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Non-Linear Visco-elastic Pounding in Multi-span Simply Supported Isolated Bridges

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Abstract

The pounding phenomena in multi-span, simply supported, isolated bridges under earthquake ground motion is investigated. The piers are considered as elastic members and the abutments as rigid ones. An isolated bridge,taken from literature, is modeled as a multi-degree of freedom system by using Matlab program and Simulink. State-of-the-art reveals that study on non-linear visco-elastic pounding inseismically isolated, simply supported bridges is limited. The present paper explores this problem of pounding in bridges by considering the bearing isolations. Results show that pounding occurs more prominently at the locations where there are changes in structural properties such as the changes in height of pier or the presence of abutment.

Keywords: Multi-span bridges, Non-linear visco-elastic pounding, Isolation

1. Introduction

During all major earthquakes, extensive damages of bridges were observed due to pounding of bridge segments. Pounding damage between adjacent bridge decks, unseating of bridge decks etc. were detected in the 1994 Northridge earthquake [1], 1999 Chi-Chi earthquake [2], 2001 Bhuj earthquake [3], 2008 Wenchuan earthquake [4], 2010 Chile earthquake[5] and 2011 Christchurch earthquake [6] etc. Due to pounding, damage of bearing supports and falling down of bridge decks was observed in Kobe earthquake 1995 [7]. It is found that the occurrence of pounding is inevitable during the life span of bridge due to the small gaps provided at expansion joints of bridge segments from the serviceability criteria of smooth traffic flow. The magnitude and the no of impact during seismic excitation depend on stiffness, mass and characteristics of out of phase movements of bridge segments. This out of phase movement of the bridge segments are occurred because of difference of stiffness of bridge components, passage of seismic wave, coherency loss during wave passage, variation of local site, soil structure interaction etc. Exact analysis and accurate prediction of pounding phenomena are not possible owing to the non-linear behavior of material of structural components like pier, bridge deck, base isolation pad etc., local damage and plastic deformation of material at the contact surface of impact, spatial variation of soil properties, earthquake incidence angle etc. However, extensive researches are accomplished to study the pounding phenomena of bridge segments under seismic excitation.

Pounding analysis of bridge segments were investigated by several research efforts [8-22]. Pounding of multiple frame bridge was studied by DesRoches and Muthukumar [8,9] and it was concluded that pounding is critical for highly out of phase frames and less pronounced for in phase frames ($T_1/T_2 \ge 0.7$). Pounding of bridge girder was investigated by P. Zhu, M. Abe and Y. Fujino [10] and validated the result of pounding analysis by numerical integration for 3-D bridge model with the experimental results. It was concluded by N. Chouw and H. Hao [13] that large non-uniform ground displacements strongly influence the impact force generated during pounding. It was referred by Z. Hai, J. Li and L. Jun-han [18] that multi support and multi-dimensional earthquake inputs increases the magnitude of pounding by 5-8 times the value of pounding under multi-support excitation.

However, in these studies, certain gaps are found in modelling of bridge against the actual bridge system. These gaps can adversely affect the global behavior of bridge for pounding analysis. For example, most of the investigation did not consider friction at bearing surface for simply supported bridge without isolation pads. Few studies [7,10,13,21,22] considered base isolation, but ignored the presence of abutments expect these investigations [10,13,22]. Most of the studies of pounding phenomena of bridge considered continuous girder [11,19] or frame structure [12,17] whereas, only a very limited study considered simply supported, multi-span bridge.

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Proceedings of the 12th Structural Engineering Convention (SEC 2022), NCDMM, MNIT Jaipur, India | 19-22 December, 2022 © 2022 The authors. Published by Alwaha Scientific Publishing Services, ASPS. This is an open access article under the CC BY license. Published online: December 19, 2022 doi:10.38208/acp.v1.544 The present study analyzes the pounding phenomena of simply supported, isolated bridge deck segments. Whole bridge, including all piers and abutments is modeled as multi degree of freedom lumped mass model, properly interconnected with spring and dashpot system. Seismic input is given to the bases of each pier and abutments as multi support excitation. Influence of excitation of all piers and abutments to the response of any bridge deck is considered according to the connected stiffness to that bridge deck by considering influence coefficient matrix.

