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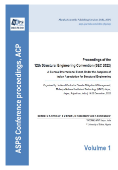

Comparison of Analytical and Finite element method of live load distribution in T Beam Girder Bridge

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Abstract

T-Beam bridges are widely used for short span of varying from 12 m to 25 m because of their cost effectiveness despite of its complexity in analysis. Live load distribution is a crucial step in analysis of any type of bridge since it gives force or moment for which bridge components need to be designed. Among the various methods of live load distribution, comparison between Courbon's Method, Morice-Little method and Finite element method has carried out herein. Present study aims to determine the most appropriate method of Live load distribution in T Beam Girder Bridge. Total eight bridge models were considered for the purpose of analysis with span varying from 12m to 24m i.e., 12m, 16m, 20m and 24m with three and four girders for each span. Finite element software SAP 2000 has used for Finite element analysis of Bridge. Finite element result were compared with Courbon's method and Morice little method results.

Keywords: Live Load Distribution, Courbon's Method, Morice-Little method, Finite element Method

1. Introduction

T Beam bridges are widely used for short span of varying from 12 m to 25 m [1],[2] because of their cost effectiveness despite of its complexity in analysis. Live load distribution is a crucial step in analysis of any type of bridge since it gives force or moment for which bridge components need be designed. Live-load distribution is a procedure to calculate each girder's carrying proportion for the applied live load.

There are several methods to calculate distribution of live load in a T Beam Girder bridge. Among the various methods of calculation lives load distribution, comparison between Courbon's Method, Morice-Little method and Finite element method has done herein. Present study aims to determine the most appropriate method to calculate Live load distribution in T Beam Girder Bridge.

Analysis of bridge can be done in three major ways. The first method divides the entire structure into a finite number of members in all direction, each with its properties of stiffness. Solving the equation of compatibility and continuity will give the solution of whole bridge. . Finite element method of bridge analysis falls into this type of analysis of bridge. Primary members are separated from reaming secondary members and the effect of secondary members on primary members considered in the second group of theories. Courbon's method of analysis falls into this type of analysis of bridge. In the third group of analysis the actual structure reduced into to an equivalent distributed system in the two principal directions i.e., a quasi-orthotropic plate. Morice-little method falls under this type of analysis of bridge

Lever-rule and the rigid-diaphragm [3] methods are two traditional methods to distribute the live load. Morice-Little method uses lever rule method to distribute the live load among the girders in which only two adjacent girders will share the applied live load. Courbon's and Finite element method of analysis of bridge rigid diaphragm method in which the load is shared by the entire girders based on rotation of cross beams.

2. Geometric Properties

Total eight bridge models are considered for the purpose of analysis with span varying from 12m to 24m i.e., 12m, 16m, 20m and 24m. Total Width of bridge kept 8.5m and Carriage way width kept 7.5m to carry the Two Lane loads as per clause 104.1.3 of IRC: 5-2015.[4] Thickness of deck slab kept as 225mm. Total depth of longitudinal girder kept 1.525m with the thickness of 0.3m and Total depth of cross girder kept 1.525m with thickness of 0.2m. Total 5 number of cross girder are given in a bridge for all spans by varying spacing of cross girder. Number of longitudinal girder kept 3 and 4 for each span as shown in Figure 1and Figure 2 respectively. Cross sectional details of bridge kept

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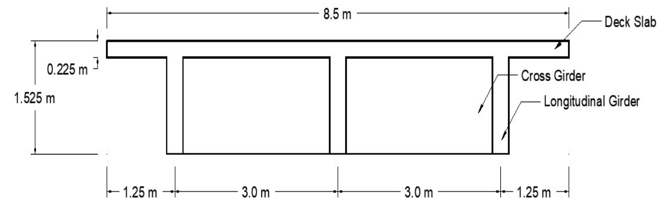

