

Higher-Order Obstacle Problems evaluation through Novel Approach

Engammagari Ganesh¹, M.K. Shrimali², P.V. Ramana^{2,*}

¹Research Scholar, National Centre for Disaster Mitigation & Management, MNIT Jaipur, India

²Professor, National Centre for Disaster Mitigation & Management, MNIT Jaipur, India

Paper ID - 030525

Abstract

Bridge problems are challenging, but this revolutionary solution streamlines the procedure. Many other difficulties were investigated, including the 4th order non-homogeneous obstacle bridge problem. A fundamental supported continuous beam with n supports and variable loading conditions is employed for this aim. Different limits were taken for various cases, solved using novel techniques, and compared with the computational method. A novel techniques methodology approach was used to multi-support and continuous bridge issues to increase the accuracy and conjunction of the abovementioned problems. Ongoing support bridge issues may be promptly remedied using creative ways. The findings of the unique technique approach gather quickly and are closer to the precise answer. This work uses the novel methodology for numerous initial value issues, such as diverse case problems. This approach has been used in a variety of engineering sectors. This approach has been used in very little research on dynamics. As a result, an effort has been undertaken to investigate the validity of such an approach. The approach provided accurate answers to the majority of the difficulties.

Keywords: Novel technique, Non-homogeneous, Displacement, Slope, Curvature, Sharpness.

1. Introduction

Bridge problems are challenging to solve, but this method may assist. It is a straightforward technique for resolving nonlinearities' initial and boundary issues [1]. Many hours have been spent on various iterative algorithms for solving differential equations. Scientists have introduced and explored many non-linear problems using the decomposing approach with higher-order convergence methods [2]. A unique technique was used to address the bridge's continuing support issues. In the 1980s, Adomian proposed a novel and creative method for obtaining an accurate explanation of many types of both linear and non-linear problems, such as algebra, derivative for traditional and partial, numerical methods, and so on [3]. This approach employs a non-linear decomposition mechanism into a succession of Adomian functions [4]. This series consists of Adomian polynomials, which are generalized. Some techniques assume that the complex structure is almost linear after similar linearization; however, this does not retain the novelty of the situation [5]. The Numerical integration method was initially proposed to address physical boundary challenges, among other things (Adomian 1992). The essential premise of the ADM is to decompose the board-dependent variables) or indefinite(s) into a valuable series while assuming minuscule parameters [6]. The approach's primary benefit is that it may be used to immediately solve any problem with homogeneous and quasi-conditions [7]. One advantage of the method is that it

significantly reduces computational labor while maintaining greater numerical accuracy [8]. The traditional approach has a deliberate procedure and conforms to several fundamental assumptions, but this method may solve the issue instantly [9]. The discretization may successfully solve a wide variety of problems, often producing a rapidly convergent serial solution [10]. It has substantial advantages over typical numerical solutions [11]. It is simple to compute, provides analytical and constant equations that do not need perturbations, interpolation, or denoising, and yields tremendous results. Since they tackle non-linear issues instead of linearizing them, the resultant solutions are much more realistic [12]. This is a practical approach for solving differential calculus sequences that arise analytically and explicitly in physical problems. It is a brilliant solution for together starting & bounded divergence complications. It can likewise resolve stochastic diverging equation classifications [13]. In contrast to various numerical approaches, the Fourier transformation with the technique of Adomian decomposition methodology needs no perturbations or simplification to discover the dynamic properties of complex dynamical systems [14]. The researchers used enhanced Taylor's coefficients to solve numerous significantly greater linear systems having derivatives [15]. The objective is to statistically solve initial and boundary problems using a novel technique solution. Understanding how such replies differ from exact solutions

*Corresponding author. Tel: +919549654189; E-mail address: 2021RNC9093@mnit.ac.in

[16,17] is vital. Thumb guidelines and tweaks for such precise answers are also examined. The finite element model must be addressed for different challenges, and its dependability in various situations must be explored [19].

2. Methodology

The general differential equation is of the form $\frac{\partial^n x}{\partial t^n} + \frac{\partial^{n-1} x}{\partial x^{n-1}} + \frac{\partial^{n-2} x}{\partial y^{n-2}} + \frac{\partial^{n-m} x}{\partial t^{n-m}} + \frac{\partial^m x}{\partial z^m} + \dots + u^2 + u^3 + \left(\frac{\partial x}{\partial z}\right)^n + \dots = f(x, y, z)$

In the first step, separate the linear, non-linear, remainder, and forcing terms.

$$K_t x + K_x x + K_y x + K_z x + \dots + D(x) = G(x, y, z)$$

Where Linear = $K_t x$, The remainder ($M(x)$) = $K_x x + K_y x + K_z x$

Linear term = $K(x) + M(x)$, Non-linear term = $D(x)$

$$K_t = \frac{\partial^n}{\partial t^n}, \quad K_x = \frac{\partial^{n-1}}{\partial x^{n-1}}, \quad K_y = \frac{\partial^{n-2}}{\partial y^{n-2}}, \quad K_z = \frac{\partial^m}{\partial z^m}.$$

$$K_t x + K_x x + K_y x + K_z x + \dots + D(x) = G(x, y, z)$$

$$K_t x + M(x) + D(x) = G(x, y, z)$$

$$K_t x = G(x, y, z) - (M(x) + D(x))$$

Apply K_t the inverse on both sides

$$K_t^{-1} K_t x = K_t^{-1} (G(x, y, z) - M(x) - D(x))$$

$$u = \text{'n' constants} + K_t^{-1} (G(x, y, z) - M(x) - D(x))$$

$$= a_0 + ta_1 + \dots + t^{n-1} a_{n-1} + Kg(x) - L_t^{-1} (M(x) + D(x))$$

u is the function of both dependent and independent terms.