Initial motivation of the present study came from the deficiency of approach for modeling of whole bridge and the limited no of approach for modeling of evaluation of impact forces during the collision of bridge decks. Stereo mechanical approach following classical theory of impact was used by R. DesRoches, S. Muthukumar [8] to evaluate impact forces during a collision. In other efforts, impact forces are determined by linear visco-elastic model by R. DesRoches, S. Muthukumar [9], Z. Hai, L. Jun-Han [18], M. S. Kim [20]. Modeling of impact force in the present study follows nonlinear visco-elastic model, in which the nonlinear spring, following the Hertz law of contact is applied together with the non-linear damper activated during the approaching period of collision. This approach simulates more accurately the process of energy dissipation during the impact.

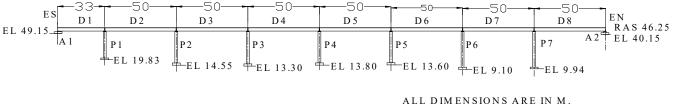
2. Numerical Simulation

2.1 Description of the benchmark bridge

In the present study, the Marga-Marga bridge, near Vina del Mar, in the Central Coasta region of Chile has been considered [23]. Detailed properties of the bridge are shown in Fig 1 to Fig 3 and Table 1 to Table 3

Pier	H (m)	H1 (m)	B (m)	Top portion	Middle Portion	Bottom Portion
Mkd						
1	21.865	1.5	10.5	A = 31.6 m2 A _{sy} = 26.86 m2 I _x = 657.385 m4 A _{sx} =26.86 m2 I _y = 10.533 m4 J = 37.92 m4 ρ = 2500 kg/m3 μ = 0.2 E = 3.3 x 10 ¹⁰	$\begin{split} A &= 6.38 \text{ m2} \\ A_{sy} &= 4.17 \text{ m2} \\ I_x &= 63.33 \text{ m4} \\ A_{sx} &= 1.25 \text{ m2} \\ I_y &= 4.18 \text{ m4} \\ J &= 12.658 \text{ m4} \\ \rho &= 2500 \text{ kg/m3} \\ \mu &= 0.2 \\ E &= 3.3 \text{ x } 10^{10} \end{split}$	$A = 57.75 \text{ m2}$ $A_{sy} = 49.088 \text{ m2}$ $I_x = 530.578 \text{ m4}$ $A_{sx} = 49.088 \text{ m2}$ $I_y = 145.578 \text{ m4}$ $J = 396.555 \text{ m4}$ $\rho = 2500 \text{ kg/m3}$ $\mu = 0.2$ $E = 3.3 \times 10^{10} \text{ N/mm}^2$
2	26.317	2	13.5	N/mm ²	N/mm ²	A = 74.25 m2
3	27.138	2	13.5			$A_{sy} = 63.113 \text{ m2}$
4	26.260	2	13.5			I _x = 1127.672 m4
5	26.082	2	13.5			$A_{sx} = 63.113 \text{ m2}$
6	30.154	2	13.5			$I_{y} = 187.172 \text{ m4}$ J = 552.531 m4 $\rho = 2500 \text{ kg/m3}$ $\mu = 0.2$ $E = 3.3 \text{ x } 10^{10} \text{ N/mm}^{2}$
7	30.086	1.5	10.5			Same as P-1

 Table 1: Geometrical dimensions and material properties of the piers [23]



A = A B U T M E N T, P = P I E RD = D E C K

Figure 1: General arrangement of Marga-Marga bridge [23]

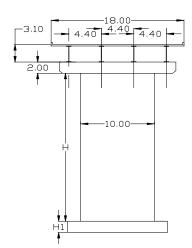


Figure 2 : Transverse view of Pier

Table 2: Properties of Bridge deck [23]

Deck Mkd	Properties of deck		
	A = 8.13 m2		
D1 to D8	$A_{sy} = 3.85 \text{ m2}$		
	$A_{sz} = 2.25 \text{ m2}$		
	$I_z = 238.6 \text{ m4}$		
	$I_y = 5.98 \text{ m4}$		
	J = 0.116 m4		
	$\rho = 2940 \text{ kg/m3}$		
	$\mu = 0.245$		
	$E = 3.3 \times 10^{10} Mpa$		

Table 3: Properties of Rubber Pads [23]

Pier/Abutment mkd	Properties of rubber Pad
	A = 1.738 m2
P1 to P7	$A_{sy} = 1.477 \text{ m2}$
	$I_z = 168.269 \text{ m4}$
	$A_{sx} = 1.477 \text{ m2}$
	$I_y = 0.099 \text{ m4}$
	J = 35.845 m4
	A = 0.906 m2
A1 (south abutment)	$A_{sy} = 0.770 \text{ m2}$
	I _z = 87.747 m4
	$A_{sx} = 0.770 \text{ m2}$
	$I_y = 0.017 \text{ m4}$
	J = 18.672 m4
	A = 1.109 m2
A2 (North abutment)	$A_{sy} = 0.943 \text{ m2}$
	I _z = 87.747 m4
	$A_{sx} = 0.770 \text{ m2}$
	$I_y = 0.017 m4$
	J = 18.672 m4

2.2 Theoretical Development

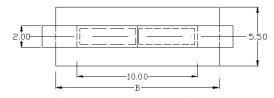


FIG: Cross section of Pier

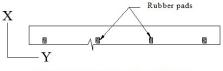


FIG: Rubber pads on top of Pier

Figure 3: (a) C/S of Pier (b) Plan view of top of Pier showing Rubber pads

2.2.1 Equation of motion

The equation of motion for the whole bridge as multidegree-of-freedom system for multi-support excitation can be written as

$$M_{ss}\ddot{x} + C_{ss}\dot{x} + Kssx = -M_{ss}\Gamma\ddot{x}_{g} \qquad \dots \dots \dots (1)$$

Where, $M_{ss} = Mass$ matrix corresponding to superstructure/non-support degrees of freedom. $C_{ss} =$ Damping force matrix corresponding to superstructure/nonsupport degrees of freedom. According to Rayleigh damping concept, it is assumed that the

$$C_{ss} = \alpha M_{ss} + \beta K_{ss} \qquad \dots \dots (2)$$

Where, the values of α and β are given by the following relations:

$$\alpha = 2\zeta \frac{\omega 1 \omega 2}{\omega 1 + \omega 2}, \ \beta = 2\zeta \frac{1}{\omega 1 + \omega 2}$$

where, $\omega 1$ and $\omega 2$ are the critical frequencies of the first two modes.

 K_{ss} = Stiffness corresponding to superstructure/non-support degrees of freedom.

 Γ = Influence coefficient matrix = - $K_{ss}^{-1}K_{sg}$

 $\ddot{\mathbf{x}}_{g}$ = ground acceleration vector

During the vibration of the bridge structure due to earthquake, if the movement of subsequent decks exceeds the gap between them, then the deck segments collides and pounding force is developed. During pounding, large impact force is generated in a short time. The process of energy transfer during pounding is highly complicated and the materials, which come in contact during collision, lose their properties. True nature of the contact surface is also uncertain. Co-efficient of friction and viscous forces are very difficult to guess. Due to all these reasons, correct evaluation of actual pounding force is very difficult. This pounding force influence the response of the bridge structure. Equation of motion catering the pounding force is inscribed below.

 $M_{ss}\ddot{x} + C_{ss}\dot{x} + K_{ss}x = -M_{ss}\Gamma\ddot{x}_g + P$ (3) Where, P is the pounding force vector. We can re-write the equation (3) as below $M_{ss}\ddot{x} = -C_{ss}\dot{x} - K_{ss}x - M_{ss}\Gamma\ddot{x}_g + P$ or, $\ddot{x} = -M_{ss}^{-1}C_{ss}\dot{x} - M_{ss}^{-1}K_{ss}x - \Gamma\ddot{x}_g + M_{ss}^{-1}P$ (4) For derivation of A matrix & B matrix of state space equation, we can write, $\dot{x} = \dot{x}$ (5)

From equation (4) and (5), we get equation (6), compatible to use in state space code in Matlab.

 $\begin{pmatrix} \ddot{x} \\ \dot{x} \end{pmatrix} = \begin{bmatrix} -\operatorname{inv}(\operatorname{Mss})\operatorname{Css} & -\operatorname{inv}(\operatorname{Mss})\operatorname{Css} \\ I & 0 \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \dot{x} \end{pmatrix} + \begin{bmatrix} I \\ I \end{bmatrix} \begin{pmatrix} -\Gamma \ddot{x}g + \operatorname{inv}(\operatorname{Mss})P \\ 0 \end{pmatrix} \dots \dots (6)$

Using the above equation & applying earthquake ground motions structural response & pounding forces are evaluated.

2.2.2 Derivation of the pounding force

Non-linear visco-elastic model is followed for determining pounding forces. This approach considers the non-linear spring, following the Hertz law of contact, and the nonlinear damper activated during the approach period of collision for deriving the pounding force. The pounding force during impact, F(t), for the non-linear viscoelastic model is expressed as referred by Jankowski [24] :

$$F(t) = \overline{\beta} \delta^{3/2}(t) + \overline{c}(t) \ \delta(t) \text{ for } \delta(t) > 0 \text{ (approach period)}$$
....... (7)

$$F(t) = \overline{\beta} \,\delta^{3/2}(t) \qquad \text{for } \delta(t) \le 0 \text{ (restitution period)}$$

...... (8)

Where, $\overline{\beta}$ is the impact stiffness parameter and $\overline{c}(t)$ is the impact element's damping, which can be obtained from the formula as prescribed by Jankowski [25]:

$$\bar{\mathbf{c}}(\mathbf{t}) = 2\bar{\xi}\sqrt{\overline{\beta}\sqrt{\delta(t)\frac{m1}{m1+m2}}}\dots \qquad (9)$$

Where, $\overline{\xi}$ denotes the damping ratio related to the coefficient of restitution, e

$$\bar{\xi} = \frac{9\sqrt{5}}{2} \frac{1 - e^2}{e(e(9\pi - 16) + 16)} \dots \dots \dots (10)$$

This nonlinear viscoelastic model is applied in the algorithm as shown in fig 6, programmed in the matlab model for calculation of pounding force during seismic excitation

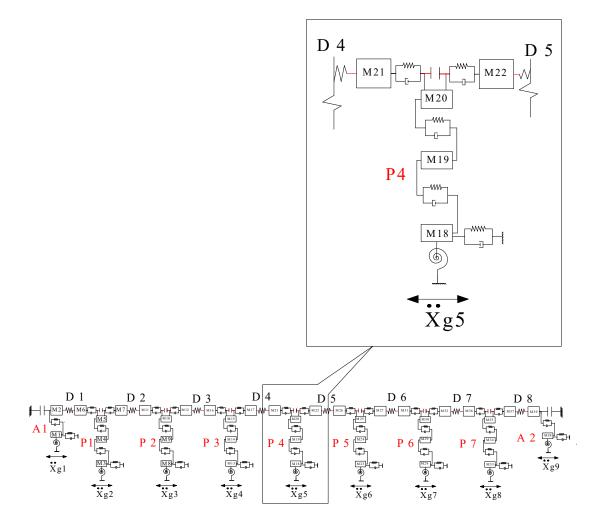


Fig 4: simplified mechanical model of margamarga bridge

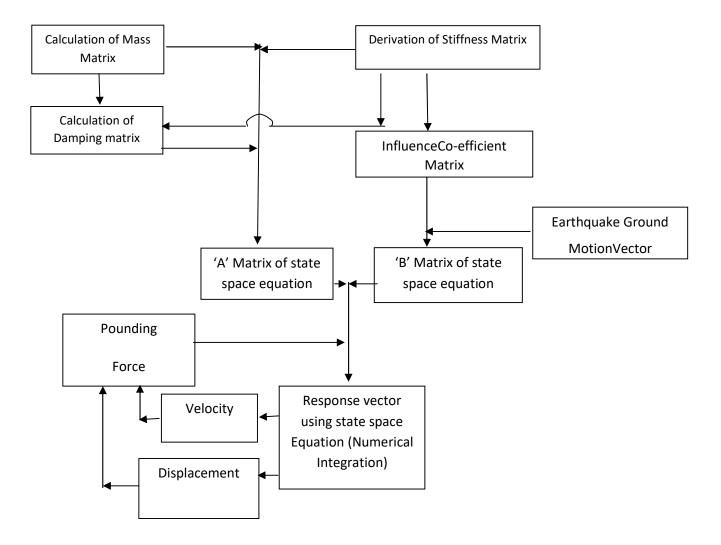


Fig 6: Algorithm for predicting the bridge response and Pounding Force

2.2.3 Analytical modeling of Structure

The whole bridge is formulated as lump mass model. Bridge model considered for simulation is shown in fig 4. The whole bridge is modeled in Matlab (using state-space equation & programing) and simulated against seismic excitations.Expansion gaps between the bridge decks are considered as 50 mm. Geometric properties of bearing pads supporting the bridge decks are mentioned in table 3. Equivalent shear modulus of the bearing pads is considered as 6 Mpaas considered by W. Dai [18] for the same bridge. Time history of Kobe earthquakeand El Centro earthquake

are applied at supports to simulate the bridge model. Simulation is executed in Matlab with the help of numerical integration through state space method.

2.2.5 Validation

First two natural frequency of the bridge model having fixed supports are derived as 0.549 Hz & 0.663 Hz and the bridge model having supports effected by soil-structure interaction are derived as 0.474 Hz and 0.599 Hz. The result is very close the first natural frequency of 0.65 Hz reported by W.

Dai [23] and 0.67 reported by Daza [25] for free decks in former studies of same bridge. Eigenvalue solution of the same bridge modeled in STAAD.Pro software reports the first two natural frequency as 0.574 Hz & 0.696 Hz for fixed base and 0.488 & 0.609 for base supports modified by SSI.

3.0 Results & Discussion

3.1 Displacements

Fig 6 shows the seismic response as displacements for the decks no 5 to deck no 8 under Kobe &ElCentro earthquake. Since, the response quantities of the right-side spans of the bridge are greater in magnitudes than the left side spans due to tall piers, results of the right side spans are shown here.

Response curves show that the displacement of the decks increases according to their proximity towards the center of the bridge length for single support excitations. Span near the abutment get the minimum displacement response. Here, displacement of deck 5 is found to be more than twice than the deck 8 for both Kobe and El Centro earthquake. Also the displacement of any deck towards the center of bridge is more than the displacements of decks located towards the abutment.

3.2 Pounding

Fig 7 shows the impact forces during pounding between deck 6 & deck 7 at pier 6 and the table 4 shows maximum impact forces at different decks.

Fig 8 and 9 represent all the major pounding incidents with their incidence time, peak and location.

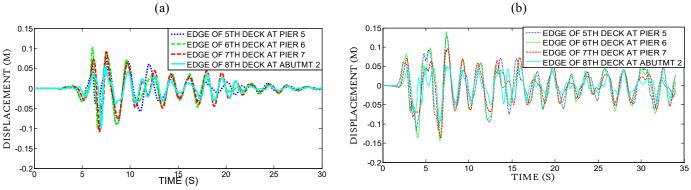


FIG 6: Displacement response of bridge decks under (a) Kobe earthquake (b) El-centro earthquake

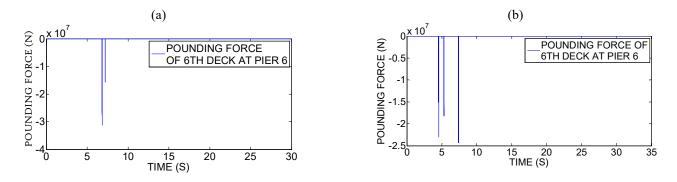


Fig 7: Pounding force of bridge decks under (a) Kobe earthquake (b) El-centro Earthquake

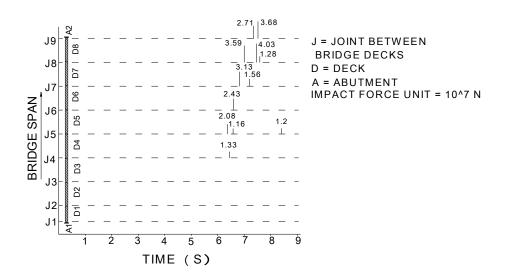


Fig 8: Sequence of peaks of impact forces at expansion joints under Kobe earthquake

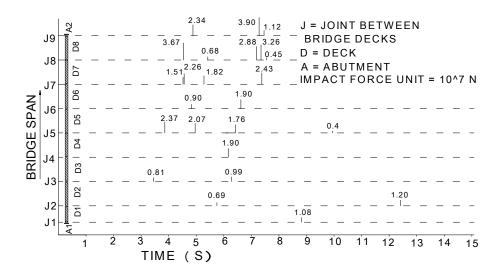


Fig 9: Sequence of peaks of impact forces at expansion joints under El-centro earthquake

Location	Maximum Pounding Force (10^7 N)		
	Kobe earthquake	El Centro earthquake	
Pier 1 (impact between D1 & D2)	0	1.203	
Pier 2 (impact between D2 & D3)	0	0.989	
Pier 3 (impact between D3 & D4)	1.338	1.905	
Pier 4 (impact betn D4 & D5)	2.088	2.37	
Pier 5 (impact between D5 & D6)	2.43	1.906	
Pier 6 (impact betn D6 & D7)	3.13	2.43	
Pier 7 (impact between D7 & D8)	4.036	3.67	
Abutment 2 (impact between D8 & fixed lateral support in horizontal direction)	3.682	3.9	

Table 5: Maximum	impact force	during po	ounding between
bride	e decks at dif	ferent Pie	rs

4 Conclusions

In this study, non-linear visco-elastic pounding phenomena is investigated in detail for long span simply supported, isolated bridges. The whole bridge is modeled and simulated in Matlab by using state-space equations. Seismic excitation is applied to all piers and abutments to study the global behavior of the bridge. This study concludes:

(1) Pounding forces are higher in magnitude at tall (flexible) pier locations. So, the bridges having tall and flexible piers are more prone to have poundings and more probable to

produce large impact forces during earthquakes than the bridges having stiff piers.

(2) After observing the sequence of pounding between the bridge decks, it is found that a series of impacts may occur at the expansion joints located sequentially within a very short period of time. During this series of poundings, transfer of energy from one deck to another deck may happen and certain bridge decks may experience a higher impact force owing to transfer and accumulation of energy.

5. List of Symbols and Abbreviations

The following symbols are used in this paper:

- 1. A = Gross cross sectional area
- 2. A_{sx} = Shear area in X direction (longitudinal direction of bridge)
- 3. A_{sy} = Shear area in Y direction (transverse direction of bridge)
- 4. B = Base width
- 5. C_{ss} = Damping matrix corresponding to superstructure
- 6. E = Young modulus of elasticity
- 7. H = Height

8.

- $I_x =$ Moment of Inertia around X axis
- 9. $I_y =$ Moment of Inertia around Y axis
- 10. [I] = Unit matrix
- 11. J = Torsional moment of inertia
- 12. K_{ss}= Stiffness matrix corresponding to superstructure
- 13. K_{sg} = Coupling stiffness matrix
- 14. M_{ss} = Mass matrix corresponding to superstructure
- 15. P = Pounding force
- 16. x = Relative displacement with respect to ground/ support
- 17. \dot{x} = Relative velocity with respect to ground/ support
- 18. \ddot{x} = Relative acceleration with respect to ground/ support

- 19. $\rho = Density$
- 20. μ = Poisson's ratio
- 21. $\omega 1 =$ Natural frequency of 1^{st} mode
- 22. $\omega^2 =$ Natural frequency of 2^{nd} mode
- 23. $\overline{\beta}$ = Impact stiffness parameter
- 24. δ = Deformation of colliding structural elements
- 25. $\dot{\delta}$ = Relative velocity between colliding structural elements
- 26. \overline{c} = Impact elements damping

Disclosures

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