

$$R = R^x + R^y \tag{13}$$

This method usually yields conservative results hence this method is not recommended by various design codes

## 3. Various types of response spectra

The response spectra are the curves plotted between the maximum response of SDOF system subjected to specific earthquake ground motion and its time period (or frequency).

The response spectra can be interpreted as the maximum response of a SDOF system for given damping ratio (ζ). The various types of response spectra considered for analysis for the structures are given as below.

*3.1 Response spectra of recorded components [SAX, SAY]*

The response spectra of the recorded components of the seismic ground motion for a given damping ratio (ζ) is considered as SAX and SAY

*3.2 Response spectra of principal components [SAXP, SAYP]*

The SAXP and SAYP are obtained by resolving the components of the original accelerogram record in the major principal direction $\theta_1$ and intermediate principal direction $\theta_2$. The angles of the major and minor principal directions $\theta_1$ and $\theta_2$ are computed from the Penzien et al. [1] idealization.

*3.3 Critical response spectra [SACRIT]*

The proposed Critical Response Spectrum is obtained by the resultant response $R_{(t)}$ of an SDOF system obtained from SRSS of the response in the x-direction $R_{x(t)}$ and response in the y-direction $R_{y(t)}$ for each natural period (T) and damping ratio (ξ). The resultant response of the SDOF system is computed by equation (14)

$$SA_{crit(T,\xi)} = R_{(t)max} \tag{14}$$

Where,

$$R_{(t)} = \sqrt{R_{x(t)}{}^2 + R_{y(t)}{}^2} \tag{15}$$

To perform a numerical study, three real ground motion time histories are considered. The details of the selected earthquake records are shown in Table. 1 and Table. 2. Time history components records and the corresponding response spectra of the three selected real earthquake records considered for analysis are shown in Fig 1 and Fig 2.

Table-1. Details of the selected earthquakes

| REC# | Name of EQ | Date | M | Focal Depth[km] | Epicenter distance [km] | Recording site |
|---|---|---|---|---|---|---|
| 1 | Gorkha Nepal | 25/04/2015 | 7.9 | 13.4 | 13.6 | Kirtipur |
| 2 | Sikkim | 18/09/2011 | 6.7 | 10.00 | 50.3 | Gangtok |
| 3 | Uttarkashi | 20/10/1991 | 6.9 | 13.2 | 73.9 | Bhatwari |

Table-2. Peak ground acceleration (PGA) of components of selected earthquake records

| REC# | Name of EQ | Ist Component | | IInd Component | |
|---|---|---|---|---|---|
| | | Comp | PGA (g) | Comp | PGA (g) |
| 1 | Gorkha Nepal | NORT | 0.157 | EAST | 0.253 |
| 2 | Sikkim | EAST | -0.145 | NORT | -0.158 |
| 3 | Uttarkashi | N85E | 0.253 | N05W | 0.246 |

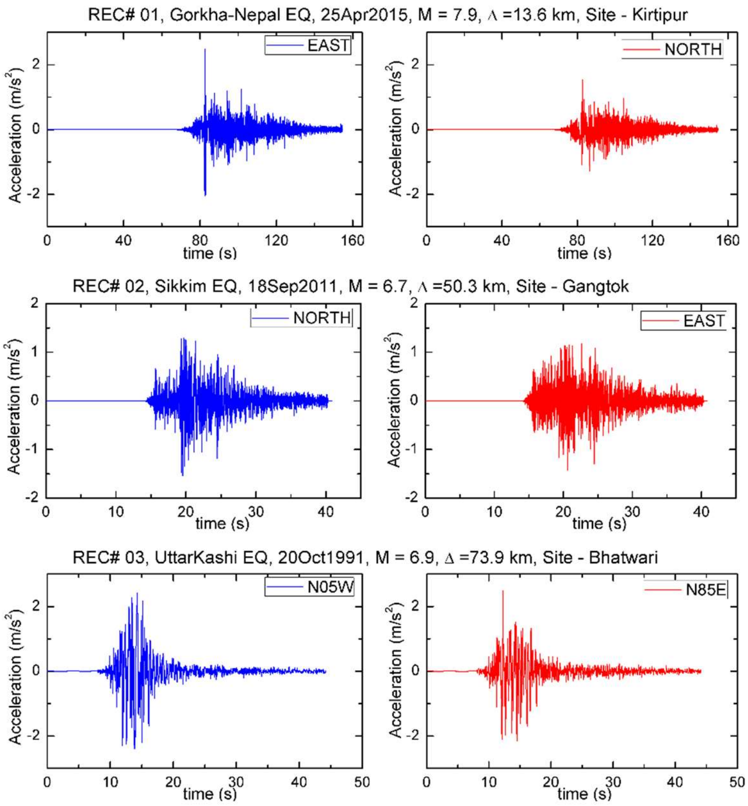


**Fig. 1. Selected recorded earthquake time history**

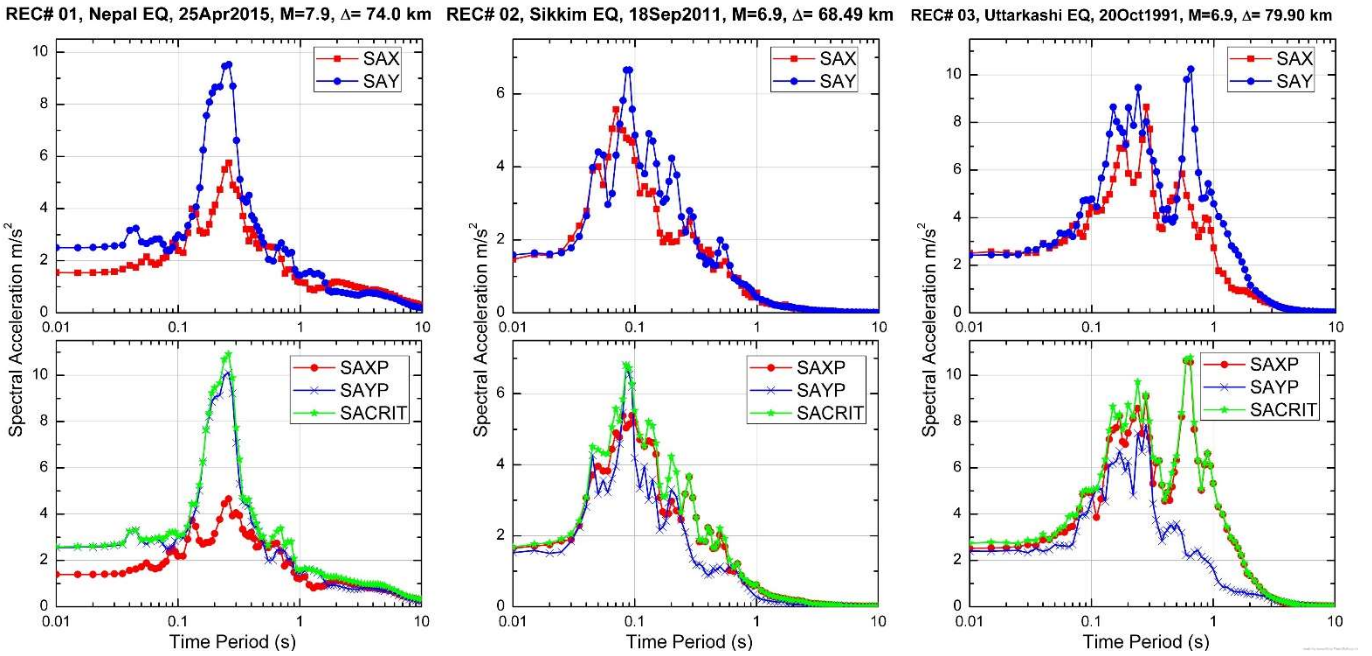


**Fig. 2. Response Spectra of selected earthquake records**

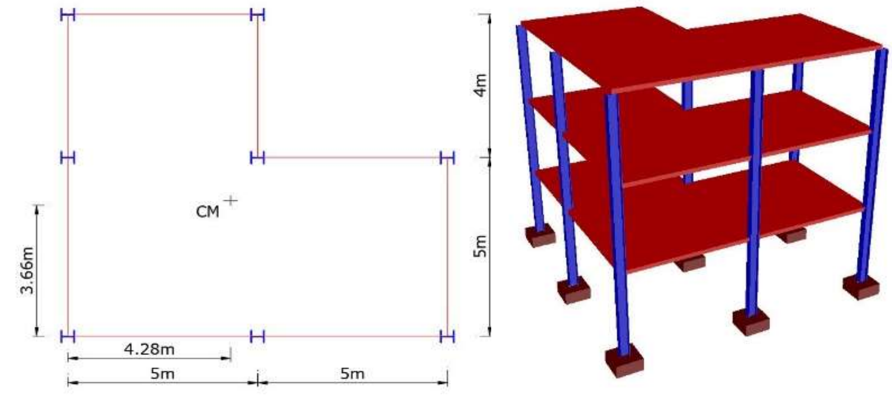


**Fig. 3. Asymmetrical steel building - Asymmecrtic building considered for analysis**

## 4. Illustrative Example

### *4.1 Example building*

A simple two-bay three-story steel building is considered for computation. Example asymmetric building is shown in Fig.3. The building is having columns ISHB 200-1 and supports 150mm thick R.C.C. slab. The arrangement of the steel columns is shown in Figure 1. The building is analyzed and designed for dead load intensity of 3.0 kN/m$^2$ and Live Load of 3.5 kN/m$^2$ acting on the intermediate floor slabs. The damping ratio considered for both the buildings is 5%. The building is assumed to have rigid diaphragms, mathematically modelled as 3 degrees-of-freedom ($\delta_x$, $\delta_y$, $\theta$) per floor considering lumped mass at story levels.

The computed Mass and Stiffness matrix for the asymmetric building is given in Equation 16 and Equation 17. The time period (s) for the asymmetrical building for the 9 modes of vibration are given in Table-3.