$u_0 = a_0 + ta_1 + t^2 a_2 + \dots + t^{n-1} a_{n-1} + Kg(x, y, z)$; u_0 is an independent variable function

$$u_1 = -K_t^{-1} (M(x_0) + D(x_0))$$

$$u_2 = -L_t^{-1} (M(x_1) + D(x_1))$$

⋮

$$u_n = -K_t^{-1} (M(x_{n-1}) + D(x_{n-1}));$$

$$u = u_0 + u_1 + u_2 + \dots + u_n [3, 10].$$

3. Problem statements

Figure 1 depicts a reinforced concrete bridge with n supports A, C, D...N, and B. Support A is hinge support, but the other three supports are all roller supports. Figure 2 depicts the beam orientation under various loading situations, which range from $[0 \leq \xi \leq 1]$, for each beam. With the forcing function $g(x)$, the limitations for support AC range from $[0 \leq \xi \leq 1]$. With the forcing function $h(x)k(x)+p(x)+m$, the limitations for support CD range from $[0 \leq \xi \leq 1]$. With the forcing function $g(x)$, the limitations for support DB range from $[0 \leq \xi \leq 1]$. Figure 3 depicts the beam alignment under various stress circumstances ranging from $[-1 \leq \xi \leq 1]$. With the forcing function $g(x)$, the limitations for support AC range from $[-1 \leq \xi \leq -\frac{1}{2}]$. With the forcing function $h(x)k(x)+p(x)+m$, the limits for support CD range from $[-\frac{1}{2} \leq \xi \leq \frac{1}{2}]$. With the forcing function $g(x)$, the limitations for support DB range from $[\frac{1}{2} \leq \xi \leq 1]$.

The assumed governing differential equation is shown in equation(1)

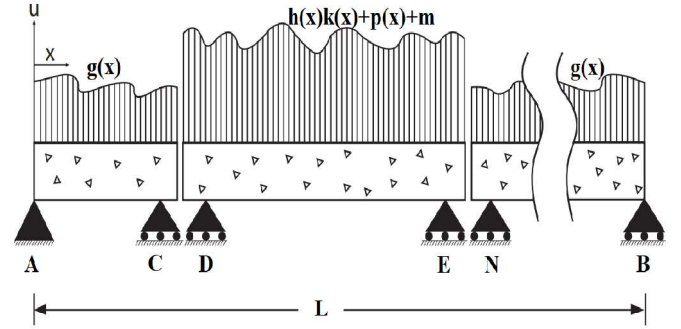


Fig. 1. Continuous Beam orientation with different loading conditions for n supports.

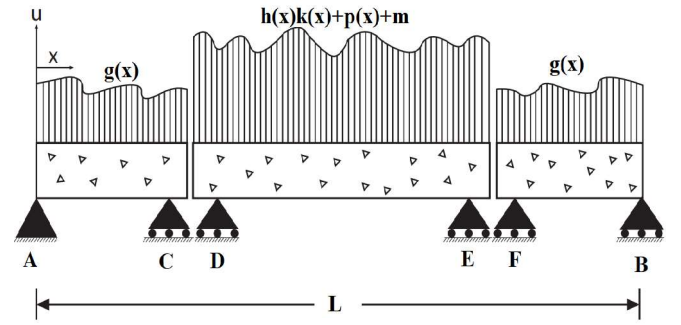


Fig. 2. Beam Orientation with different loading conditions limits varies from $[0 \leq \xi \leq 1]$ for individual beams.

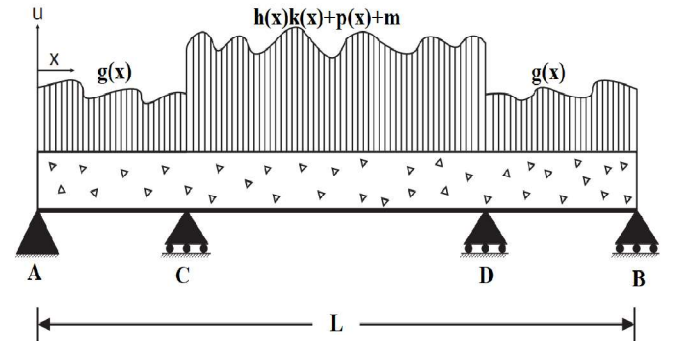


Fig. 3. Beam Orientation with different loading conditions varies from $[-1 \leq \xi \leq 1]$.

$$EI \frac{d^n u}{dx^n} = u^{(n)} = \begin{cases} g(x), & A \leq x \leq C, \\ h(x)k(x) + p(x) + m, & C \leq x \leq D, \\ g(x), & D \leq x \leq B, \end{cases} \quad (0)$$

with the boundary and permanence conditions

$$\begin{cases} u(A) = u(B) = A_{11} u''(A) = u''(B) = A_{22} \\ u(C) = u(D) = B_{11} u''(C) = u''(D) = B_{22} \end{cases}$$

In the above equation (0) assumed $EI = 1$, $n = 4$, $g(x) = p(x) = 1 + u$, $k(x) = (\pi^4 - 1) \sin(\pi x)$, $r = -1$, $h(x) = 1$. Fig.3 The limits taken for span AC, CD, and DB are $-1 \leq \xi \leq -\frac{1}{2}$, $-\frac{1}{2} \leq \xi \leq \frac{1}{2}$, and $\frac{1}{2} \leq \xi \leq 1$. Then equation (1) changes to

$$u^{(4)} = \begin{cases} 1 + u, & -1 \leq \xi \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq \xi \leq 1, \\ (\pi^4 - 1)\sin(\pi x) + u, & -\frac{1}{2} \leq \xi \leq \frac{1}{2}. \end{cases}$$

The above fourth-order equation problems are solved from case 1 to case 3.