Figure 1. Typical cross section of 3Girder Bridge

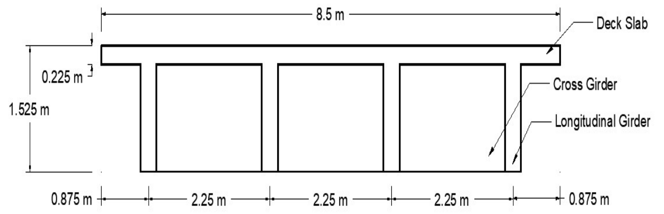

Figure 2. Typical cross section of 4Girder Bridge

Table 1. Geometry of Bridge

Bridge Index	Span (m)	No of Longitudinal Girder	Spacing of Longitudinal Girder (m)	Spacing of Cross Girder (m)
12m-3G	12	3	3	3
12m-4G	12	4	2.25	3
16m-3G	16	3	3	4
16m-4G	16	4	2.25	4
20m-3G	20	3	3	5
20m-4G	20	4	2.25	5
24m-3G	24	3	3	6
24m-4G	24	4	2.25	6

constant and are listed in **Table 1**. and given in **Figure 1**. and **Figure 2**. Live loads are considered as per Table 6. Of IRC: 6-2016.[5] According to which 1Lane of 70R Tracked, 1Lane of 70R Wheeled and 2Lane of Class A Loading has to be considered for 2Lane bridge. Edge distance, clearance between successive vehicles and adjacent vehicles are considered as per Table 6 of IRC: 6-2016

3. Courbon's Method

Courbon's method is one of the earliest and simplest methods which are based on comparatively gross assumptions. Indian road congress recommends this method because of its simplicity. This method falls into the category of diaphragm method according to which the wheel load is linearly distributed into each girder based on to rigid rotation of the cross beams or transverse diaphragm. Courbon's method assumes the linear variation of deflection in transverse direction. The assumption made here is not realistic since the deflection profile is non linear in nature but this assumption reduces the complexity in mathematical modeling to greater extent. This assumption leads to cause maximum deflection in exterior beam on the side of eccentricity of loading on the other hand minimum deflection in exterior beam on the opposite side of eccentricity of loading.

Bending moment shared by any girder can be calculated by multiplying Total bending moment over entire bridge with the reaction factor. The reaction factor (Ri) can be calculated as follows:

$$R_i = \frac{PI_i}{\Sigma I_i}\left[1 + \frac{\Sigma I_i}{\Sigma I_i d_i^2} ed_i\right] \quad (1)$$

Where, P = Total Live load, Ii= moment of inertia of longitudinal girder i, e = eccentricity of the live load, d_i= distance of girder I from the axis of the bridge

3.1 Limitations of Courbon's method

Courbon's method can be used when the following condition satisfied,

- The span to width ration should be greater than 2 but less than 4.
- The longitudinal beams are interconnected by at least five symmetrically spaced cross girders/or diaphragms.
- Total depths of cross girders should be kept at least 3/4th of the total depth of the longitudinal girders.

4. Morice-Little method

This method was firstly proposed by Gyuon [6] for torsion less grillage and for slabs and has been generalized by Massonet [7] . Gyuon initially considered a quasi orthotropic plate and derived an equation for deflection based on Lengrange equations. Advantage of quasi orthotropic plate is that it can be used for almost any structural shape in the same general terms. The method has become known as the distribution coefficient method. Major disadvantage of this method is it cannot be used for skew bridge.

In order to get the bending moment for each girder, the mean bending moment has to be multiplied by a distribution factor K. The distribution factor K can be obtained from curves prepared by Guyon and tabulated by Massonet. Gyuon developed set of curves to find distribution factors by assuming that there is no effect of poison's ratio and the same were tabulated by Massonet. Later when Morice compared the experimental results with the results obtained by Gyuon curves he found that difference were considerable. Then Morice [8] modified the curves by including g poison's effect. The differences between theoretical and practical were in the order of 10 percent. Guyon's suggestion is that, for practical purposes it s necessary to increase the theoretical values by 10%.

