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On Flexure-Shear Interaction in Fibre Model for the Analysis of RC Section

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Abstract

This paper investigates the effects of flexure-shear interaction on the force-deformation response of RC section. The analysis is performed using an improved version of fibre section model to consider the interaction of shear force. The model uses a supplementary shear strain field, in addition to the 'plane-section' hypothesis, for section kinematics. The concrete is modelled using a bi-axial stress-strain model based on a rotating, smeared-crack model with equivalent uniaxial stress-strain relation in principal directions. The confining effect of stirrups is considered by using Mander model. The reinforcing bars are modelled using a uniaxial stress-strain relation. The use of bi-axial model allow the flexure-shear interaction to reflect at the material point. The fibre section model is used to analyse an RC section to study the moment-curvature and shear force-shear strain response for increasing levels of shear forces and bending moments. The results shows that the moment-curvature and shear force-shear strain diagrams are significantly influenced by the presence oflarge shear force and bending moment respectively.

Keywords: fibre section model, flexure-shear interaction,concrete smeared-crackmodel, moment-curvature diagram, shear force- shear strain diagram

1. Introduction

Fibre models are widely used for the nonlinear analysis of RC framed structures as a general method for computing the force-deformation behaviour (CEB 1996). In a fibre model, cross-section is discretised into a large number of small areas (called fibres). The cross-section behaviour is obtained from the contribution of all the fibres. In a typical displacement based method, the general analysis procedure consists of finding the fibre strains from section kinematics. Subsequently, the fibre stresses are integrated to determine the section resultants (CEB 1996).The fibre stress and strain are related by an appropriate material model.

The most of the early development in fibre models were aimed towards the element formulation and their solution procedure (Menegotto and Pinto 1973; Ciampi and Carlesimo 1986; Zeris and Mahin 1988; Spacone et al. 1996; Neuenhofer and Filippou 1997; De Souza 2000). In these models, the shear is usually neglected. The cross-sectional behaviour is computed using the Bernoulli's 'plane section' hypothesis and uniaxial material stress-strain relation for flexure. However, the assumptions are only reasonable for well-detailed, slender RC members (large shear span ratio), where the fibre are predominantly subjected to uniaxial stress (CEB 1996). In general, the presence of shear force produces a multi-axial state of stress inside an RC member. The normal stress (axial and flexural stress) and shear stress interact among themselves and produce complex member behaviour (Park and Paulay 1975) which is more evident in RC members with small shear span ratio and in RC members within sufficient transverse reinforcement. The use of simplifying assumption of the traditional fibre model for these type of members is not appropriate and leads to wrong prediction. Therefore, the development of fibre model with normal-shear interaction is important and necessary to improve the applicability of the fibre model to wide range of RC member.

In last three decades, many researcher have attempted to extend the fibre model to consider the normal-shear interaction (Ceresa et al. 2007). The models differ, mostly in terms of the adopted multi-axial material model and the section kinematic assumption (Ceresa et al. 2007). Based on the section kinematics, two modelling approaches can be identified to account shear interaction in a sectional model (Bairan and Mari 2007): (a) fixed-pattern based approach, and (b) inner-fibre equilibrium approach. In fixed-pattern based approach, a fixed shear strain/stress distribution is assumed for the whole analysis process. The usual choice for the a priori distribution is either a uniform distribution or a shape corresponding to the elastic solution (e.g., a parabolic shear distribution for rectangular section) (Vecchio and Collins 1988;Petrangeli et al. 1999; Ceresa et al. 2009; Ferreira et al. 2014). Whereas, in inner fibre equilibrium approach, the shear stress is estimated from the fundamental stress equilibrium condition (Vecchio and Collins 1988; Benz 2000; Bairan and Mari 2006; Kagermanov and Ceresa 2017). In general, the inner fibre equilibrium approach is comparatively more accurate and computationally more involving. A qualitative comparison of both the approaches can be found in Vecchio and Collins

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(1988). The above approaches have been used with different displacement-based and force-based element formulations. Most of the applications usually prefer a fixed-pattern based approach, due to their simplicity (e.g., Petrangeli et al. 1999; Filippou and Saritas 2006; Ceresa et al. 2009; Mullapudi and Ayoub 2009; Guner and Vecchio 2010; Navarro-Gregori et al. 2013; Ferreira et al. 2014; Feng et al. 2017; Feng et al. 2019). Some of these models give good agreement with the experimental results but are, in general, computationally costly for use in large scale analysis. Therefore, there is a need of further development in fibre model to account the effect of shear interaction efficiently.