Fig.2 The limits taken for span AC, CD, and DB are $0 \leq \xi \leq 1$, $0 \leq \xi \leq 1$, and $0 \leq \xi \leq 1$. assumed $EI = 1000$, $n = 4$, $g(x) = p(x) = 1 + u$, $k(x) = (\pi^4 - 1)\sin(\pi x)$, $r = -1$, $h(x) = 1$. Then equation (0) changes to

$$u^{(4)} = \begin{cases} 1 + u, & 0 \leq \xi \leq 1 \text{ and } 0 \leq \xi \leq 1, \\ (\pi^4 - 1)\sin(\pi x) + u, & 0 \leq \xi \leq 1. \end{cases}$$

The above fourth-order equation problems are solved from case 4 to case 6.

4. Results and Discussion

4.1. Case 1: The governing differential reckoning of the fourth order equation is $u^{(4)} = 1 + u$, and the Boundary conditions are $u(-1) = 0$, $u(-\frac{1}{2}) = 0$, $u''(-1) = 0$, $u''(-\frac{1}{2}) = 0$, and differs from $-1 \leq \xi \leq -\frac{1}{2}$.

4.1.1. Novel technique solution:

Displacement, $u = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.3776 \cos(x) + 0.3518 \sin(x) + 0.5131 \exp(x) + 1.1449 \exp(-x) - 1 \quad (1)$$

Slope, $\frac{du}{dx} = -a \sin(x) + b \cos(x) + c \exp(x) - d \exp(-x)$

$$= -0.3776 \sin(x) + 0.3518 \cos(x) + 0.5131 \exp(x) - 1.1449 \exp(-x) \quad (2)$$

Curvature, $\frac{d^2u}{dx^2} = -a \cos(x) - b \sin(x) + c \exp(x) + d \exp(-x)$

$$= -0.3776 \cos(x) - 0.3518 \sin(x) + 0.5131 \exp(x) + 1.1449 \exp(-x) \quad (3)$$

Sharpness function, $\frac{d^3u}{dx^3} = a \sin(x) - b \cos(x) + c \exp(x) - d \exp(-x)$

$$= 0.3776 \sin(x) - 0.3518 \cos(x) + 0.5131 \exp(x) - 1.1449 \exp(-x) \quad (4)$$

Impendent function, $\frac{d^4u}{dx^4} = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x)$

$$= 0.3776 \cos(x) + 0.3518 \sin(x) + 0.5131 \exp(x) + 1.1449 \exp(-x) \quad (5)$$

4.1.2. Exact solution:

The exact solution, $u_e = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.3776 \cos(x) + 0.3518 \sin(x) + 0.5131 \exp(x) + 1.1335 \exp(-x) - 1 \quad (6)$$

4.1.3. Discussion of Results for case 1:

Fig. 4 depicts the displacement, slope, curvature, sharpness, and error plot for both novel and computational methods for a beam length varying from $[-1 \leq \xi \leq -\frac{1}{2}]$.

Displacement is initially 0 at -1 units and zeroes at -0.5 units for both procedures, which varies parabolically. At -1

units, the displacement inaccuracy is -2×10^{-3} , and it grows exponentially to -3×10^{-3} at -0.5 units. Both approaches have

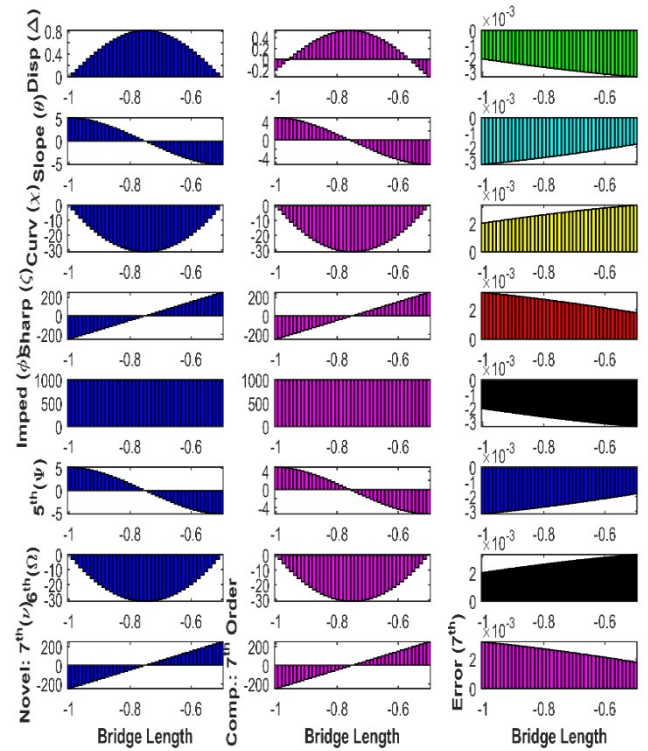


Fig.4. Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order and error (Novel & Computational) for $u^{(4)} = 1 + u$ varies from $-1 \leq \xi \leq -\frac{1}{2}$.

a slope & 5th order of 5 at -1 unit, which falls to -5 at -0.5. At -1 units, the slope error is -3×10^{-3} and decreases exponentially up to -2×10^{-3} . For both approaches, the curvature & 6th order is 0 at -1 units and 0 at -0.5 units. Curvature error is 2×10^{-3} at -1 units, growing exponentially to 3×10^{-3} at -0.5 units. For both techniques, sharpness & 7th order is -200 at -1 units and increases to 200 at -0.5 units. Sharpness error is 3×10^{-3} at -1 unit and rapidly decreases to 2×10^{-3} at -0.5. For both methods, the impendent function is 1000 at -1, -0.5 units. The impendent error is -2×10^{-3} at -1 units, growing exponentially to -3×10^{-3} at -0.5 units. The novel and computational solutions nearly coincide, and the maximum displacement, slope, curvature, sharpness, and impendent values observed are 0.8, 5, -30, 200, and 1000 units.