In order to get distribution factor K for any bridge the flexural properties and Torsional properties of the bridge has to be expressed in a unique parameter ϴ and α as shown in equation 2 and 3 respectively. The Geometrical properties of the bridge deck under analysis are defined in terms of some special parameters. The effective width is equal to the bridge deck is defined as 2B where 2B is defined as mp, where m is the number of girders for the bridge deck consists of a number of longitudinal girders at a spacing p.

$$\theta = \frac{b}{2a}\left(\frac{i}{j}\right)^{0.25} \quad (2)$$

$$\alpha = \frac{G\left(i_o + j_o\right)}{2E\sqrt{ij}} \quad (3)$$

Where,

i = longitudinal moment of inertia of the equivalent deck per unit width = I/P,

j = transverse moment of inertia of equivalent deck per unit length =J/Q ,

I = moment of inertia of area of each longitudinal girder,

P =spacing of longitudinal girders

J = Moment of inertia of area of each transverse cross beam

Q = Spacing of cross beam,

E = Young's modulus of material of deck

i_o= torsional stiffness constant of a longitudinal girder

j_o=torsional stiffness constant of a cross beam

In any bridge, when a concentrated load is placed, transverse section of the deck will be distorted due to varying deflection across the width. For a given bridge this deflection depends upon longitudinal und transverse position of load and flexural and structural properties of deck.

Morice little divided the effective width into eight equal segments, the nine boundaries which are known as standard position as shown in Fig Initially the average moment by the uniformly distributed load across the width will be find then the actual moment at each of this nine points will be find by multiplying distribution factor K. The distribution factor K at each nine station for the corresponding bridge will be find using the curves then these values will be interpolated to get the moment at actual location of load and Girder position.

5. Finite Element Method (FEM):

Finite element method is a general approach to structural analysis. Its generality makes it the most suitable one for analysis of structures with arbitrary properties or geometries. The element stiffnesses are assembled in the proper manner to form the structure stiffness matrix. After proper modifications for the boundary conditions have been made, the rows of the structure stiffness matrix provide the coefficients relating the nodal point displacements to the applied nodal point loads. Thus, for the given nodal point loads, the displacements corresponding to the assumed degrees of freedom at the nodes can be evaluated. The element strains and stresses can then be obtained using the calculated nodal point displacements, the assumed displacement variations, and the material elastic properties of the element. Cross et al [9] conducted experiment test on Illinois bridges and compared the results with results obtained by Finite element results and concluded that Finite element results were in good agreement especially regarding the load distribution of live load through bridge.

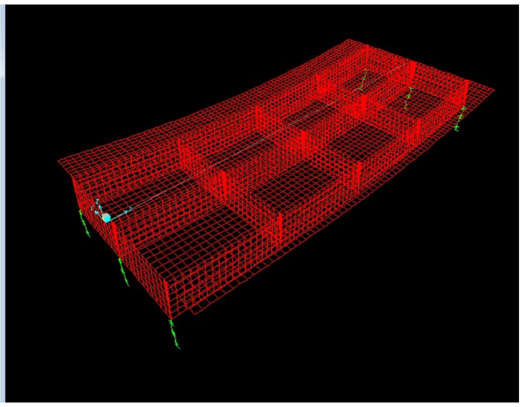

Figure 3. Typical Finite Element Modeling 3Girder Bridge

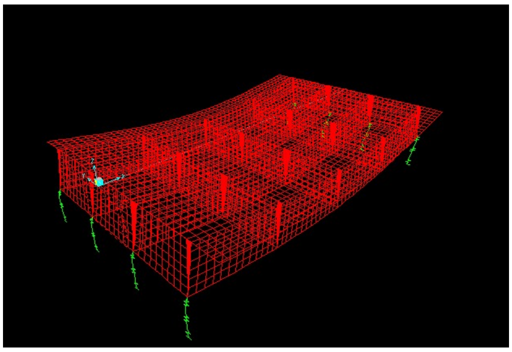

Figure 2. Typical cross section of 4Girder Bridge

Finite element software SAP 2000 has used for Finite element analysis of Bridge. Four noded thin shell elements with six degree of freedom at each node have used for Slab and Girder. Geometric dimension given Table 1 are used for Finite element modeling. Figure 4 and Figure 5 shows typical finite element modeling of Three Girder and Four Girder Bridge. Nakin Suksawang and Hani H. Nassif [10] suggest that boundary conditions of bearing can be simulated by restricting displacement. Roller connections were used to represent expansion bearings and pinned connections were used to represent pinned bearings. The rollers allowed for movement in the x direction and rotation about the y axis, whereas the pin only allowed for rotation about the y axis. The remaining degrees of freedom were fixed for each support.

Results and Discussion

Considering Finite element analysis results are more accurate and more closed to filed values the same are compared with Courbon's method of results and Morice Little method results. BM values at mid span of different span and different Loadings are represented in Table from Table 3-Table11.

1Lane of 70R Tracked Loading outer Girder results, 1Lane of 70R Tracked Loading Inner girder results, 1Lane of 70R Wheeled Loading outer Girder results, 1Lane of 70R Wheeled Loading Inner girder results, 2Lane of Class A Loading outer Girder results, 2Lane of Class A Loading Inner girder results are represented in Table 2, Table 3, Table 4, Table 5, 6, respectively. It can be observed from the results that FEM results are lesser in comparison of

Courbon's Method results and Morice Little method results except in case of 70R tracked Loading. The reason for this would be dynamic effect of distributed load is more in case of area load in FEM analysis than Point Load since the 70R Tracked load area load unlike in case of 70R wheeled load and Class A loading which are point load in nature.

It is also can be observed that percentage difference between FEM and Courbon's method is more in case of 4 Girder bridge and less in case 3 Girder Bridge Where as in case of inner Girder FEM results of 12m and 16m are higher than Courbon's method results and lesser for 20m and 24m.

From Table 5 and 6 it can be observed that % difference between FEM and Courbon's method is less in case of 4 Girder bridge and higher in case 3 Girder in case of Outer Girder where as in case of Inner Girder percentage Difference between FEM results and Courbon's methods are more in case of 4Girder and Less in case 3Girder bridge. The reason behind this can be in case of 3 Girder bridge $\Sigma I_i d_i^2$ reduces in comparison to 4Girder since d_i is zero for middle Girder in equation 1. Since $\Sigma I_i d_i^2$ term is in denominator with reduced value increase the R_i for the outer Girder. In case of inner girder loading d_i is zero for 3Girder which makes R_i lesser for Inner girder and which is not equal to zero in 4Girder bridge which leads to higher R_i.
Load is nearer to the Inner Girder both in case of 3Girder and 4Girder Bridge which makes more deviation of Morice Little method from FEM results since eccentricity of loading place vital role in Morice Little method.
From Table 7 and 8 it can be observed that % difference between FEM and Courbon's method is less in case of 4 Girder bridge and higher in case 3 Girder both in case of Outer Girder Inner Girder. For outer Girder reason is as same as in case of 70R Wheeled Loading where as incase of inner Girder loading is nearer to inner compared to above 2 loading cases which makes more eccentricity for outer girder thereby increasing R_i for outer Girder. In case of Morice Little method since there is 2lane of Class A loading which is distributed over wider portion of deck unlike in above two cases makes the results more reasonable since reference station and actual loading station are almost near to each other.
Over all it can be observed that deviation of results from FEM results in case of Courbon's method is no dependent on span where as in case of Morice method deviation is increasing with the span. The reason for this can be cross

Table -2 Results and comparison of 70R Tracked Loading Outer Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	1003.00	961.14	4.17	1021.87	-1.88
12m-4G	700.04	731.97	8.73	785.73	2.03
16m-3G	1421.10	1356.87	4.51	1454.29	-2.33
16m-4G	1133.43	1033.34	8.83	1127.72	0.50
20m-3G	1856.62	1752.60	5.60	1878.44	-1.18
20m-4G	1446.17	1334.72	7.71	1456.62	-0.72
24m-3G	2186.86	2148.33	1.76	2301.17	-5.23
24m-4G	1699.74	1636.10	3.74	1788.27	-5.21

Table -3 Results and comparison of 70R Tracked Loading Inner Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	991.63	980.68	1.10	1082.22	-9.13
12m-4G	604.19	570.92	5.51	668.61	-10.66
16m-3G	896.93	894.45	0.28	1007.08	-12.28
16m-4G	789.57	805.98	-2.07	922.68	-16.85
20m-3G	1118.22	1155.32	-3.32	1300.80	-16.33
20m-4G	1016.74	1041.05	-2.39	1191.78	-17.22
24m-3G	1362.22	1416.19	-3.96	1589.40	-16.68
24m-4G	1216.20	1276.11	-4.93	1457.47	-19.84

