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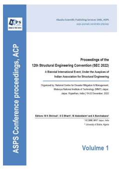

Influence of moment modification factor in lateral torsional buckling of doubly and monosymmetric sections

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Abstract

Simply supported doubly symmetric prismatic beam subjected to uniform moment along the unsupported length is the simplest and basic case in the buckling analysis. The closed form solution for the elastic critical moment was derived based on the assumptions of perfect beam with standard restraint conditions at both ends of the beam. A perfect beam will be having linear-elastic behaviour without any type of geometrical or material imperfections. The standard conditions of restraint at each end of the beam in basic case are restrained against lateral movement, restrained against rotation about the longitudinal axis and free to rotate on plan. But in reality, loading conditions and conditions of restraint will vary widely from the basic case. An equivalent uniform moment factor is typically applied to elastic critical moment to account for the effects of variable moment and end restraints along an unbraced girder length. The values are derived numerically and varies with specifications. In this article equivalent uniform moment factor for IS 800-2007 and Eurocode 3 provisions are compared. The study is also extended to monosymmetric I sections.

Keywords: *Lateral torsional buckling, Elastic critical moment, equivalent uniform moment factor, monosymmetric sections.*

1. Introduction

Structural members that support the transverse loads by bending and shearing action are referred as beams. The deflection of a beam in the plane of loading is defined as flexural deflection whereas that in the perpendicular plane is called buckling deformation. When an unrestrained beam is subjected to transverse loading, lateral deflection occurs along with twist about the shear centre of the cross-section. This phenomenon is known as Lateral Torsional Buckling (LTB). [1,2] The primary reason for this combined flexural and torsional stability failure criteria is due to (i) the bending compressive stress tries to destabilise the beam in lateral direction and (ii) the bending tensile stress tries to stabilise the beam cross-section by anchoring the tension flange resulting in twisting deformation.

The buckling behaviour of beams are generally classified as three types, (i) elastic buckling that occurs in long members when the cross-section is fully elastic, (ii) inelastic lateral torsional buckling that occurs in beams of intermediate length and the bending moment is just sufficient to cause portions of the member to yield and, (iii) local buckling that occur in stocky members where the beam is able to reach its local buckling capacity before failure by lateral torsional buckling.[3] Lateral torsional buckling can occur in elastic or inelastic range which in turn depends upon the laterally unsupported length of the section.

In the case of lateral torsional buckling of beams, an upper limit to the load carrying capacity of the beam is provided by elastic critical moment for constant moment or equivalent constant moment. However, the design resistance will be lower than the elastic critical moment. The factors influencing elastic critical moment are distribution of moment along unsupported length, position of load with respect to shear center, boundary conditions at the support, restraint along member length and buckling interaction. [1,9] Distribution of moment along the unsupported length is accounted by moment modification factor. End boundary conditions and effect of loading condition are taken into account using effective length factors for doubly symmetric beam subject to constant moment. The design provisions exemplify doubly symmetric beams but not monosymmetric beam.

This article details the LTB behaviour of doubly symmetric and monosymmetric beams as per IS 800-2007 [12], Eurocode 3 [11] and the influencing parameters are addressed carefully.

2. Limit state of LTB

The limit state of LTB is considered in the determination of design bending strength of laterally unrestrained beam.The methodologies adopted for the same in Eurocode 3 and IS 800-2007are discussed in this section.

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Table-1.Recommended values of imperfection factors for lateral torsionalbuckling curves

Buckling curve	a	b	c	d
Imperfection factor	0.21	0.34	0.49	0.76

Table-2.Recommended values for lateral torsional buckling curves for cross-sections in general case.

Cross-section	Limits	Buckling curve
Rolled I sections	$h/b \leq 2$ $h/b > 2$	a b
Welded I sections	$h/b \leq 2$ $h/b > 2$	c d
Other cross-sections		d

2.1. Eurocode 3

According to EC3 [11], the design buckling resistance moment of a laterally unrestrained beam isdeterminedfrom EQ.1.

$$M_{bRd} = \frac{\chi_{LT} W_y f_y}{\gamma_{mo}} \quad (1)$$

For Class-1 and Class-2 cross sections Wy is the plastic section modulus and for Class-3 cross section, it is the elastic section modulus. The EC3 specifies two methods for calculating the reduction factor-General case and Specific case. The general case is a simple, conservative method and it can be applied to any type of beam sections. The reduction factor according to general case can be calculated using equation 2.