4.2. Case 2: The prevailing differential equivalence of the 4th order equation is $u^{(4)} = (\pi^4 - 1)\sin(\pi x) + u$, and the Boundary conditions are $[u(-\frac{1}{2}) = 0, u(\frac{1}{2}) = 0, u''(-\frac{1}{2}) = 0, u''(\frac{1}{2}) = 0]$, and diverges from $[-\frac{1}{2} \leq \xi \leq \frac{1}{2}]$.

4.2.1. Novel technique solution:

Displacement, $u = a \sin(x) + b \sin(\pi x) + c \exp(x) + d \exp(-x) + e \sin(x + gx) + f \sin(x - hx)$

$$= -11.3361 \sin(x) - 4.4348 \sin(\pi x) + 4.2553 \exp(x) - 4.2553 \exp(-x) + 11.2544 \sin(x+5.8196x) + 2.1416 \sin(x-4.1416x) \quad (7)$$

$$\text{Slope, } \frac{du}{dx} = a \cos(x) + b \pi \cos(\pi x) + c \exp(x) - d \exp(-x) + e \cos(x+gx) (g+1) - f \cos(x-hx) (h-1) \\ = -11.3361 \cos(x) - 4.4348 \pi \cos(\pi x) + 4.2553 \exp(x) + 4.2553 \exp(-x) + 76.7505 \cos(x+5.8196x) - 6.7281 \cos(x-4.1416x) \quad (8)$$

$$\text{Curvature, } \frac{d^2u}{dx^2} = -a \sin(x) - b \pi^2 \sin(\pi x) + c \exp(x) + d \exp(-x) - e \sin(x+gx) (g+1)^2 - f \sin(x-hx) (h-1)^2 \\ = 11.3361 \sin(x) + 4.4348 \pi^2 \sin(\pi x) + 4.2553 \exp(x) - 4.2553 \exp(-x) - 523.4077 \sin(x+5.8196x) - 21.1370 \sin(x-4.1416x) \quad (9)$$

$$\text{Sharpness function, } \frac{d^3u}{dx^3} = -a \cos(x) - b \pi^3 \cos(\pi x) + c \exp(x) - d \exp(-x) - e \cos(x+gx) (g+1)^3 - f \cos(x-hx) (h-1)^3 \\ = 11.3361 \cos(x) + 4.4348 \pi^3 \cos(\pi x) + 4.2553 \exp(x) + 4.2553 \exp(-x) - 3569.4311 \cos(x+5.8196x) + 66.4039 \cos(x-4.1416x) \quad (10)$$

$$\text{Impendent function, } \frac{d^4u}{dx^4} = +a \sin(x) + b \pi^4 \sin(\pi x) + c \exp(x) + d \exp(-x) + e \sin(x+gx) (g+1)^4 + f \sin(x-hx) (h-1)^4 \\ = -11.3361 \sin(x) - 4.4348 \pi^4 \sin(\pi x) + 4.2553 \exp(x) - 4.2553 \exp(-x) + 2434.2092 \sin(x+5.8196x) - 208.6144 \sin(x-4.1416x) \quad (11)$$

4.2.2. Exact solution:

The exact solution, $u_e = a \sin(x) + b \sin(\pi x) + c \exp(x) + d \exp(-x) + e \sin(x+gx) + f \sin(x-hx)$

$$= -11.3361 \sin(x) - 4.4348 \sin(\pi x) + 4.2553 \exp(x) - 4.2127 \exp(-x) + 11.2544 \sin(x+5.8196x) + 2.1416 \sin(x-4.1416x) \quad (12)$$

4.2.3. Discussion of Results for case 2:

Fig. 5 illustrates the displacement, slope, curvature, sharpness, and error plots for both new and computational approaches for a beam length changing from $[-\frac{1}{2} \leq \xi \leq \frac{1}{2}]$. Displacement is initially 0 at 50 units and zeroes at 0 units for both procedures, and it varies sinusoidally. At -0.5 units, the displacement inaccuracy is 6×10^{-3} , and it falls exponentially to 3×10^{-3} at 0.5 units. Both approaches have a slope of -200 at -0.5 units, which climbs parabolically to 200 at 0 units. At -0.5 units, the slope error is -6×10^{-3} and falls exponentially to -3×10^{-3} . For both approaches, the curvature is 200 at -0.3 units and 0 at 0. Curvature error is 6×10^{-3} at -0.5 units and gradually decreases to 3×10^{-3} at 0.5

units. For both techniques, sharpness is 1000 at -0.5 units and declines exponentially to -1000 at 0 units. Sharpness error is -6×10^{-3}