Table -4 Results and comparison of 70R Wheeled Loading Outer Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	966.44	980.68	-1.43	1082.22	-11.98
12m-4G	741.98	747.52	-0.75	837.48	-12.87
16m-3G	1486.31	1538.59	-3.52	1704.49	-14.68
16m-4G	1150.58	1172.77	-1.92	1328.49	-15.46
20m-3G	1984.44	2110.88	-6.37	2338.50	-17.84
20m-4G	1535.46	1609.00	-4.79	1822.64	-18.70
24m-3G	2456.74	2723.13	-10.84	3013.00	-22.64
24m-4G	1910.13	2075.68	-8.67	2352.29	-23.15

Table -5 Results and comparison of 70R Wheeled Loading Inner Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	582.01	634.13	-8.95	726.52	-24.83
12m-4G	527.11	576.57	-9.38	691.00	-31.09
16m-3G	891.69	994.88	-11.57	1117.46	-25.32
16m-4G	799.19	904.58	-13.18	1061.08	-32.18
20m-3G	1208.32	1364.94	-12.96	1533.12	-26.88
20m-4G	1099.11	1241.05	-12.91	1455.76	-32.45
24m-3G	1564.16	1760.84	-12.57	1971.56	-26.05
24m-4G	1406.36	1601.01	-13.84	1873.68	-33.22

Table -6 Results and comparison of Class A Loading Outer Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	774.16	784.68	-1.35	850.49	-9.86
12m-4G	586.00	593.45	-1.27	644.87	-10.05
16m-3G	1126.00	1158.93	-2.92	1274.96	-13.23
16m-4G	874.86	876.49	-0.19	977.07	-11.68
20m-3G	1563.19	1567.28	-0.26	1724.20	-10.30
20m-4G	1183.00	1185.32	-0.20	1321.36	-11.70
24m-3G	1991.89	2025.92	-1.71	2229.57	-11.93
24m-4G	1520.00	1532.19	-0.80	1713.78	-12.75

Table -7 Results and comparison of Class A Loading Inner Girder

Bridge Index	Finite Element Method	Courbon's Method		Morice Little Method	
	BM (kN-m)	BM (kN-m)	% Diff	BM (kN-m)	% Diff
12m-3G	555.48	594.36	-7.00	726.52	-30.79
12m-4G	473.17	503.31	-6.37	691.00	-46.04
16m-3G	800.36	877.84	-9.68	1117.46	-39.62
16m-4G	689.36	743.36	-7.83	836.00	-21.27
20m-3G	1150.36	1187.15	-3.20	1533.12	-33.27
20m-4G	1002.36	1005.29	-0.29	1455.76	-45.23
24m-3G	1496.36	1534.55	-2.55	1971.56	-31.76
24m-4G	1279.36	1299.47	-1.57	1873.68	-46.45

girder spacing is increasing as the span increases as mentioned in Table1. Since the Courbon's method is not dependent on cross girder spacing deviation and the same is independent from cross girder spacing whereas in case of Morice little method the basic flexural and torsional properties are more dependent on spacing of girders causing the increasing deviation of loading with increase of span.

6. Conclusions

Total 8 bridge models are analyzed with spgfvtgggtfan varying from 12m to 24m (i.e., 12m, 20m and 24m) bridge is more suitable for this span range by Courbon's method , Morice Little method and Finite element Analysis Method with the different loading as prescribed in IRC6:2016. From the results we can concluded that

1. Courbon's method, Morice Little Method Bending Moment values are always conservative compared to FEM method results except in case of 70R Tracked results.
2. Though the Courbon's method simplest method it gives almost nearer results as FEM results. The percentage difference Courbon's method varies from +5.51% to 13.18%
3. Increase of 10% of BM values for practical purpose in IRC loading cases is not required as the Morice Little method BM values are always higher than FEM BM values.
4. Span and Cross Girder spacing plays vital role in Morice method. As the span and spacing of girder (both Longitudinal Girder and Cross Girder) increase deviation from FEM results increases.

Disclosures

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