The objective of the present study is to demonstrate the importance of considering flexure-shear interaction in the analysis of RC members. The paper presents a fibre model based on fixed-pattern approach. The model uses the 'plane-section' hypothesis and a supplementary shear strain field for section kinematic. The concrete behaviour is expressed using a bi-axial, smeared-crack model based on modified compression field theory, MCFT (Vecchio and Collins 1986) together with confining effect of stirrups (Mander et al. 1988). Uniaxial material models are used for longitudinal and transverse reinforcing bars. The fibre model is used to compute moment-curvature and shear force-shear strain response of an RC section. The effect of the interaction on the sectional responses is shown.

2. Sectional Model

The section is discretised into concrete and longitudinal reinforcing steel fibres as shown in Figure 1(b). The transverse steel is smeared over the section and is expressed in terms of steel reinforcement ratio ρ_{sy} associated with each concrete fibre. It is assumed that the concrete fibres resist the normal stress and shear stress thus are subjected to a biaxial stress state as shown in Figure 1(d). The shear resistance of steel fibre is neglected and these are assumed to carry the axial stress only. The sectional model is formulated based on a variation of Timoshenko beam theory with parabolic shear strain distribution. The stress resultants are obtained from the fibre stresses following the model proposed in Filippou and Saritas (2006) which is briefly discussed here for the sake of notation.

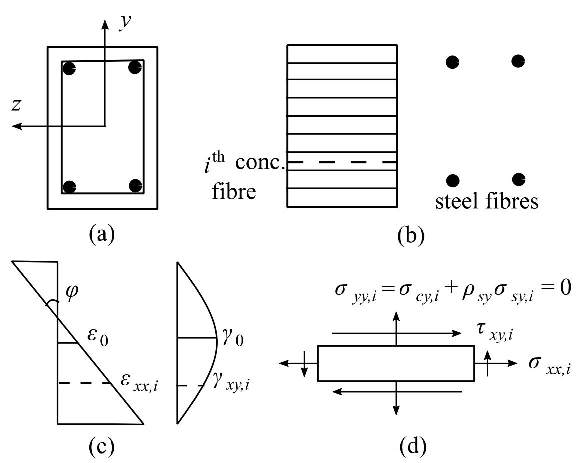

Figure 1. (a) RC cross-section, (b) concrete fibres and reinforcing steel fibres, (c) axial strain and shear strain distribution, (d) stresses in a concrete fibre.

Consider, the curvature (ϕ), the shear strain at the centroid (γ_0) and the axial strain at the centroid (ε_0) as the generalized section deformations. The axial strain and shear strain at a fibre location, y_i, are related to the section deformations, by kinematic relations as

$$\varepsilon_{xx,i} = -y_i\phi + \varepsilon_0 \tag{1}$$

$$\gamma_{xy,i} = \psi\left(y_i\right)\gamma_0 \tag{2}$$

where ψ_i denotes the shape of the assumed shear distribution. For parabolic shear strain distribution it is calculated as (Filippou and Saritas, 2006)

$$\psi_i = \frac{5}{4}\left(1-4\left(\frac{y_i}{H}\right)^2\right)$$

In vector notation,

$$\mathbf{e}_i = \begin{bmatrix}\varepsilon_{xx,i} & \gamma_{xy,i}\end{bmatrix}^T = \mathbf{a}_{s,i}\mathbf{d} \tag{3}$$

$$\mathbf{a}_{s,i} = \begin{bmatrix}-y_i & 0 & 1\\ 0 & \psi_i & 0\end{bmatrix} \tag{4}$$

$$\mathbf{d} = \begin{bmatrix}\phi & \gamma_0 & \varepsilon_0\end{bmatrix}^T \tag{5}$$