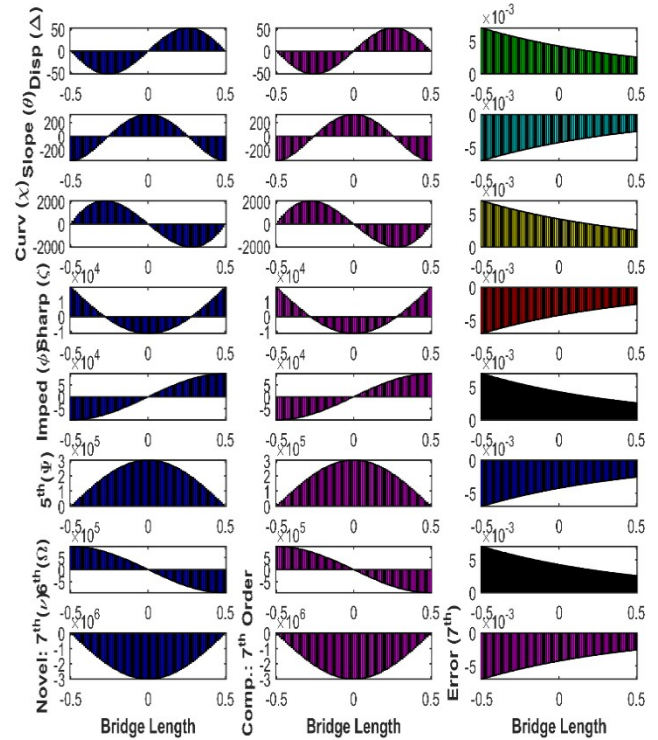


Fig.5. Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order (Novel & Computational) for $u^{(4)} = (\pi^4 - 1) \sin(\pi x) + u$ differs from $-\frac{1}{2} \leq \xi \leq \frac{1}{2}$.

and falls exponentially up to -3×10^{-3} . Impendent and 6th order have the same values but are opposite at -0.5. It is -5×10^{-3} at -0.5 units and gradually decreases to 3×10^{-3} at 0.5 units. 5th and 7th order have the same values, but opposite at -0.5, 0 at -0.5, and 0.5 units and gradually decrease to 3×10^{-3} at 0.5 units. The maximum displacement, slope, curvature, sharpness, and impendent values measured are 50, 200, $200, 1 \times 10^4$, and 4×10^4 units, respectively.

4.3. Case 3: The presiding differential equality of the fourth order equation is $u^{(4)} = 1+u$, and the Boundary stipulations are $[u(1) = 0, u(\frac{1}{2}) = 0, u''(1) = 0, u''(\frac{1}{2}) = 0]$, and contrasts from $\frac{1}{2} \leq \xi \leq 1$.

4.3.1. Novel technique solution:

$$\text{Displacement, } u = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e \\ = 0.3776 \cos(x) + 0.3518 \sin(x) + 0.1145 \exp(x) + 0.5131 \exp(-x) - 1 \quad (13)$$

$$\text{Slope, } \frac{du}{dx} = -a \sin(x) + b \cos(x) + c \exp(x) - d \exp(-x) \\ = -0.3776 \sin(x) + 0.3518 \cos(x) + 0.1145 \exp(x) - 0.5131 \exp(-x) \quad (14)$$

$$\text{Curvature, } \frac{d^2u}{dx^2} = -a \cos(x) - b \sin(x) + c \exp(x) + d \exp(-x)$$

$$= -0.3776 \cos(x) - 0.3518 \sin(x) + 0.1145 \exp(x) + 0.5131 \exp(-x) \quad (15)$$

Sharpness function, $\frac{d^3 u}{dx^3} = a \sin(x) - b \cos(x) + c \exp(x) - d \exp(-x)$

$$= 0.3776 \sin(x) - 0.3518 \cos(x) + 0.1145 \exp(x) - 0.5131 \exp(-x) \quad (16)$$

Impendent function, $\frac{d^4 u}{dx^4} = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x)$

$$= 0.3776 \cos(x) + 0.3518 \sin(x) + 0.1145 \exp(x) + 0.5131 \exp(-x) \quad (17)$$

4.3.2. Exact solution:

The exact solution, $u_e = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.3776 \cos(x) + 0.3518 \sin(x) + 0.1145 \exp(x) + 0.5080 \exp(-x) - 1 \quad (18)$$

4.3.3. Discussion of Results for case 3:

Fig. 6 illustrates the displacement, slope, curvature, sharpness, and error plots for both innovative and computational approaches. The innovative and computational solutions nearly correspond to a beam length change from $[\frac{1}{2} \leq \xi \leq 1]$. For both techniques, displacement is initially 0 at 0.5 unit and zero at 1 unit, varying parabolically. The displacement error is -3×10^{-5} at 0.5 units and reduces exponentially to -2×10^{-5} at 1 unit. The slope for both procedures is five at 0.5 units, then drops exponentially to -5 at 1 unit. The slope error is -0.05 at 0.5 units, increasing rapidly up to 0.05. Both approaches have a Curvature & 6th order of -30 at 0.74 and 0 at 0.5, 1 units. Curvature error is 0 at 0.5 units and increases to 0.3×10^{-3} and decreases to 0 at 1 unit. Sharpness & 7th order is -200 at 0.5 units and increases exponentially to 200 at 1 unit for both techniques. Sharpness error is 2×10^{-3} at 0.5 unit and declines rapidly to -2×10^{-3} at 1 unit. For both methods, the impendent function is 1000 at -1, -0.5 units. The impendent error is -2×10^{-3} at -1 units, and it grows exponentially to -3×10^{-3} at -0.5 units, and the maximum displacement, slope, curvature, sharpness, and impendent values measured are 800, 5, -30, 200, and 1000 units, respectively.

4.4. Case 4: The reigning differential contrast of the 4th order equation is $u^{(4)} = 1 + u$, and the Boundary prerequisites are $[u(0) = 0, u(1) = 0, u''(0) = 0, u''(1) = 0]$, and varies from $[0 \leq \xi \leq 1]$.