$$M = 10^3 \begin{bmatrix} 60.65 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 60.65 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 48.17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 60.65 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 60.65 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 48.17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 838.51 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 838.51 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 665.88 \end{bmatrix} \quad (16)$$

Table-3. Natural modes and frequencies ($\omega_n$) of Asymmetric Building

| Mode No. | Mode Description | Time Period [s] | Natural frequency ($\omega_n$) [rad/s] | Cummulative Mass participation (x-direction) |
|---|---|---|---|---|
| 1 | Translation in y-direction | 1.336941 | 4.699672 | 0.87% |
| 2 | Translation in x-direction | 0.692135 | 9.077979 | 80.13% |
| 3 | Torsion about the vertical axis | 0.63415 | 9.908039 | 91.87% |
| 4 | Translation in y-direction | 0.485174 | 12.95036 | 91.87% |
| 5 | Translation in y-direction | 0.344841 | 18.22055 | 91.87% |
| 6 | Translation in x-direction | 0.251175 | 25.01518 | 98.13% |
| 7 | Torsion about the vertical axis | 0.230132 | 27.30248 | 99.04% |
| 8 | Translation in x-direction | 0.178524 | 35.19518 | 99.88% |
| 9 | Torsion about the vertical axis | 0.163568 | 38.41331 | 100% |

Table 4. Cases Considered for Dynamic Analysis

| Case# | Case Details |
|---|---|
| 1 | Time history analysis using time history of recorded components of ground-motion for the angle of incidence from $0^0$ to $180^0$ with an interval of $1^0$. |
| 2 | Response spectrum analysis using critical response spectra [13] of the components of real ground motion and combining response using SRSS rule (Present Study) |
| 3 | Response spectrum analysis using response spectra of the principal components [1] and combining response using CQC3 Rule for critical response [7, 4] |
| 4 | Response spectrum analysis using response spectra of the recorded components (considering real components as principal components) and combining response using CQC3 Rule for critical response [7, 4] |
| 5 | Response spectrum analysis using response spectra of the recorded components of ground-motion and combining response using [Max+30%] rule. [3] |
| 6 | Response spectrum analysis using response spectra of the recorded components of ground-motion and combining response using [Max+40%] rule. [2] |
| 7 | Response spectrum analysis using response spectra of the recorded components of ground-motion and combining response using SUM rule. |

$$K = 10^6 \begin{bmatrix} 51.32 & -25.66 & 0 & 0 & 0 & 0 & -17.41 & 8.71 & 0 \\ -25.66 & 51.32 & -25.66 & 0 & 0 & 0 & 8.71 & -17.41 & 8.71 \\ 0 & -25.66 & 25.66 & 0 & 0 & 0 & 0 & 8.71 & -8.71 \\ 0 & 0 & 0 & 13.75 & -6.88 & 0 & 1.23 & -0.61 & 0 \\ 0 & 0 & 0 & -6.88 & 13.75 & -6.88 & -0.61 & 1.23 & -0.61 \\ 0 & 0 & 0 & 0 & -6.88 & 6.88 & 0 & -0.61 & 0.61 \\ -17.41 & 8.71 & 0 & 1.23 & -0.61 & 0 & 862.71 & -431.35 & 0 \\ 8.71 & -17.41 & 8.71 & -0.61 & 1.23 & -0.61 & -431.35 & 862.71 & -431.35 \\ 0 & 8.71 & -8.71 & 0 & -0.61 & 0.61 & 0 & -431.35 & 431.35 \end{bmatrix} \quad (17)$$

*4.2 Numerical Analysis and Results*

A detailed numerical analysis is carried out for the asymmetric building as shown in Fig 3. The dynamic analysis performed for seven different analysis cases. Details of these analysis cases are given in Table 4. In the present study, time history analysis (Case-1) is considered as the benchmark method. The response obtained by response spectrum superposition methods (Case-2 to Case-7) are compared with the time history solution. In the proposed critical spectrum method the modal response due to simultaneous action of multi-component seismic ground motion is evaluated based on the critical spectra as defined in section 3 and the modal response quantities are combined using SRSS rule. The story response quantities such as displacements, base shear force and bending moment are tabulated in Table-5. The normalised displacement, shear force and bending moments plots are shown in Fig 4.

Table 5. Story response results

| REC# | Case No. | Story Displacement (mm) | | | Base Shear Force (kN) | | | Bending Moment (kNm) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Story -1 | Story -2 | Story -3 | Story -1 | Story -2 | Story -3 | Story -1 | Story -2 | Story -3 |
| 1 | Case-1 | 19.03 | 35.23 | 44.19 | 483.01 | 410.76 | 245.84 | 8694.24 | 3696.85 | 737.53 |
| | Case-2 | 19.43 | 33.41 | 40.72 | 487.55 | 376.08 | 230.21 | 8775.85 | 3384.74 | 690.62 |
| | Case-3 | 15.60 | 27.47 | 33.39 | 392.03 | 303.74 | 190.44 | 7056.53 | 2733.69 | 571.31 |
| | Case-4 | 15.14 | 25.78 | 31.46 | 379.59 | 292.59 | 188.52 | 6832.59 | 2633.28 | 565.55 |
| | Case-5 | 15.15 | 25.81 | 31.49 | 379.93 | 292.85 | 188.66 | 6838.70 | 2635.64 | 565.99 |
| | Case-6 | 15.16 | 25.81 | 31.50 | 380.04 | 292.94 | 188.71 | 6840.74 | 2636.44 | 566.13 |
| | Case-7 | 15.19 | 25.86 | 31.56 | 380.73 | 293.47 | 189.00 | 6853.04 | 2641.20 | 567.01 |
| 2 | Case-1 | 6.28 | 11.46 | 14.41 | 159.21 | 136.22 | 79.10 | 2865.69 | 1225.97 | 237.30 |
| | Case-2 | 6.77 | 11.80 | 14.35 | 171.22 | 132.67 | 74.68 | 3082.01 | 1194.02 | 224.03 |
| | Case-3 | 6.32 | 11.04 | 13.43 | 158.98 | 123.06 | 68.67 | 2861.63 | 1107.56 | 206.02 |
| | Case-4 | 5.71 | 9.97 | 12.13 | 145.10 | 112.35 | 65.94 | 2611.75 | 1011.16 | 197.83 |
| | Case-5 | 5.72 | 9.98 | 12.14 | 145.24 | 112.46 | 66.00 | 2614.29 | 1012.15 | 198.01 |
| | Case-6 | 5.72 | 9.99 | 12.14 | 145.29 | 112.50 | 66.02 | 2615.15 | 1012.48 | 198.07 |
| | Case-7 | 5.73 | 10.01 | 12.17 | 145.57 | 112.72 | 66.15 | 2620.30 | 1014.48 | 198.44 |
| 3 | Case-1 | 59.14 | 105.20 | 128.13 | 1482.32 | 1153.08 | 573.77 | 26681.80 | 10377.70 | 1721.31 |
| | Case-2 | 59.54 | 105.49 | 128.14 | 1495.75 | 1160.96 | 583.03 | 26923.50 | 10448.70 | 1749.10 |
| | Case-3 | 58.23 | 103.22 | 125.39 | 1463.20 | 1135.92 | 568.23 | 26337.50 | 10223.30 | 1704.69 |
| | Case-4 | 56.52 | 100.13 | 121.63 | 1418.12 | 1100.65 | 553.42 | 25526.20 | 9905.81 | 1660.26 |
| | Case-5 | 56.55 | 100.17 | 121.69 | 1418.76 | 1101.14 | 553.67 | 25537.60 | 9910.25 | 1661.02 |
| | Case-6 | 56.56 | 100.19 | 121.71 | 1418.97 | 1101.30 | 553.76 | 25541.40 | 9911.73 | 1661.27 |
| | Case-7 | 56.61 | 100.27 | 121.81 | 1420.24 | 1102.29 | 554.26 | 25564.30 | 9920.59 | 1662.79 |

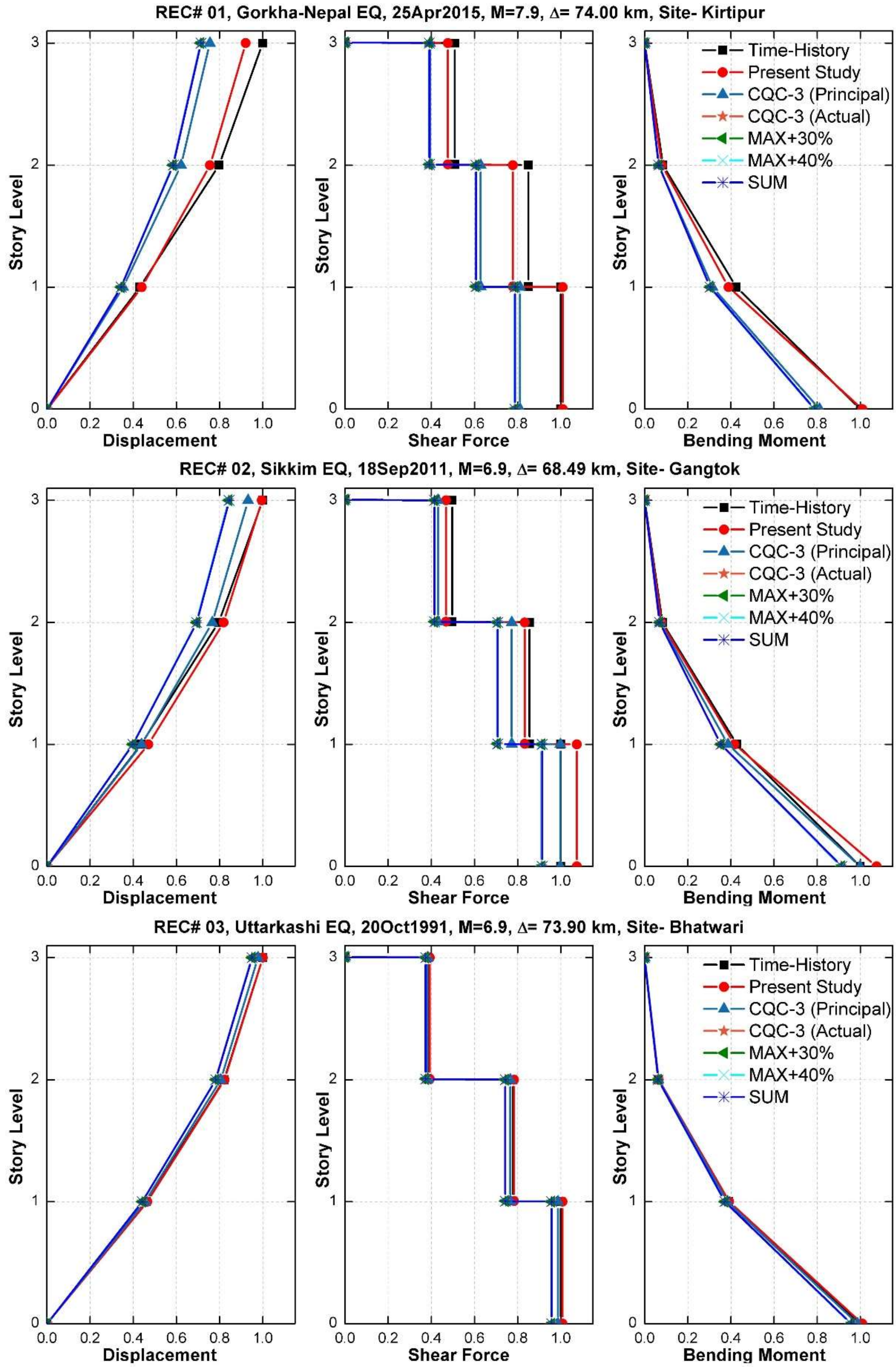


**Fig. 4. Normalised response plot**

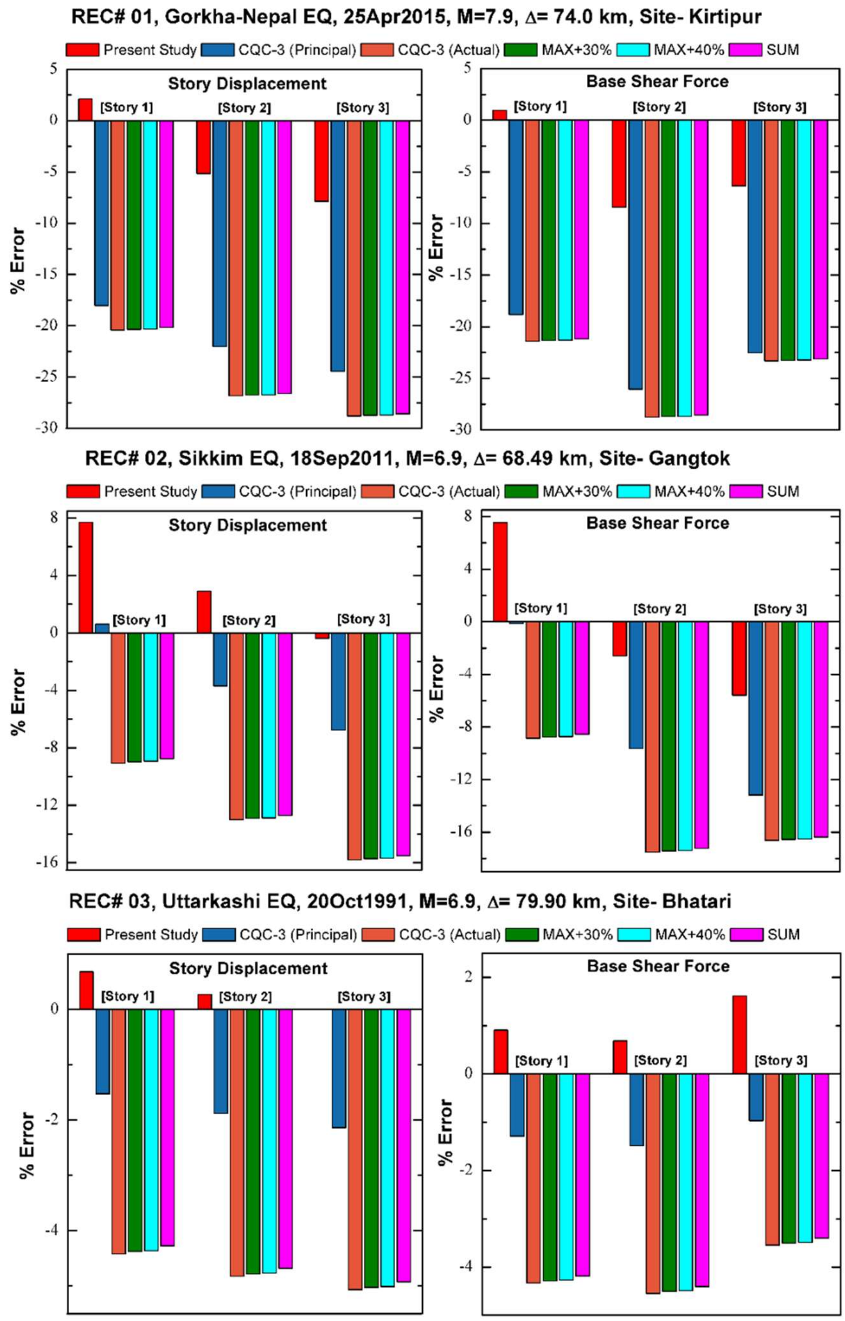


**Fig. 5. % Error in story response**

To illustrate the efficacy of the proposed method over the existing methods plots with percentage error is shown considering the time history method (Case-1) as the benchmark method as shown in Fig.5. As shown in Fig 5, It can be seen that the percentage rules viz Max+30%, Max+40% rules and Sum method underestimate the response. The main reason for lower estimates of the response is due to the non-consideration of the incident angle of the earthquake in the simplified percentage rule methods. The CQC3 rule [7, 4] method underestimates the response when compared with the time history method although the critical angle of earthquake incidence is considered while evaluating the response. However, it can be seen that the proposed critical spectrum method (Case-2) provides a very good estimate of the desired response without the need for consideration of any earthquake incident angle. The response quantities obtained by the proposed method are more consistent when compared with other methods of combining response due to multi-component seismic ground motion. The proposed method can be very easily implemented in earthquake analysis and design software.

## 5. Conclusion

The existing simplified percentage rule methods for combining the response for building under the simultaneous action of multi-component seismic ground motion underestimates the desired response in all the cases. The main reason for underestimating response in percentage rule methods is due to the non-account for the incident angle of the earthquake. The CQC-3 method considers the critical angle for a response, but in most of the cases, the response obtained by this method underestimates the response when compared with the exact time history solution.

In this paper, a new method is presented to find the critical response of the buildings under the simultaneous action of multi-component seismic ground motion. The proposed method provides a convenient approach to estimate the critical response without the need for an incident angle of the earthquake. The results obtained by the proposed method are consistent with the time history method. The proposed method can be very easily implemented through existing design software.

## Disclosures

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