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Moment Curvature of Reinforced Concrete beams- An analytical and experimental validation

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Abstract

Performance of Reinforced Concrete (RC) beam sections depend on their ability to undergo large inelastic deformation. Nonlinear analysis methods such as Pushover analysis demands inputs like moment curvature relationships of critical sections. This paper proposes analytical and experimental study of RC rectangular statically determinate beam sections. Beam specimens were tested under flexural loading. Behaviour of concrete and steel were also captured. An experimental study has been validated with various confinement models available in various research papers.

Keywords: Moment curvature, strain, reinforced concrete beam, load- deflection

1. Introduction

Reinforced concrete (RC) is the one of the most widely used structural material. Due to the non-homogeneity of the reinforced concrete, various methods for its analysis and design are being used. However, linear behaviour is valid for a region of small response. At ultimate load condition material exceeds their elastic limit and linear relationship no longer valid. Thus the moment curvature relationship is used for non linear analysis of section. The Moment-Curvature also give as the clear view of strength reduction beyond the peak point and degradation of the flexural rigidity. For under reinforced section material linearity is valid till steel is not yielded. With increasing grade of concrete result in increasing brittleness and lack of ductility [1].

To obtain Moment- Curvature relationship, RC beam specimens of M30 concrete grade are used. Experimental results are compared with analytical results obtained through computer program prepared in MATLAB. Various Stress-Strain models such as IS 456 model [2], IRC 112:2011 model [3], Hognestad model [4] and Kent and Park model [4] are used to obtain analytical results.

2. Analysis of Moment Curvature for RC beam elements

MATLAB code is used for the analytical approach. The entire program is based on the formal mathematical models which have been coded for stress-strain block. Results obtain by analytical method are compared to the experimental results. There are two methods to evaluate Moment Curvature of RC section. (i) Transform area method and (ii) Strain Compatibility method [5]. Transform area method is simple and approximate method whereas Strain Compatibility method is accurate and iterative method which uses Stress-Strain models for steel and concrete.

Using Transform are method, Moment Curvature is found out at three different stages, (a) at yielding, (b) at cracking and (c) at ultimate. Full Moment Curvature graph is obtained for Strain Compatibility method. Strain Compatibility method is based on few assumptions similar to theory of pure bending. Procedure of Strain compatibility method is given below.

First of all, the extreme fiber concrete compressive strain and neutral axis depth are assumed. For the assumed value of neutral axis, compressive forces C_c in concrete is calculated using Stress- Strain models for concrete. Strain in tension and compression and steel are calculated based on the strain compatibility. Based on strain in the tension steel, corresponding stresses are calculated from the stress strain curve of steel. After that force in tension steel (T) is calculated. Same procedure is applied to calculate compressive force in compression steel. The total compression force $C\left(C = C_c + C_t\right)$ acting in the compression zone is calculated. If $C = T$, assumed neutral axis is correct otherwise same procedure is repeated for another assumed neutral axis. Same procedure is repeated till the assumed strain is equal to the ultimate strain.

Total moment about neutral axis is, $M = M_c + M_{cs} + M_t$. Where, M_c =Moment of compressive force in concrete about the neutral axis, M_{cs} = Moment of force in

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compressive steel about the neutral axis and M_t = Moment of force in tensile steel about the neutral axis.

Bouafia et.al. [6] have given analytical and experimental evaluation of RC beam elements. Concrete compressive strength, the ratio, diameter, the configuration of transverse bars, the diameter & ratio of longitudinal bars and sections geometry are taken into account for circular and square shape of element. Kwak and Kim [7], Sunita et. al. [8], Srikanth et. al. [5], Zhao et.al. [9] have also obtained analytical and/or experimental evaluation of Moment Curvature for RC beam specimens and which are referred for the preparation of test specimens and their analytical validation.

3. Analytical evaluation of Moment – Curvature:

Geometric specification of beam, results obtained through Transform area method, Strain Compatibility method and Response 2000 software are given in this section.