4.4.1. Novel technique solution:

Displacement, $u = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \exp(x) + 0.3655 \exp(-x) - 1 \quad (19)$$

Slope, $\frac{du}{dx} = -a \sin(x) + b \cos(x) + c \exp(x) - d \exp(-x)$

$$= -0.5000 \sin(x) + 0.2732 \cos(x) + 0.1345 \exp(x) - 0.3655 \exp(-x) \quad (20)$$

Curvature, $\frac{d^2 u}{dx^2} = -a \cos(x) - b \sin(x) + c \exp(x) + d \exp(-x)$

$$= -0.5000 \cos(x) - 0.2732 \sin(x) + 0.1345 \exp(x) + 0.3655 \exp(-x) \quad (21)$$

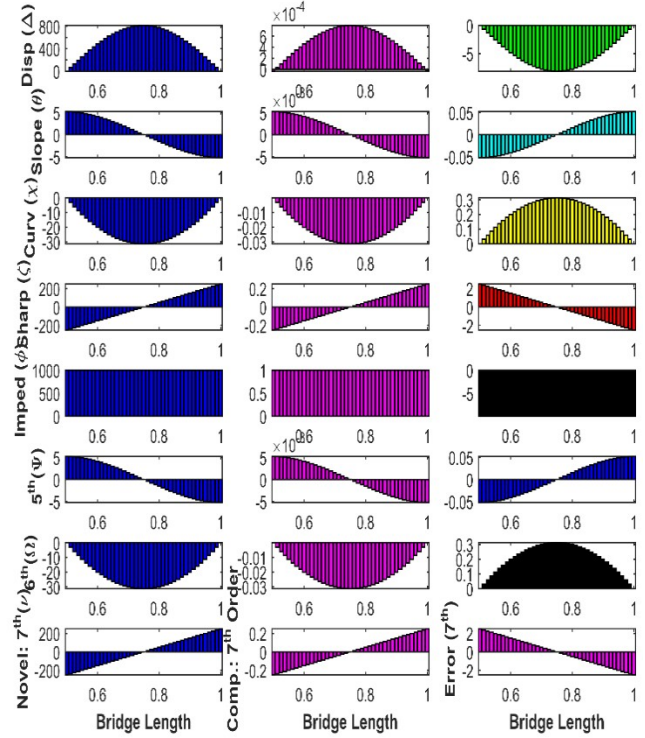


Fig.6. Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order (Novel & Computational) for $u^{(4)} = 1 + u$ varies from $\frac{1}{2} \leq \xi \leq 1$.

Sharpness function, $\frac{d^3 u}{dx^3} = a \sin(x) - b \cos(x) + c \exp(x) - d \exp(-x)$

$$= 0.5000 \sin(x) - 0.2732 \cos(x) + 0.1345 \exp(x) - 0.3655 \exp(-x) \quad (22)$$

Impendent function, $\frac{d^4 u}{dx^4} = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x)$

$$= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \exp(x) + 0.3655 \exp(-x) \quad (23)$$

4.4.2. Exact solution:

The exact solution, $u_e = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \exp(x) + 0.3619 \exp(-x) - 1 \quad (24)$$

4.4.3. Discussion of Results for case 4:

Fig. 7 illustrates the displacement, slope, curvature, sharpness, and error plots for both novel and computational methods for a beam length change from $[0 \leq \xi \leq 1]$. For both ways, displacement is initially 0 at 0, 1 unit, and it varies parabolically. The displacement inaccuracy is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. The slope & 5th order for both techniques is 0.04 at 0 unit; then it decreases exponentially to -0.04 at 1 unit and zeroes at 0.5 unit. The slope & 5th order error is 2×10^{-5} at 0 unit and decreases exponentially up to 1×10^{-5} . Both

approaches have a curvature & 6th order of -0.1 at 0.5 units and 0 at $0,1$. Curvature & 6th order error is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. Sharpness & 7th

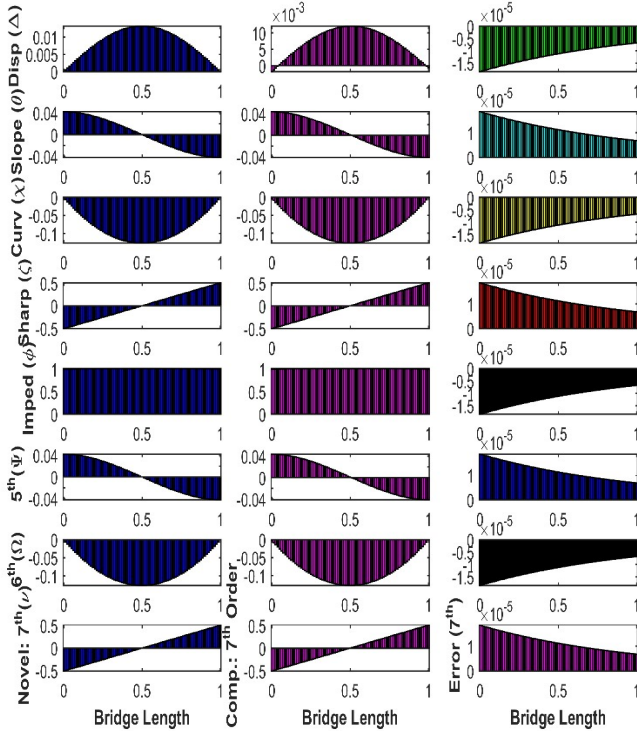


Fig.7. Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order, and error (Novel & Computational) for $u^{(4)} = 1+u$ differs from $0 \leq \xi \leq 1$.

order is -0.5 at 0 units and grows exponentially to 0.5 at 1 unit and zero at 0.5 units for both approaches. Sharpness error is 2×10^{-5} at 0 unit and drops exponentially to 1×10^{-3} at 1 unit. For both methods, the impendent function is one at 0 and 1 unit. The impendent error is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. The novel and computational solutions closely correspond, and the maximum displacement, slope, curvature, sharpness, and impendent values measured are 0.01 , 0.04 , -0.1 , 0.5 , and 1 unit, respectively.

4.5. Case 5: The overseeing differential contrast of the fourth order equation is $u^{(4)} = (\pi^4 - 1) \sin(\pi x) + u$, and the Boundary specifications are $[u(0) = 0, u(1) = 0, u''(0) = 0, u''(1) = 0]$, and differs from $[0 \leq \xi \leq 1]$.

4.5.1. Novel technique solution:

Displacement, $u = a \cos(\pi x) + b \sin(\pi x) + c \sin(\pi x) + d \sin(\pi x) + e$

$$= 5.7614 \cos(4.1416x) - 5.7614 \sin(4.1416x) - 4.4348 \sin(\pi x) + 11.2544 \sin(x + 2.1416x) \quad (25)$$

Slope, $\frac{du}{dx} = -a \sin(\pi x) + b \cos(\pi x) + c \pi \cos(\pi x) + d \cos(x + \pi x) (1 + \pi)$

$$= -23.8614 \sin(4.1416x) - 23.8614 \cos$$

$$(4.1416x) - 4.4348 \pi \cos(\pi x) + 35.3568 \cos(x + 2.1416x) \quad (26)$$

Curvature $\frac{d^2u}{dx^2} = -a \cos(\pi x) - b \sin(\pi x) - c \pi^2 \sin(\pi x) - d \sin(x + \pi x) (1 + \pi)^2$

$$= -98.8244 \cos(4.1416x) + 98.8244 \sin(4.1416x) + 4.4348 \pi^2 \sin(\pi x) - 111.0769 \sin(x + 2.1416x) \quad (27)$$

Sharpness function, $\frac{d^3u}{dx^3} = a \sin(\pi x) - b \cos(\pi x) - c \pi^3 \cos(\pi x) - d \cos(x + \pi x) (1 + \pi)^3$

$$= 409.2911 \sin(4.1416x) + 409.2911 \cos(4.1416x) + 4.4348 \pi^3 \cos(\pi x) - 348.9592 \cos(x + 2.1416x) \quad (28)$$

Impendent function, $\frac{d^4u}{dx^4} = a \cos(\pi x) + b \sin(\pi x) + c \pi^4 \sin(\pi x) + d \sin(x + \pi x) (1 + \pi)^4$

$$= 1695.1200 \cos(4.1416x) - 1695.1200 \sin(4.1416x) - 4.4348 \pi^4 \sin(\pi x) + 1096.2902 \sin(x + 2.1416x) \quad (29)$$

4.5.2. Exact solution:

The exact solution, $u_e = a \cos(\pi x) + b \sin(\pi x) + c \sin(\pi x) + d \sin(x + \pi x)$

$$= 5.7614 \cos(4.1416x) - 5.8196 \sin(4.1416x) - 4.4348 \sin(\pi x) + 11.2544 \sin(x + 2.1416x) \quad (30)$$

4.5.3. Discussion of Results for case 5:

Fig. 8 The displacement, slope, curvature, sharpness, and error plots for both novel and computational approaches are shown. Length varies for a beam $[0 \leq \xi \leq 1]$. For both methods, displacement starts at zero and increases parabolically. At 0.38 units, the displacement inaccuracy is -3×10^{-4} , increasing parabolically to 5×10^{-3} at 1 unit. Both approaches have a 200 at 0 unit slope, which declines parabolically to -200 at 1 unit and zero at 0.5 unit. The slope error is -1.5×10^{-3} at 0 unit, increasing parabolically to 2.4×10^{-3} at 0.7 , and finally to zero at 1 unit. Curvature & 6^{th} order error is -800 at 0.5 units and 0 at 0 , and 1 unit for both techniques. Curvature & 6^{th} order error is 0 at 0 , increasing parabolically to 6×10^{-3} at 0.35 and -5×10^{-3} at 1 unit. Sharpness & 7^{th} order starts at -20000 and increases to 20000 at 1 unit and zero at 0.5 for both approaches. At 0 unit, the sharpness & 7^{th} order error is 20×10^{-3} and declines to -20×10^{-3} at 0.65 unit. The maximum displacement, slope, curvature, sharpness, and impendent values observed are 800 , 200 , -800 , 2×10^4 , and 10×10^5 units, respectively.

4.6. Case 6: The governing differential reckoning of the 4^{th} order equation is $u^{(4)} = 1 + u$, and the Boundary provisions are $[u(0) = 0, u(1) = 0, u''(0) = 0, u''(1) = 0]$, and diverges from $[0 \leq \xi \leq 1]$.

4.6.1. Novel technique solution:

Displacement, $u = a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e$

$$= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \exp(x) + 0.3655 \exp(-x) - 1 \quad (31)$$

Slope, $\frac{du}{dx} = -a \sin(x) + b \cos(x) + c \exp(x) - d \exp(-x)$

$$\begin{aligned}
 x) \\
 &= -0.5000 \sin(x) + 0.2732 \cos(x) + 0.1345 \\
 &\exp(x) - 0.3655 \exp(-x) \quad (32)
 \end{aligned}$$

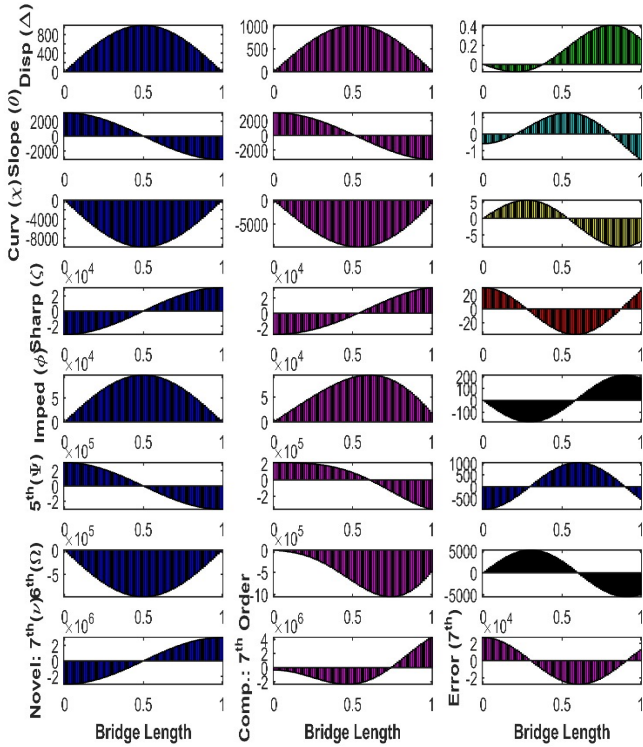


Fig.8. Displacement, slope, curvature, sharpness and error (Novel & Computational) for $u^{(4)} = (\pi^4 - 1) \sin(\pi x) + u$ varies Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order and error from $0 \leq \xi \leq 1$.

$$\begin{aligned}
 \text{Curvature, } \frac{d^2 u}{dx^2} &= -a \cos(x) - b \sin(x) + c \exp(x) + d \exp(-x) \\
 &= -0.5000 \cos(x) - 0.2732 \sin(x) + 0.1345 \\
 &\exp(x) + 0.3655 \exp(-x) \quad (33)
 \end{aligned}$$

$$\begin{aligned}
 \text{Sharpness function, } \frac{d^3 u}{dx^3} &= a \sin(x) - b \cos(x) + c \exp(x) - d \exp(-x) \\
 &= 0.5000 \sin(x) - 0.2732 \cos(x) + 0.1345 \\
 &\exp(x) - 0.3655 \exp(-x) \quad (34)
 \end{aligned}$$

$$\begin{aligned}
 \text{Impendent function, } \frac{d^4 u}{dx^4} &= a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) \\
 &= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \\
 &\exp(x) + 0.3655 \exp(-x) \quad (35)
 \end{aligned}$$

4.6.2. Exact solution:

$$\begin{aligned}
 \text{The exact solution, } u_e &= a \cos(x) + b \sin(x) + c \exp(x) + d \exp(-x) + e \\
 &= 0.5000 \cos(x) + 0.2732 \sin(x) + 0.1345 \\
 &\exp(x) + 0.3619 \exp(-x) - 1 \quad (36)
 \end{aligned}$$

4.6.3. Discussion of Results for case 6:

Fig. 9 illustrates the displacement, slope, curvature, sharpness, and error plots for both novel and computational methods for a beam length change from $[0 \leq \xi \leq 1]$. For both ways, displacement is initially 0 at 0,1 unit and varies parabolically. The displacement inaccuracy is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. The

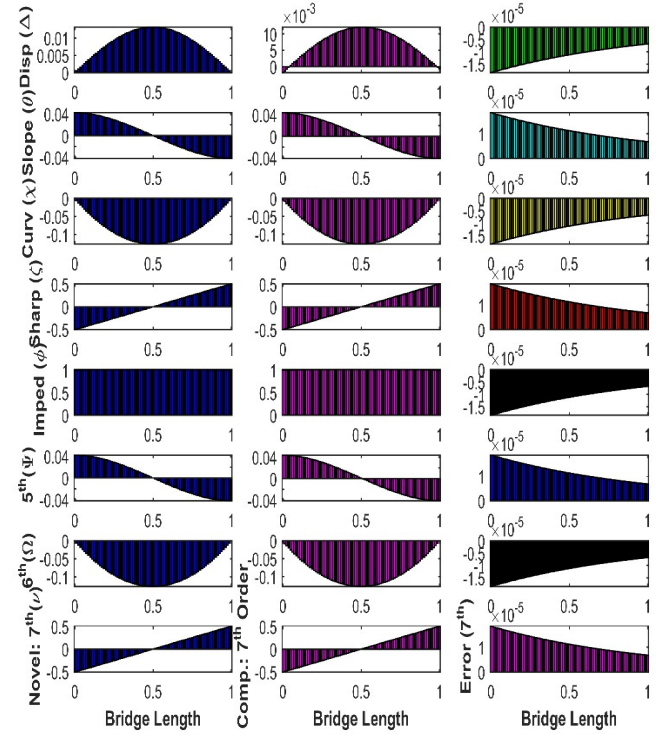


Fig.9. Displacement, slope, curvature, sharpness, impendent, fifth, sixth, seventh order, and error (Novel & Computational) for $u^{(4)} = 1 + u$ differs from $0 \leq \xi \leq 1$.

slope & 5th order for both techniques is 0.04 at 0 unit; then it decreases exponentially to -0.04 at 1 unit and zeroes at 0.5 unit. The slope & 5th order error is 2×10^{-5} at 0 unit and decreases exponentially up to 1×10^{-5} . