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Dynamic Formulation for Cable Stayed Bridge and It's Validation

J.P. Variyavwala [1,*], A.K. Desai[2]

[1] Department of Civil Engineering, Saradar Vallabhbhai National Institute Technology, Surat, 395 007, India,
[2] Department of Civil Engineering, Professor, Saradar Vallabhbhai National Institute Technology, Surat, 395 007, India.

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Abstract

Cable stayed bridge is the most sensitive structure because of its higher flexibility. Exact bridge behavior can be obtained by proper dynamic bridge formulation. Cable stayed bridge deck and pylon can be modelled as abeam element. Researchers used to model cables with simple truss element and letter by parabolic cable element; although they are not presenting the exact cable performance under different lateral loads. In this study, a complete comprehensive study of modelling scheme for cable stayed bridge with improved catenary cable element is adopted by Equivalent modulus approach. The Modelling strategy involved finite element modelling with a dynamic stiffness method. A real CS Bridge is modelled in Midas-civil and results are compared. This improved innovative modelling approach agrees well with the bridge analysis results of the research article.

Keywords: Dynamic formulation, cable stayed bridge, catenary cable element

1. Introduction

Cable stayed bridge is the most favourite structure of researchers because of its aesthetic appearance, structural efficiency, and enhanced stiffness as compared to suspension bridges. CS Bridge allows the high span length with the slenderest section, this trend requires high strength material and an advance computation approach. The exact analysis for mode shape and time period for CS bridge is fundamental of the dynamic response of bridge due to lateral loads i.e., earthquake, wind and traffic loads. Earlier cables were modelled with a simple truss element. [1-7] This analysis approach is quite simple but provides inadequate dynamic results because it basically excludes the transverse cable vibration. In another way, resulting mode shapes are accounting for bridge deck, and tower only. The dynamic interaction between cable-deck and cable-pylon is completely eliminated. The cable, deck and pylon interaction is the most recent topic of research. [8-11]

More recent, earthquake load is the most predominant lateral load. This encounter the most efficient finite element method (FEM) analysis approach in which cables were modelled as a small string element. [12-17] outcome of this analysis provides more specific vibration modes. Coupled deck-cable interaction modes provide torsional and bending movement of the deck as the same time it give vertical and swinging motion of cables. This study encourages scrupulous multiple cable element modelling towards more reliable results.

Eventually, finite element approach is adopted with different meshing techniques which involve single element cable modelling and multiple element cable modelling. The outcome of such a study proved the requirement of a more accurate cable modelling approach; because cables are highly sensitive during seismic activity. [17-19]

Nowadays, the CS Bridge analysis become more precise because of the highly efficient FEM analytical software. The mathematical approaches are involved in precise prediction of closely spaced mode shapes. Boundary induced vibration completely changes the bridge performance.

In this article, CS Bridge is analysed by ordinary FEM and dynamic stiffness method (DSM) unitedly. The pylon and deck are modelled as an Euler-Bernoulli beam element. While adopting the dynamic stiffness method, each cable should act as an extensible sagging element. The outcome of DSM tolerates less lack of precision. The modelling of cable with improved catenary cable element by Equivalent modulus approach mitigates all basic analysis requirements of the bridge with a high precision.

2. Dynamic formulation of CS Bridge

The Bridge deck behaves as an Euler-Bernoulli beam element. So, let assume the small beam element i-j having L length with six-degree freedom system, whose stiffness is incorporate with the summation of stiffness due to bending and prestressing force. The governing equations are given for the stiffness calculation individually.

*Corresponding author. Tel: +919924860200; E-mail address: variyavwalajigar8@gmail.com

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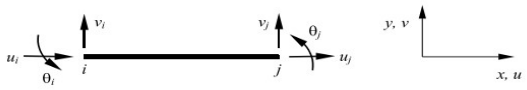

Figure 1 Beam element with six DOF

six end displacements or DOF a cubical variation in displacement is expressed by

$$v = Aa \tag{1}$$

A= (x_1 x_2 x_3 x_4 x_5 x_6) and a^T= (a_1 a_2 a_3 a_4 a_5 a_6) (Displacement variation within element. There are six DOF corresponding to the displacements u_i, u_j, v_i, v_j and the rotations θ_i , θ_j is given by,

q = Ca (Nodal displacements) (2)

With equations (1) and (2) we have,

$$V = AC^{-1}q \tag{3}$$

Where, C is the connectivity matrix and q^T= (v_i u_i θ_i v_j u_j θ_j). I is the moment of inertia and E is Young's modulus then bending moment M for the element is given by

$$M = D\frac{\partial^2 v}{\partial x^2} = DBC^{-1}q \tag{4}$$

Here, D =EI(x) and B= $d^2a/_{dx^2}$

2.1 Beam Stiffness as a result of bending

The potential energy U_B due to bending is

$$U_B = \frac{1}{2}\int_0^l \frac{d^2v}{dx^2} M dx \tag{5}$$

And the stiffness is given by

$$k_b = \frac{\partial^2 U_B}{\partial d^2} \tag{6}$$

By equations (5) and (6) we get,

$$k\bar{b} = \int_0^l B^T DBdx \quad \text{(Elemental)} \tag{7}$$

$$Kb = (C^{-1})^T k\bar{b} C^{-1} \quad \text{(Assembled)} \tag{8}$$

2.2 Beam Stiffness as a result of prestressing force

The potential energy of prestressing force is expressed by,

$$U_Q = \frac{1}{2}\int_0^l Q\left(\frac{\partial v}{\partial x}\right)^2 dx \tag{9}$$

So, the stiffness is presented by,

$$k_Q = \frac{\partial^2 U_Q}{\partial d^2} \tag{10}$$

By putting equation (9) in (10) we get,

$$\bar{k}_Q = \int_0^l A^T k_Q A dx \tag{11}$$

$$K_Q = (C^{-1})^T k_Q C^{-1} \tag{12}$$

Ultimately, the whole stiffness is given by

$$K = K_b + K_q \tag{13}$$

The Mass matrix for an element is equivalent to the nodal mass which represents the actual distributed mass of the element. Those are represented as the kinetic energy of the element.

$$T = \frac{1}{2}\int_0^l (\dot{v})^T \rho dV \dot{v} \tag{14}$$

Where, ρ = mass density and $\dot{v}$ = Lateral velocity

$$T = \frac{\rho}{2}(\dot{q})^T (C^{-1})^T \left\{\int_0^l A^T hxAdx\right\}(C)^{-1} q \tag{15}$$

Then, the mass matrix is given by,

$$m = (C^{-1})^T \overline{m} C^{-1} \tag{16}$$

$$\overline{m} = \rho \int_0^l A^T hxA\ dx \tag{17}$$

An expression for the free vibration of the beam is,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \tag{18}$$

An expression for forced vibration of the beam is,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f\} = [N]^{-1} f_0 \tag{19}$$

For cable stayed bridge, cables are thinner and the effect of prestressing force is comparatively high so, the effect of Prestress can be calculated as per equation 19.

3. Vibration of Cables

3.1 Original modulus approach

An alternative approach to calculate the transverse cables vibrations is the original modulus. The element incremental stiffness matrix in local coordinates can be written as

$$K_C = \frac{E_C A_C}{L_i(L_i+e)^2}\begin{vmatrix} (L_i+u_c)^2 & V(L_i+u_c) & -(L_i+u_c)^2 & -V(L_i+u_c) \\ V(L_i+u_c) & V_c^2 & -V(L_i+u_c) & -V_c^2 \\ -(L_i+u_c)^2 & -V(L_i+u_c) & (L_i+u_c)^2 & V(L_i+u_c) \\ -V(L_i+u_c) & -V_c^2 & V(L_i+u_c) & V_c^2 \end{vmatrix} + \frac{T}{(L_i+e)^3}\begin{vmatrix} V_c^2 & -V(L_i+u_c) & -V_c^2 & V(L_i+u_c) \\ -V(L_i+u_c) & (L_i+u_c)^2 & V(L_i+u_c) & (L_i+u_c)^2 \\ -V_c^2 & V(L_i+u_c) & V_c^2 & -V(L_i+u_c) \\ V(L_i+u_c) & -(L_i+u_c)^2 & -V(L_i+u_c) & (L_i+u_c)^2 \end{vmatrix} \tag{20}$$

Here, T and e are basic tension and extension of the element to the element longitudinal axis respectively given by,

$$T = T_i + E_C A_C / L_i e \tag{21}$$

$$e = \sqrt{V_c^2 + (L_i+u_c)^2} - L_i \tag{22}$$

T_i and L_i are the original cable pre-tension and the original cable length respectively, u_c and v_c are comparative displacements of single node acting along and perpendicular to the cable chord respecting the other node.

3.2 Equivalent modulus approach

Initially, In the CSB analysis, cables are modelled as a single truss element with an equivalent modulus to allow a sag effect as suggested by F. T. K. Au et al. [3]. The stiffness matrix in local coordinates for a cable element can be written as,

$$k_c = \frac{A_c E_{eq}}{l_c}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

The equivalent modulus of elasticity is,

$$E_{eq} = \frac{E_c}{1 + (wH_c)^2 A_c E_c / 12T^3} \tag{24}$$

Where, l_c, H_c, A_c, E_c are the chord length, the horizontal projection length, the cross-sectional area, and the effective material modulus of elasticity respectively. W stands for the weight per unit length and T is the updated cable tension of the cable. After getting the equivalent modulus, the profile will not affect the final analysis.

3.3 Mass matrix for cable

The cable element mass matrix is the same for both the single-element and multiple-element modelling methods. The mass matrix is given as follows,

$$m_c = \frac{m_{cc}}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \tag{25}$$

In which m_{cc} is the total mass of the cable element

3.4 Vibration of stay cables

To know local cable vibrations, each cable was analysed with an inclined profile and different support conditions. But in reality, the end anchorages are movable. The symmetric in-plane vibration and anti-symmetric in-plane vibration frequencies ω (radians/second) can be expressed, as given below suggested by F. T. K. Au et al. [3]

$\omega = \frac{\omega^*}{l}\sqrt{\frac{T_\theta}{m}}$ For symmetric in-plane vibration and,

$\omega = \frac{2\pi}{l}\sqrt{\frac{T_\theta}{m}}$ For anti-symmetric in-plane vibration,

Where, $\tan\left(\frac{\omega^*}{2}\right) = \left(\frac{\omega^*}{2}\right) - \frac{4}{\lambda^2}\left(\frac{\omega^*}{2}\right)^3$ (26)

Here, l & m represents the chord length, static cable tension and the mass of cable per unit length respectively.

4. Cable stayed bridge model validation

The geometry of the cable-stayed bridge chosen for this study is similar to that of the Quincy Bayview Bridge crossing the Mississippi River, the bridge is located in Illinois, United States of America. The bridge consists of two H-shaped concrete towers, double-plane fan type cables, and a composite concrete-steel girder bridge deck consisting of steel edge girders, steel floor beams and a reinforced concrete slab deck. The bridge has 274 meters main span and two equal 134 meters side spans. So, the total length of bridge is 542 meter. The tops of the towers are 71 meters from the waterline. There are a total of 56 cables, 28 supporting the main span and 14 supporting each side span. The width of the deck from center to center of cables is 12 meters.

4.1 Material and sectional properties of cable stayed bridge

For the bridge modelling modulus of elasticity are taken as 2 x 10^8 KN/m2 & 3 x 10^7 KN/m^2 for steel & Concrete respectively. Unit weight and Poisson ratios for steel & concrete are 76.97 KN/m^3 & 0.3 and 23.56 KN/m^3 & 0.25 respectively. Different nodes are provided at the place where element property change and the cable connection locations. The Bridge Tower modelled using 28 linear elastic beam elements. Bridge deck includes the concrete slab, main steel girder, five stringer steel beam and parapet which convert into a spine beam.

Table 1 Sectional Properties of Bridge

	Bridge component	Area [m^2]	I_{XX} [m^4]	I_{YY} [m^4]	J [m^4]
1	Cable- 1	0.0089	0.0	0.0	0.0
	Cable- 2	0.0067	0.0	0.0	0.0
	Cable- 3	0.0053	0.0	0.0	0.0
	Cable- 4	0.0034	0.0	0.0	0.0
2	Girder	0.8268	0.3409	19.76	0.027
3	Pylon (sec. 3)	17.88	129.46	16.39	13.69
	Pylon (sec. 2)	08.77	3.5386	15.311	9.8824
	Pylon (sec. 1)	07.06	3.4092	14.025	7.6988
4	Girder beam	0.0499	0.0031	0.0447	0.1331
5	lower strut	6.3638	7.3708	13.37	13.835
6	Upper strut	7.2464	8.2339	13.999	14.983

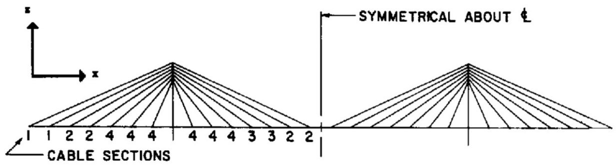

Figure 2 Cable Arrangement

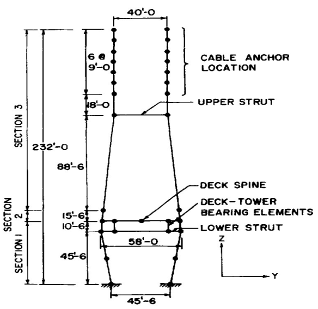

Figure 3 Cross Section of Pylon

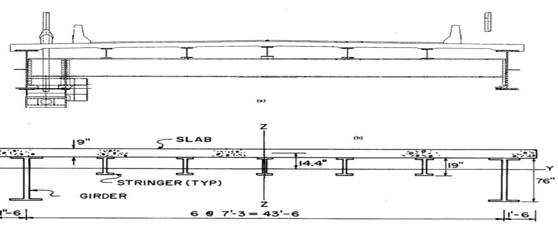

Figure 4 Bridge deck: (a) Split cross section of actual deck, left at cable anchor right between anchor, transverse floor beam is also shown, (b) Cross section used for FE Modelling.

4.3 Validation Problem Results Comparison

	As per author	Result Obtained	% difference
First Mode frequency	0.371 Hz	0.390 Hz	5.12 %
Time period	2.695 sec.	2.563 sec.	4.88 %

Analysis result is compared with wilson et. al. [22] research paper titled modelling of CS Bridge for dynamic analysis.

5. Conclusions

The dynamic formulation of cable stayed bridge with by stiffness method provided the nearest performance solution. While cable modelling by the Equivalent modulus approach provides the actual sag effect and interactive behaviour of cables during lateral loading.

Disclosures

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