3.1 Geometric specification of Beam:

Three M30 grade of beam specimens are prepared. Steel is of grade Fe500. Beams are designed as singly under reinforced beam according to IS 456:2000. Beam dimension is $120\,mm \times 150\,mm \times 1350\,mm$ and clear length of beam is 1200 mm. Cover is 20 mm and neutral axis depth is 52.62 mm. Area of steel is 157.07 mm^2. Results obtained through Transform area method are given in Table 1. Same values are plotted in graph and given in Fig 1.

For Strain compatibility method, Stress-strain curve of IS 456 model, Hognestad model, Kent and Park model and IRC 112 model are used. These models are selected from various literatures. Results obtained through Strain Compatibility method using various models are plotted in graph as in Fig 2.

Table 1. Moment Curvature using Transform area method

Grade of concrete	Moment (M) in kN.m	Curvature (ϕ)
$M-\phi$ at cracking	2.0061	0.0019
$M-\phi$ at yielding	9.022	0.0304
$M-\phi$ at ultimate	9.21	0.1431

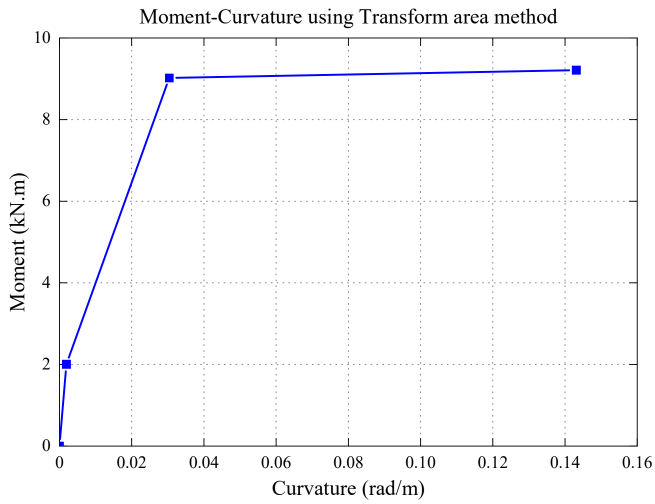


Fig 1. Moment-Curvature using Transform area method

In addition to these, two analytical methods, Response 2000 software is also used to verify the values obtained. It uses King's model for steel and Sandor Popovics model for concrete. Results obtained through Response 2000 are plotted in graph given in Fig 3.

4. Experimental evaluation of Moment-Curvature:

Three RC beam specimens are prepared using M30 grade of concrete and Fe500 grade of steel. Concrete mix design is selected by preparing three concrete cubes of size 150 mm × 150 mm× 150 mm. Tensile test is performed on 8 mm and 10 mm reinforcement bar to evaluate Tensile Strength and Modulus of Elasticity of steel. Flexural test is performed to evaluate Modulus of Rupture which comes 4.33 for M30 grade of concrete. Modulus of Elasticity of concrete is also evaluated experimentally and the value obtained is 34543.47 MPa. Reinforcement details for beam is shown in Fig 4.

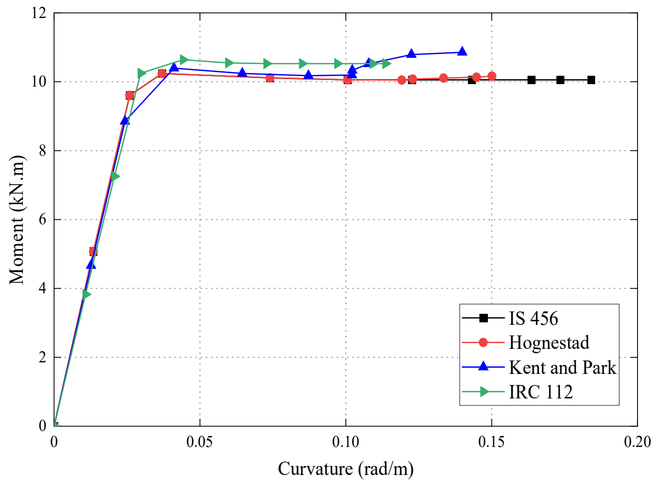


Fig 2. Moment-Curvature using Strain Compatibility method

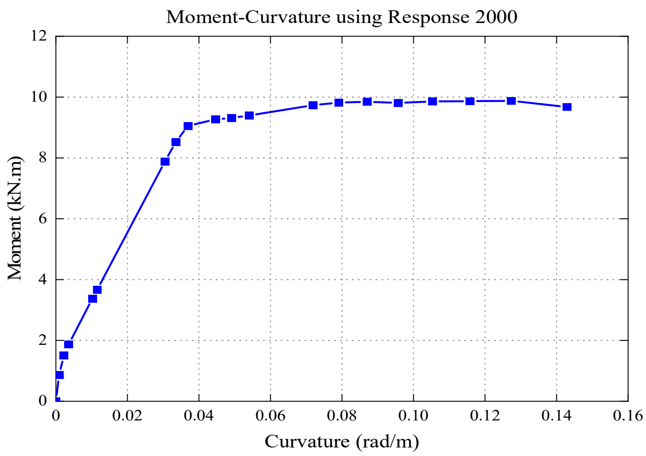


Fig 3. Moment-Curvature using Response 2000

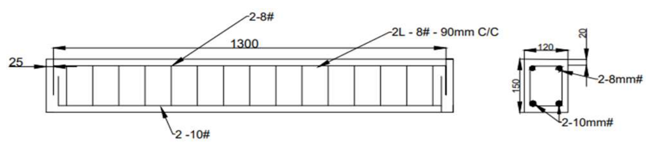


Fig 4. Reinforcement details for beam test specimen

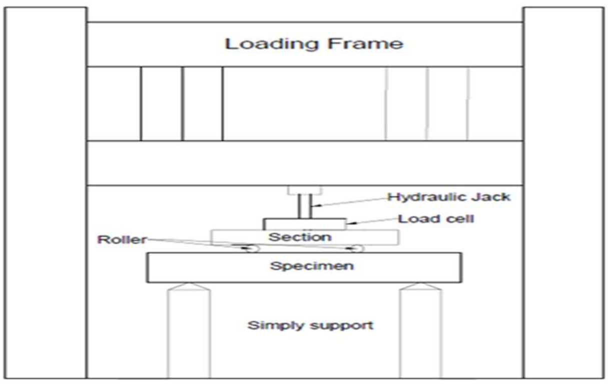
Fig 5. Installation and strain gauge resistant measurement

4.1 Instrumentation for Flexural test on RC beam specimens:

Instruments like hydraulic jack, load cell, dial gauge, data taker and loading frame are used to perform two point load test on RC beam specimens. Capacity of hydraulic jack is 250 kN. Load cell used for this test has a capacity of 100 kN. Strain gauges for this test are of two types, 5 mm length for steel and 90 mm length for concrete. Data taker (DT80) is also used for the test which transmit signals from load cell and strain gauges to computer attached with the test set up.

4.2 Loading set up:

RC beams are tested under loading frame by two-point load flexure test. Two-point loads are applied in a middle third portion of RC beam. Bending moment is constant in this zone so it is called as 'Pure Bending Zone'. Curvature is defined as the strain gradient so it can be measured by strain in steel and concrete. Schematic diagram and actual test set up are given in Fig 6.



(a)

(b)

Fig 6. (a) Schematic diagram and (b) actual test set up

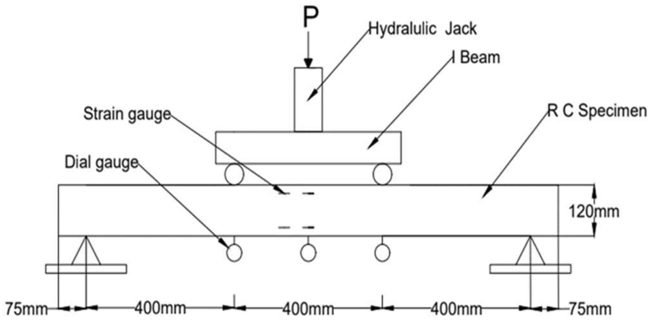


Fig 7. Two-point loading and position of Concrete and steel strain gauges

To measure strain in concrete of 90mm gauge length and for steel 5 mm gauge length strain gauges are used. Both strain gauges are checked for with 120*Ω* resistance. Two concrete strain gauges are applied on concrete surface from 20mm top of RC beam in pure bending zone as well as two steel strain gauges are applied on bottom steel reinforcement in pure bending zone. Clear distance between steel and concrete strain gauge was 110 mm. Two-point loading is applied on middle third length (400 mm) of the beam. To transfer load for hydraulic jack to beam one I section steel beam is placed on two rollers. These two rollers are placed in middle third portion of beam. Position of loads and strain gauges are shown in Fig 7.

4.3 Test results:

During the testing of RC beam specimen load and corresponding strain in steel and concrete are measured. In addition to this defection of beam at three different positions are measured using dial gauges. Deflected shape profile of one of the beams is shown in Fig 8.

Load vs Strain results are plotted in Fig 9. Concrete and Steel strain are showing tensile and compressive nature.

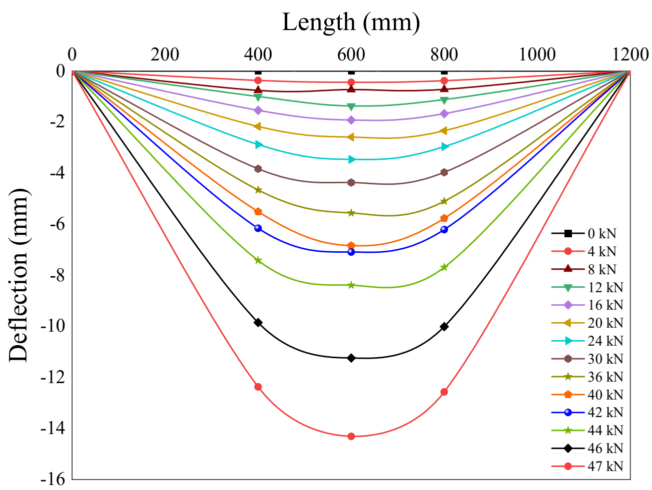


Fig 8. Deflected shape profile of beam specimen

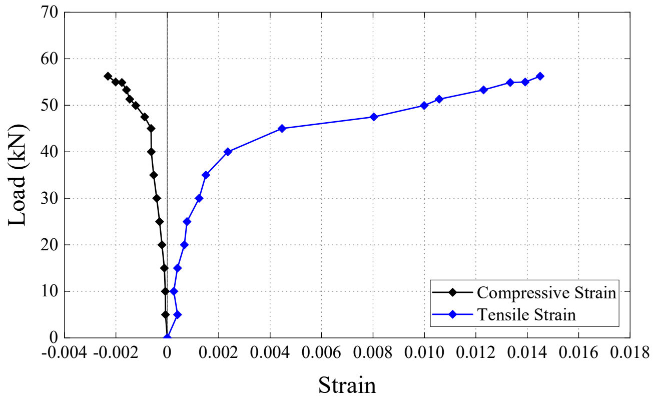


Fig 9. Load vs strain results

Load vs deflection curve of two tested beams are plotted in Fig 10. Plotted results show similarity in results for both beams. During the loading strain gauges got detached in third beam so its results are not shown here.

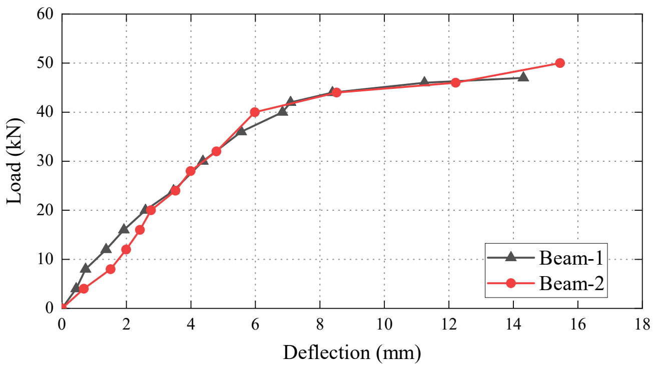


Fig 10. Load vs deflection curve of tested beam specimens

Moment –Curvature obtained through experiment for tested beams are shown in Fig 11. Moment is calculated at pure bending portion and curvature is calculated using strain in steel and concrete.

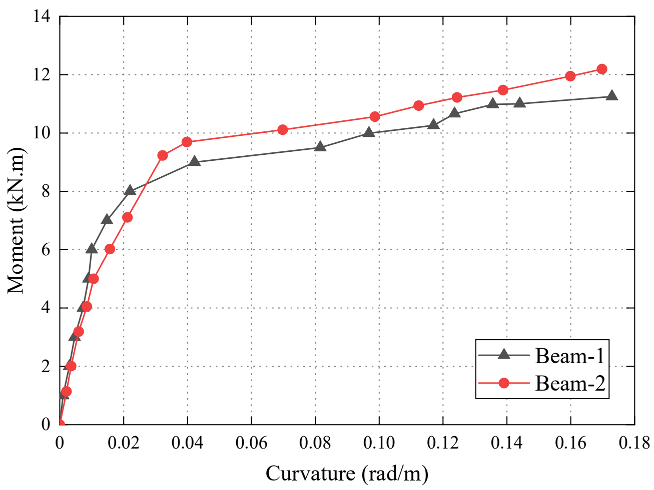


Fig 11. Moment-Curvature obtained through experiment

Comparison of curvature at yield and at ultimate for different analytical methods used and experimentally obtained values are given in Table 4. Curvature ductility is also shown in the table given.

Fig 12. Failure of beam specimen during testing

Table 4. Curvature at yield, at ultimate and curvature ductility

Stress-Strain models	θ		Curvature ductility
	θ_y	θ_u	
IS 456:2000	0.037	0.1841	4.97
Hognestad	0.037	0.1501	4.05
Kent and Park	0.041	0.1399	3.40
IRC 112: 2011	0.044	0.1136	2.57
Transform Area method	0.030	0.1431	4.70
Response 2000	0.037	0.1429	3.87
Experimental (Avg)	0.041	0.1429	4.17

During testing it is observed that, concrete at the top surface crushed slightly and beam has flexural cracks. In addition to crushing of concrete at top surface there are more numbers of flexural cracks. It is also observed that initial cracks were near to mid-point and after that they developed towards the supports with increment of load. Sample failure of beam specimen is shown in Fig 12.

5. Conclusion:

From the experiments conducted on specimens and with the analytical solution following concluding remarks are made.

- From the analytical study of Moment Curvature relationship by different models it has been observed that with increase in grade of concrete, there is an increment in the moment carrying capacity of section but substantial reduction in curvature ductility.
- Experimental results are very close to IRC 112 model up to the yield and then it follows IS 456 model in inelastic zone.
- With increase in the strain, flexure rigidity decreases which lead to degradation of stiffness.
- Flexure cracks are observed in tension zone and no shear cracks has been observed due to limitation of load applied.
- There is brittle failure in all three beam specimens. Steel reinforcement experiences large strain before failure that shows good bond between steel and concrete.
- Strain Compatibility method is accurate than Transform Area method because it gives Moment curvature at every strain value and values are more closer to experimentally obtained values.

Disclosures

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