$$\chi_{LT} = \frac{1}{(\varphi_{LT} + (\varphi_{LT}^2 - \lambda_{LT}^2)^{0.5})} \leq 1.0 \quad (2)$$

$$\varphi_{LT} = 0.5\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2\right] \quad (3)$$

$$\lambda_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \quad (4)$$

α_{LT} takes into account initial member imperfections, residual stresses and other non-linear effects.The imperfection factor depends upon the type of buckling curve, which in turn depends upon the cross-sectional parameters. Recommended values of imperfection factors and buckling curves are specified in Table 2 and Table 3.

2.2. IS 800-2007

In IS 800-2007[12], the design bending strength of laterally unsupported beam, M_d is determinedby Eq. (5).

$$M_d = \beta_b Z_p f_{bd} \quad (5)$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}} \quad (6)$$

where β_b =1 for plastic and compact section and $\beta_b = Z_e/Z_p$ for semi compact section.The design bending compressive stress which is given by Eq. (6)depends upon the reduction factor and partial safety factor for the material. Reduction factor can be calculated by using equations (2) and (3). The imperfection factor α_{LT} has only two values as per IS 800-2007. The values are 0.21 for rolled section and 0.49 for welded section. The non-dimensional slenderness is given by

$$\lambda_{LT} = \sqrt{\frac{\beta_b Z_p f_y}{M_{cr}}} \leq \sqrt{\frac{1.2 Z_e f_y}{M_{cr}}} \quad (7)$$

3. Determination of elastic critical moment

3.1. Doubly symmetric section

Simply supported doubly symmetric prismatic beam subjected to uniform moment along the unsupported length is the simplest and basic case in the buckling analysis. The closed form solution of elastic critical moment was derived based on the assumptions of perfect beam with standard boundary conditions at beam ends. Perfect beam will be having linear elastic behaviour without any type of geometrical or material imperfections. The standard conditions of restraint at each end of the beam in basic case are restrained against lateral movement, restrained againstrotation about the longitudinal axis and free to rotate on plan. The expression for elastic critical moment is as follows

$$M_{cr} = \frac{\pi^2 E I_y}{L^2} \sqrt{\frac{I_w}{I_y} + \frac{G I_t L^2}{\pi^2 E I_y}} \quad (8)$$

The loading pattern varies widely from the basic case in reality.The effect of moment distribution along an unbraced girder length is accounted with the help of an equivalent uniform moment factor,c_1.The expressions derived numerically for finding out equivalent uniform moment factor can be divided into three groups. This factor is depending on moment gradient/ bending moment variation and lateral support condition

The boundary conditions are specified in terms of lateral and warping restraints and incorporated in the above equation usingeffective length parameter in IS 800-2007 [12]. The destabilizing effect due to top flange loading is also taken care in the calculation of effective length parameter.The equation (8) is now modified as.

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(L_{LT})^2} \sqrt{\frac{I_w}{I_y} + \frac{G I_t (L_{LT})^2}{\pi^2 E I_y}} \quad (9)$$

Guidelines for finding out elastic critical moment are not present in Eurocode 3,because of the complex nature of lateral torsional buckling. It is only specified that M_{cr} should be calculated based on gross cross-sectional properties and it should consider the loading conditions, the real moment distribution and lateral restraints.Whereas it is detailed in ECCS technical committee report, Annexure B[8]The effect of boundary condition is incorporated by the introduction of lateral bending coefficient K and warping restraint factor K_w in the report.

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2} \sqrt{\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y}} \qquad (10)$$

The equation gets modified with the introduction of a new factor c_2 which depends on distance between the point of application of load and the shear centre of the cross section, y_g. The value of y_g is positive when the load is acting towards the shear centre from the point of application.