Define $\mathbf{s}_i$ and $\mathbf{k}_i$ as the fiber stresses and tangent matrix corresponding to $\mathbf{e}_i$. The section resultants $\mathbf{S}_{\text{int}}$ and stiffness $\mathbf{K}_s$ are obtained, from the virtual work principle as

$$\mathbf{S}_{\text{int}} = \begin{bmatrix}M & V & N\end{bmatrix}^T = \int_A \mathbf{a}_{s,i}{}^T\mathbf{s}_i dA \tag{6}$$

$$\mathbf{K}_s = \frac{\partial \mathbf{S}_{\text{int}}}{\partial \mathbf{d}} = \int_A \mathbf{a}_{s,i}{}^T\mathbf{k}_i\mathbf{a}_{s,i} dA \tag{7}$$

where

$$\mathbf{s}_i = \begin{bmatrix}\sigma_{xx,i} & \tau_{xy,i}\end{bmatrix}^T = f\left(\mathbf{e}_i\right) \tag{8}$$

$$\mathbf{k}_i = \frac{\partial \mathbf{s}_i}{\partial \mathbf{e}_i} \tag{9}$$

The process of determining the stresses ($\mathbf{s}_i$) and tangent matrix ($\mathbf{k}_i$) of concrete and reinforcing steel fibre is described in the following sections.

2.1 Concrete Fibre State Determination

The concrete fibres are assumed to resist axial stress and shear stress. The state of the concrete fibre is determined using a bi-axial constitutive relation based on modified compression field theory (MCFT) developed by Vecchio and Collins (1986).The MCFT is a rotating smeared-crack model. It assumes cracked concrete as an orthotropic material with equivalent uniaxial stress-strain relationship in principal strain directions. The stress-strain relationship are defined in terms of average stresses and strains (Vecchio and Collins, 1986). In this paper, the stress-strain relation for concrete in compression in principal direction is modelled using Mander model (Mander et al. 1988). This model is adopted to takes care of confining effect of stirrups on concrete constitutive model.The other assumptions of MCFT are: (i) the reinforced concrete as a composite made of plane concrete and smeared steel,(ii) The reinforcing steel are perfectly bonded to the surrounding concrete, (iii) the steel shear stress is neglected,(iv) the principal concrete stress direction and principal strain direction are assumed to coincide(Vecchio and Collins, 1986).

Here, according to the MCFT, the concrete fibre is considered as a composite, made of plane concrete and smeared transverse steel (stirrup area).For, each concrete fibre is a transverse reinforcement ratio (ρ_{sy}) is defined and considered it as an internal property of the concrete fibre (Vecchio and Collins 1988). The behavior of the concrete fiber is obtained by adding the contribution from both the components. The following paragraphs briefly describe the process of concrete fibre state determination.

The implementation of the MCFT on a concrete fibrerequires a complete strain tensor, to estimate the principal direction θ. The axial strain and shear strainare known from the assumed section kinematics relations. The remaining unknown value transverse strain ε_{yy} is determined by imposing an equilibrium constrainton the total transverse stress (Rericha 1991).

$$\sigma_{cy} + \rho_{sy}\sigma_{sy} = 0 \tag{10}$$

where σ_{cy} , σ_{sy} are the stress in concrete and stirrup in the transverse direction, ρ_{sy} is the ratio of transverse reinforcement (stirrup area). As the transverse stress and strain values are dependent on each other, an iterative procedure is followed to estimate the required transverse strain value. The iterative procedure at the concrete fibre to determine the transverse strain value is given in Figure 2. An initial value transverse strain ε_{yy} is assumed to start the calculation and it is subsequently updated until the transverse equilibrium constraint is satisfied within a specified tolerance.

Once the state of the fibre is established, the fibre stresses (σ_{xx} , σ_{yy} , τ_{xy}) and the tangent material matrix $\mathbf{D}$ in

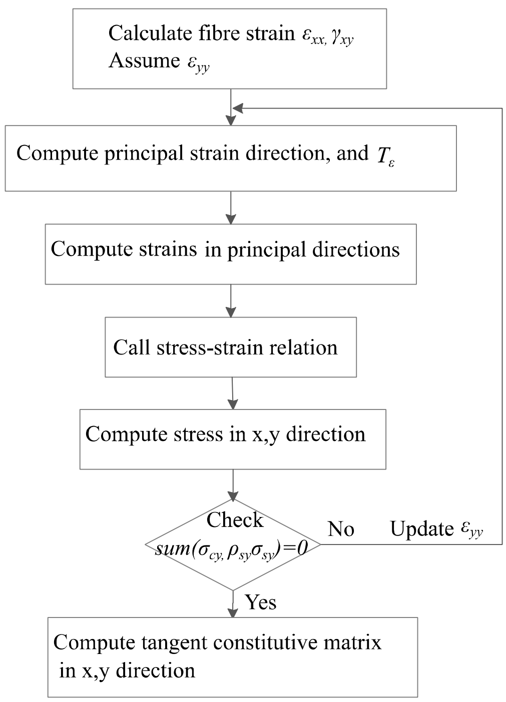

Figure 2. Flow chart for concrete fibre state determination

x-y direction are determined by adding the contribution of concrete and smeared steel as follows

$$\begin{bmatrix}\sigma_{xx} & \sigma_{yy} & \tau_{xy}\end{bmatrix}^T = \mathbf{T}_\varepsilon{}^T\begin{bmatrix}\sigma_{c1} & \sigma_{c2} & 0\end{bmatrix}^T + \begin{bmatrix}0 & \rho_{sy}\sigma_{sy} & 0\end{bmatrix} \tag{11}$$

$$\mathbf{D} = \mathbf{T}_\varepsilon{}^T\mathbf{D}_{c,1-2}\mathbf{T}_\varepsilon + \mathbf{D}_{sy} \tag{12}$$

where $\mathbf{T}_\varepsilon$ is the transformation matrix, σ_{c1}, σ_{c2} are the principal stresses in concrete, $\mathbf{D}_{c,1-2}$ is the concrete tangent matrix in principal direction and $\mathbf{D}_{sy}$ tangent matrix of smeared steel

$$\mathbf{T}_\varepsilon = \begin{bmatrix}\cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -\sin 2\theta & \sin 2\theta & \cos 2\theta\end{bmatrix} \tag{13}$$

$$\mathbf{D}_{c,1-2} = \begin{bmatrix}E_{c1} & 0 & 0 \\ 0 & E_{c2} & 0 \\ 0 & 0 & G_c\end{bmatrix} \tag{14}$$

$$\mathbf{D}_{sy} = \begin{bmatrix}0 & 0 & 0 \\ 0 & \rho_{sy}E_{sy} & 0 \\ 0 & 0 & 0\end{bmatrix} \tag{15}$$

where $\theta$is principal strain direction, E_{c1} , E_{c2} are the tangent modulus of concrete in principal directions, and E_{sy} is the tangent modulus of smeared steel, G_c is the approximate shear modulus, represented as (Vecchio and Collins 1986)

$$G_c = \frac{E_{c1}E_{c2}}{E_{c1} + E_{c2}} \tag{16}$$

The computation of the section resultant ($\mathbf{S}_{\text{int}}$) and stiffness ($\mathbf{K}_s$)in Equations 6 and 7,requires reduced stress ($\mathbf{s}_i$) and stiffness matrices ($\mathbf{k}_i$). Therefore, static reduction is performed on the fibre stresses (σ_{xx} , σ_{yy} , τ_{xy}) and the tangent material matrix ($\mathbf{D}$) to condense out the contribution in the transverse direction, before applying them in section resultant and stiffness calculation. This study differs from earlier similar formulation (Filippou and Saritas 2006; Guner and Vecchio 2010; Navarro-Gregori et al. 2013) in terms of using concrete constitutive model which considers the confining effect of stirrups.

2.2 Steel Fibre State Determination

The reinforcing steel fibres are assumed to resist only axial stresses. The state of the fibers are determined from the usual axial strain and uniaxial stress-strain relation. The estimated axial stress and tangent modulus are used in the section resultant and stiffness computation (Equation 6, 7).

$$\mathbf{s}_i = \begin{bmatrix}\sigma_{xx,i} & 0\end{bmatrix}^T \tag{17}$$

$$\mathbf{k}_i = \begin{bmatrix}\dfrac{\partial\sigma_{xx,i}}{\partial e_{xx,i}} & 0 \\ 0 & 0\end{bmatrix} \tag{18}$$

2.3 Uniaxial Material Stress-Strain Relations

The average stress-average strain relation for concrete in compression in the principal axes is modelled by Mander model (Mander et al. 1988) to account the confining effect of stirrups. The effect of transverse tensile strain ε_{c1} on the compression stress-strain relation (known as compression softening effect), is accounted using the factor β (Vecchio and Collins 1986)

$$\beta = \frac{1}{0.8 - 0.34\left(\dfrac{\varepsilon_{c1}}{\varepsilon_0}\right)} \leq 1 \tag{19}$$

where ε_0 is the concrete strain corresponding to the concrete compressive strength.

The average stress-average strain relation for concrete in tension in the principal axes is modelled as linear elastic up to cracking followed by the tension stiffening curve, represented as (Vecchio and Collins 1986)

$$\sigma_c = \frac{\sigma_{cr}}{1+\sqrt{200\varepsilon_c}}, \text{ for } \varepsilon > \varepsilon_{cr} \tag{20}$$

The reinforcing steel and stirrups stress-strain relationship is expressed using bilinear (linear elastic-linear strain hardening) stress-strain relation.

3. Computational Algorithm

The fibre model discussed in the previous section for sectional response analysis has been implemented in MATLAB (ver. R2014a). The displacement-controlled algorithmis used to calculate the force-deformation diagram of an RC section (Crisfield 1991). The flowchart for calculating the section shear force- shear strain diagram is shown in Figure 3.

For section moment-curvature diagram corresponding to the changes in shear force is carried out in the similar manner. Keeping the axial force and shear force constant, moment is calculated for the incremental curvature as input iteratively. Thus, moment-curvature diagram and shear force-shear strain diagram has been generated for a given section.

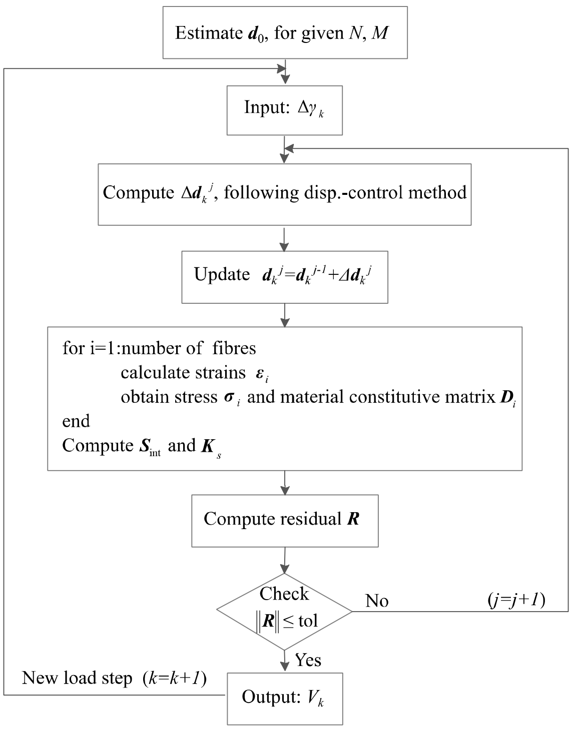

Figure 3. Flow chart for calculation of shear force-shear strain diagram

4. Numerical Study

The cross-section studied to calculate moment-curvature and shear force-shear strain diagram is the shear-critical short column (specimen 1-1) tested by Bett et al. (1985) in double curvature. The dimensions of the column is given here for ready reference, the detailed description can be found in Bett et al. (1985). The length of the column is 914 mm (3’). The column is subjected to a constant axial load of 288 kN (64.8 kip). The cross-section dimension are 305 mm × 305 mm (12” × 12”). The concrete strength is 29.9 MPa (4333 psi). Longitudinal reinforcement are eight bars of 19 mm (0.75”) diameter and yield stress of 462 MPa (67 ksi). Stirrups are of 6 mm diameter spaced at 203 mm (8”) and 414 MPa (60 ksi) yield stress.

The moment-curvature and shear force-shear strain diagrams calculated for different shear force and moment levels are shown in Figure 4(a) and Figure 5(a) respectively. The axial load is the kept constant equal to the applied load in all the calculations. It is observed that for zero-shear condition the section has the maximum moment capacity. Similarly for zero-moment the section has the maximum shear capacity. On comparing, the lateral load levels corresponding to maximum moment and shear capacity, it is found that the section has amaximum moment capacity of164kNm (Figure 4),that corresponds to a lateral load of359kN $(V = 2M/L)$. The maximum shear capacity of the section is 244 kN (Figure 5). This indicates that the section is critical in shear.

Further, for the peak load of 220 kN (49 kip)the moment at the column end section is estimated to be 100.5 kNm (Bett et al. 1985). The shear capacity corresponding to this moment predicted from Figure 5(b) is found to be around 230 kN. The value is close to the reported value in the experiment. This shows the shear interaction model developed in this study is in good agreement with the experimental result.

The moment-curvature diagram of the section for different constant shear forces is shown in Figure 4(a). It is observed that the shape of the moment-curvature diagrams are severely influenced by the presence of large shear force. The presence of shear force reduces the initial stiffness and moment carrying capacity of the section. The variation of moment capacity in plotted in Figure 4(b). It is observed that the reduction in moment capacity is negligible for small values shear force, but are significant in presence of higher-level shear force. These observations are similar to Bairan and Mari (2004).

The effect of constant moment on the shear force vs shear strain diagram is shown in Figure 5 (a) and (b). It is seen that the shear strength of the section decreases with increasing moment. The initial stiffness is also decreases with the increase in moment level. Similar observations have been made by Bairan and Mari (2004). Thus, flexure-shear interaction model presented in this work is able to capture the observed behavior of beam-column section.

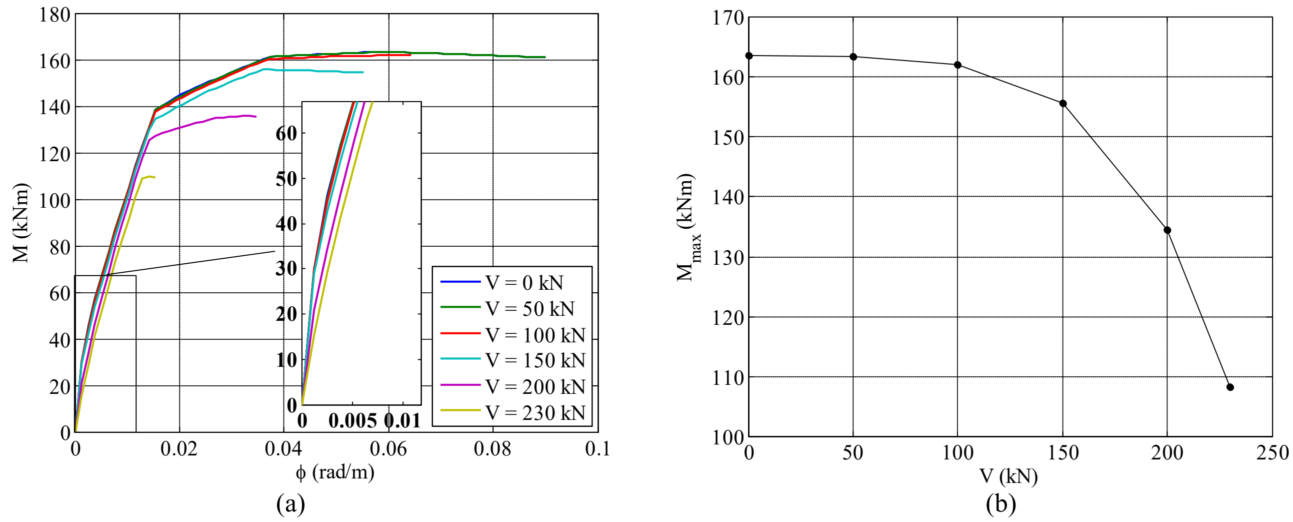

Figure 4. (a) Moment-curvature diagrams for different shear force, (b) variation of moment capacity with shear force

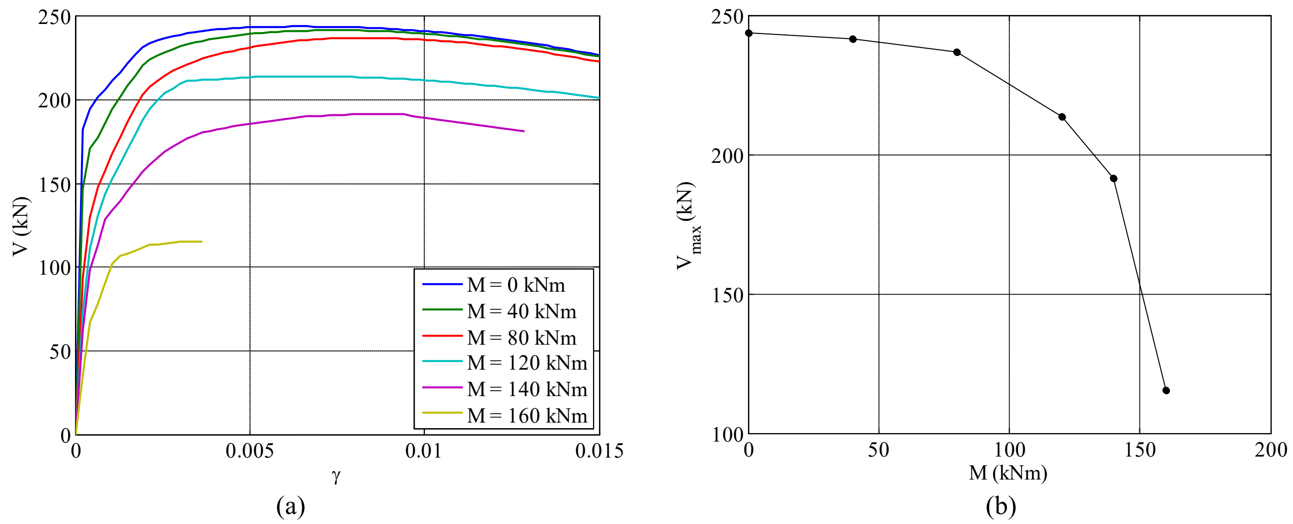

Figure 5. (a) Shear force-shear strain diagram for different moment, (b) variation of shear capacity with bending moment

5. Conclusions

A fibre model is implemented for analysis of RC section considering the flexure-shear interaction. The model uses the ‘plane-section’ hypothesis and a supplementary shear strain field for section kinematics. The concrete stress-strain behavior is based on a bi-axial rotating smeared-crack model. The model also includes the strength enhancement due to confining effect of stirrup, strength reduction due to transverse tensile strain for concrete in compression and tension stiffening effect for concrete in tension. The model is used to study the effect of flexure-shear interaction on the moment-curvature and shear force-shear strain response of an RC section.

It is observed that the moment-curvature and the shear force-shear strain responses are affected by the presence of shear and moments respectively. The peak load capacity and initial stiffness are found to be reduced in both cases. For small values of shear or moment, the reductions are small and can be neglected. However, for high values of shear and moment the reductions are quite significant. This shows the importance of the considering flexure-shear interaction in RC member analysis. Neglecting this effect will affects the accuracy of the prediction of RC members.

Disclosures

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