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2} \sqrt{\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + (C_2 y_g)^2} - C_2 y_g \qquad (11)$$

3.2. Monosymmetric section

Elastic critical moment values are higher for sections which are symmetrical only about the minor axis and bending about major axis when compared with bisymmetric sections of same area. [4,6]The non-coincidence of shear centre and centroid causes a torque, which changes the effective torsional rigidity of the section. The effective torsional rigidity increases when the larger flange is in compression which increases the buckling resistance. The equation for finding out the elastic critical moment values of monosymmetric sections are derived from bisymmetric sectionswith the help of monosymmetry parameter.

$$M_{cr} = \frac{\pi^2 EI_y}{L^2} \{ \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \frac{I_w}{I_y} + \frac{GI_t L^2}{\pi^2 EI_y}} \} \qquad (12)$$

$$\beta = \frac{1}{I_z} \int_A (z^2 + y^2)\, y dA - 2y_s \qquad (13)$$

The value of y_s is positive when the shear centre is on the compression side of centroid and y, z are the co-ordinates of the elemental area with respect to centroid of the section. The computation of monosymmetric parameter with the above equation is difficult and hence approximate formulae have been used commonly. The equation (12)have been modified and used in ECCS 2006 [8] as well as in IS 800-2007[12].

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(L_{LT})^2} \left\{ \left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (L_{LT})^2}{\pi^2 EI_y} + \left(c_2 y_g - c_3 y_j\right)^2 \right]^{0.5} - (c_2 y_g - c_3 y_j) \right\} \qquad (14)$$

$$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y dA / I_z \qquad (15)$$

where $c_1$accounts for moment gradient with respect to lateral end restraints, $c_2$accounts for load position with respect to moment gradient and lateral end restraints, $c_3$considers the influence of monosymmetry with respect to moment gradient and the end restraints.y_j is the monosymmetry parameter.

Both IS 800-2007 and Eurocode 3 have been using the approximate formulae for the calculation of monosymmetric parameter. The formulas presented in IS 800-2007 for plain flanges are

$$y_j = 0.8(2\beta_f - 1)\frac{h_y}{2} \quad (\text{when } \beta_f > 0.5) \qquad (16)$$

$$y_j = 1.0(2\beta_f - 1)\frac{h_y}{2} \quad (\text{when } \beta_f \leq 0.5) \qquad (17)$$

$$\beta_f = \frac{I_{yc}}{(I_{yc} + I_{yt})} \qquad (18)$$

where h_y is the distance between shear centre of the two flanges of the cross section and β_f is the degree of monosymmetric parameter which depends upon moment of inertia of the compression flange about the minor axis of the entire section

The formulas presented in Eurocode 3 (ECCS 2006) for plain flanges are given in equations (19) and (20). The concept of degree of monosymmetric parameter is adopted with the introduction of a factor ψ_f given as

$$y_j = 0.8\psi_f \frac{h_y}{2} \quad (\text{when } \psi_f \geq 0) \qquad (19)$$

$$y_j = 1.0\psi_f \frac{h_y}{2} \quad (\text{when } \psi_f < 0) \qquad (20)$$

$$\psi_f = \left(\frac{I_{yc} - I_{yt}}{I_{yc} + I_{yt}}\right) \qquad (21)$$

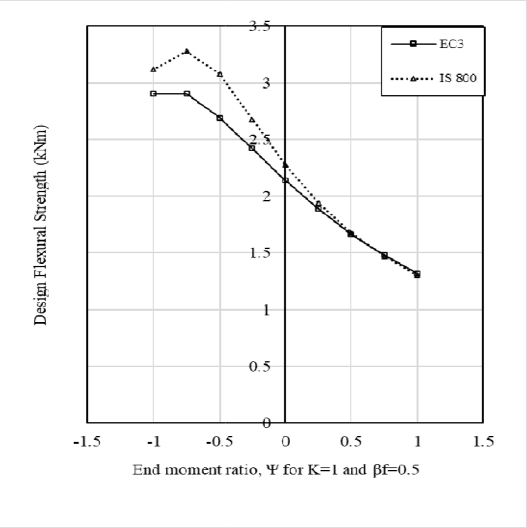

Fig-1.Design flexural strength values for various end moments of doubly symmetric cross section with K=1

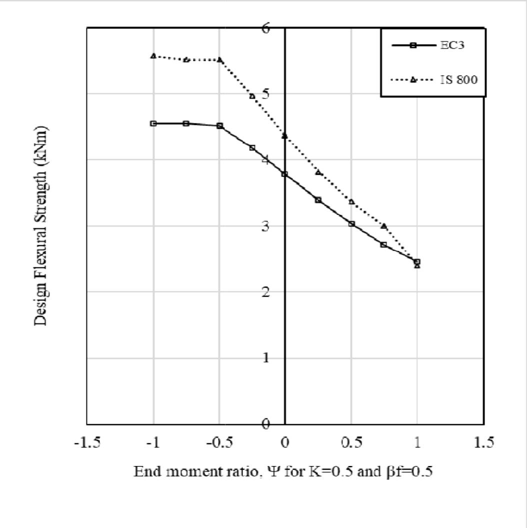

Fig-2.Design flexural strength values for various end moments of doubly symmetric cross section with K=0.5

4. Influence of c_1, c_2 and c_3 factors

4.1. The c_1 factor

The c_1 factor, also called the equivalent uniform moment factor was introduced to take into account the effect of moment distribution along unbraced length. IS 800-2007 and Eurocode 3 gives coefficients for beams subjected to varying end momentsor transverse loading.. For linear distribution of bending moment and simply supported boundary condition(K=1) Eurocode 3 uses the equation (22) for calculating c_1[10].

$$1.77 - 1.04\Psi + 0.27\Psi^2 \leqslant 2.6 \qquad (22)$$

4.2. The c_2 factor

The basic equation for limit state of lateral torsional buckling assumes that load is acting at the shear center. The load applied above the shear center produces an additional overturning moment and hence increases the destabilizing effect. However, the load applied below the shear center produces a stabilizing effect. The effect of load height for various transverse loads in lateral boundary condition are considered using c_2 factor in the elastic critical moment calculation.It should be noted that the the c_2 values are not applicable for end moments without transverse loads which are irrelevant with respect to point of application. For transverse loading cases the $c_2$value is approximately 0.4 times c1 factor in both Eurocode 3 and IS 800-2007.

4.3. The $c_3$factor

With respect to loading and end lateral restraint the c3 factor modifies the monosymmetry parameterwhich develops an additional twisting moment. The degree of asymmetry in a cross section is represented by monosymmetry parameter. The monosymmetry parameter gives a positive value when the larger flange is on the compression side and a negative value when smaller flange is on the compression side. This monosymmetry parameter reduces to zero for a doubly symmetric section. The c3values are presented in IS 800-2007 and Eurocode 3. For linear distribution of bending moment and simply supported boundary condition (K=1) Eurocode 3 uses the equation (23) for calculating c3 [10].

$$c_3 = (1 + \Psi)c_1 \qquad (23)$$

5. Numerical calculations

The determination of design flexural strength of beams are explained using example problems. At first, a welded doubly symmetric I beam of length 5 m with the following properties are considered.

Depth of the section = 150 mm
Thickness of the web = 3 mm
Thickness of flange = 4.6 mm
Width of flange = 50 mm
Area of the section = 882.4 mm2
Yield stress = 250 N/mm2
Young's modulus of Elasticity = 2×105 N/mm2
Poisson's ratio = 0.3

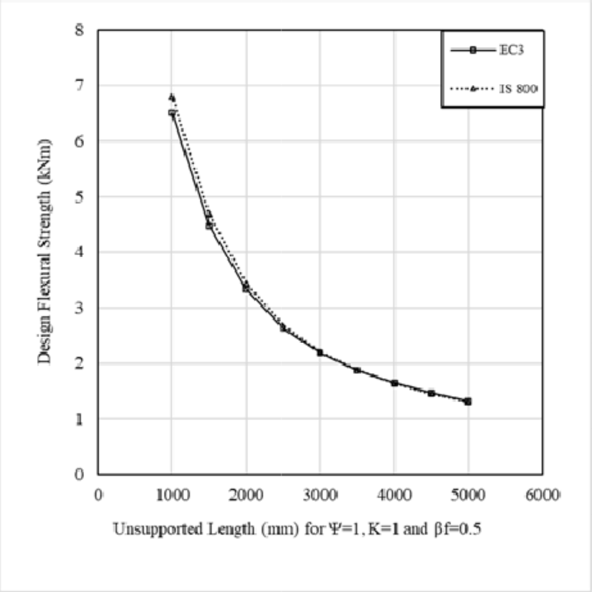

Fig-3.Design flexural strength values of doubly symmetric cross section with K=1 for various values of unsupported length.

The design flexural strength values corresponding to IS 800-2007 and Eurocode 3 are found out for varying end moment condition with Ψ varying from 1 to -1. Simply supported boundary condition (K=1) and simply supported with lateral bending prevented condition (K=0.5) are also considered. It can be seen from Figure 1 and Figure 2 that design flexural strength increases as the bending pattern changes from single curvature to double curvature. Design flexural strength values are also plotted for various range of unsupported length. As the unsupported length increases, design flexural strength values decrease. (Figure 3 and Figure 4) Except for the initial lengths the EC3 and IS 800-2007 results are same, because both the methods ideally usethe same set of equations with minor variations in imperfection factor. Design of doubly symmetric beams is a straight forward procedure but design of monosymmetric beam bend about non-symmetry axis involves design complications as detailed in the next example.

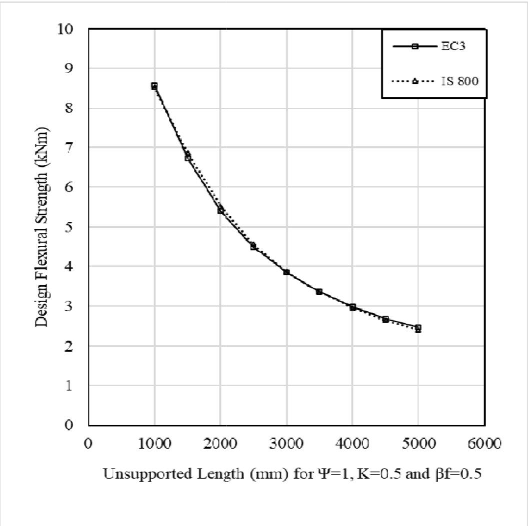

Fig-4.Design flexural strength values of doubly symmetric cross section with K=0.5 for various values of unsupported length.

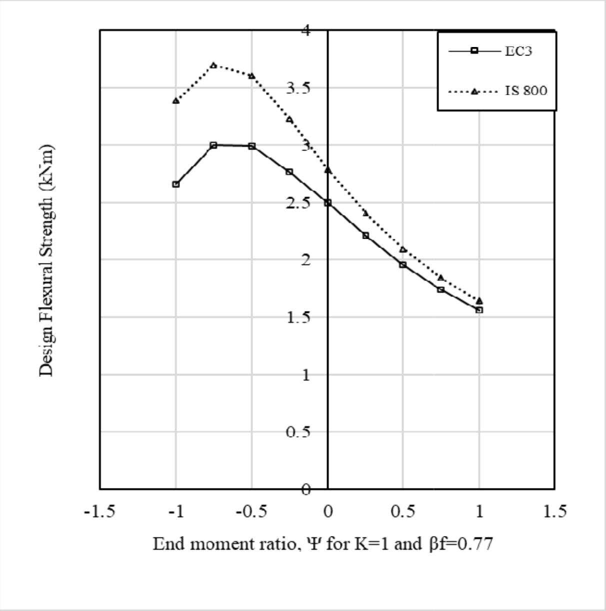

Fig-5.Design flexural strength valuesfor various end moments of mono symmetric cross section with K=1 and $\beta_f = 0.77$

The degree of monosymmetry is varied by proportioning the top and bottom flange areas as detailed in the next example. For comparing the results obtained for doubly symmetric cross section subjected to varying bending moment, monosymmetric I section of same area, material properties and length having a degree of monosymmetry value of 0.77 is considered [5].

Depth of the section = 150 mm
Thickness of the web = 3 mm
Thickness of flange = 4.6 mm
Width of top flange = 60 mm
Width of bottom flange = 40 mm
Area of the section = 882.4 mm^2

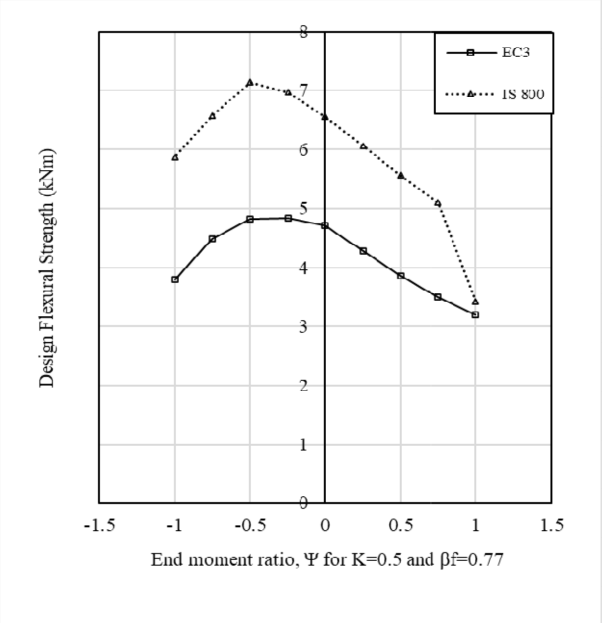

Fig-6.Design flexural strength values for various end moments of mono symmetric cross section with K=0.5 and $\beta_f = 0.77$

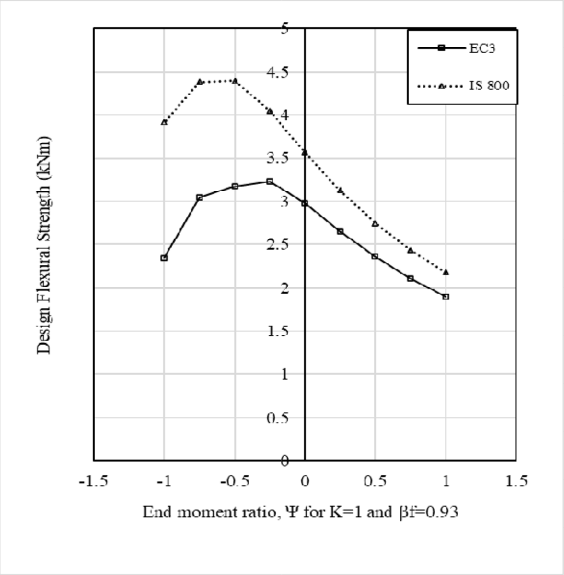

Fig-7.Design flexural strength values for various end moment ratios of mono symmetric cross section with K=1 and $\beta_f = 0.93$

The design flexural strength increases as the end moments change from single curvature to double curvaturesimilar to doubly symmetric cross section but the values are higher in monosymmetric cross section. (Figure 5 and Figure 6)The degree of monosymmetry parameter is further increased to 0.93 by keeping width of top flange and bottom flange as 70 mm and 30 mm respectively. All the other parameters remain constant. It can be seen from Figure 7 and Figure 8 that as the degree of monosymmetry parameter is increased, design flexural strength value also increases.

The degree of monosymmetry parameter is decreased to 0.35 by keeping width of top flange and bottom flange as 45 mm and 55 mm respectively.It can be seen from Figure 9 and Figure 10 that degree of monosymmetry parameter is decreased, design flexural strength value decreases.

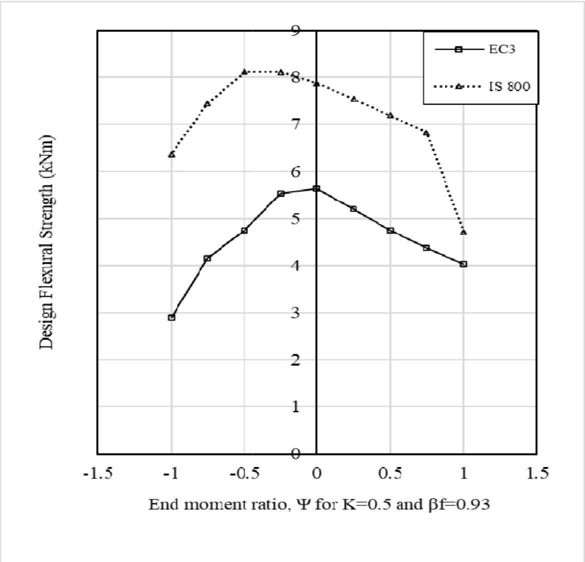

Fig-8.Design flexural strength values for various end moment ratios of mono symmetric cross section with K=0.5 and $\beta_f = 0.93$

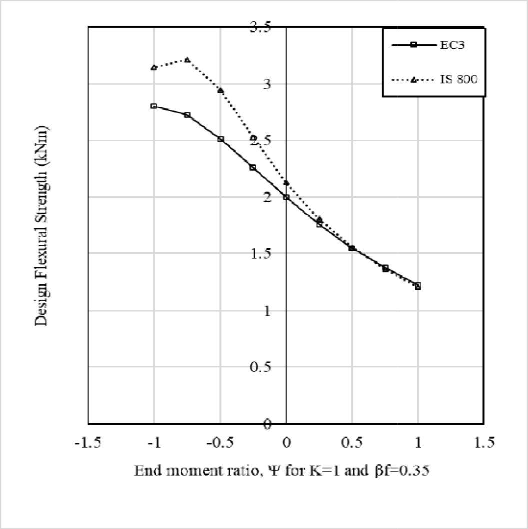

Fig-9.Design flexural strength values for various end moment ratios of mono symmetric cross section with K=1 and $\beta_f = 0.35$

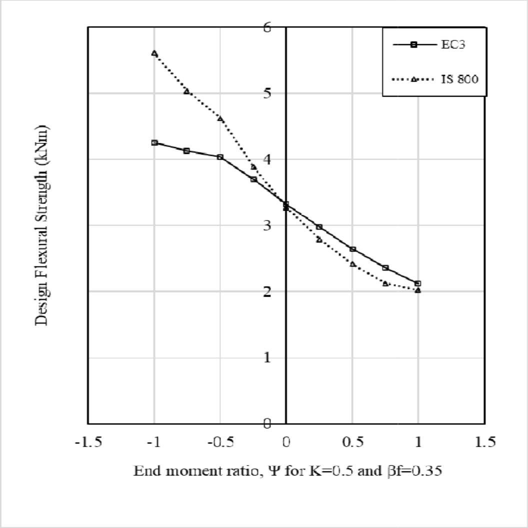

Fig-10.Design flexural strength values for various end moment ratios of mono symmetric cross section with K=0.5 and $\beta_f = 0.35$

6. Conclusions

In this article the design flexural strength for lateral torsional buckling limit state is compared for EC3 and IS 800-2007. The variation in bending strength with respect to cross section symmetry, end moment variation, lateral support condition and member length are discussed in detail. The design strength predictions according to IS:800-2007 are almost closer with EC3 findings for the range of length of members carrying constant moment along the length. However, for varying bending moments the IS:800-2007 results are higher than EC3 predictions. Although the fundamental equation to predict the design strength according to EC3 and IS:800-2007 are same, the variation in c1, c2 and c3 values result in higher strength for IS:800-2007 than EC3 except for lower monosymmetry values with single curvature bending. Albeit the IS:800-2007 suggests constant c3 values irrespective of compression flange second moment area which could be larger or smaller than tension flange second moment of area, EC3 includes this variation in double curvature bending and provides set of equations for c3 values. It should also be noted that, (i) design flexural strength value increases for a monosymmetric section with larger flange in compression when compared with that of a doubly symmetric section of same cross-sectional area and (ii) design flexural strength values increase when simply supported beam with lateral bending prevention condition is considered.

Disclosures

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Notations

E -	Modulus of elasticity.
G -	Modulus of rigidity.
I_y	Moment of inertia about minor axis

I_{yc}	Moment of inertia of compression flange about minor axis
I_{yt}	Moment of inertia of tension flange about minor axis
$I_{y,Top}$	Moment of inertia of the top flange about the minor axis.
I_t	Torsion constant of the cross section.
I_w	Warping constant.
L	Unbraced length against lateral torsional buckling.
M_p	Plastic bending moment
M_y	Yielding moment at the extreme fiber.
M_{cr}	Elastic critical bending moment
W_y	Appropriate section modulus.
Z_p	Plastic section modulus.
Z_e	Elastic section modulus.
b_{fc}	Width of the compression flange.
d	Depth of the section.
f_{bd}	Design bending compressive stress.
f_y	The minimum yield stress
h	Depth of the web.
h_o	Distance between the flange centroids.
t_{fc}	Thickness of the compression flange.
t_w	Thickness of web
χ_{LT}	Reduction factor to account for lateral torsional buckling
y_s	Co-ordinate of shear centre with respect to centroid.
Υmo	Partial safety factor for material
α_{LT}	Imperfection factor for lateral torsional buckling
λ_{LT}	Non-dimensional slenderness for lateral torsional buckling
β	Monosymmetric parameter
Ψ	End moment ratio.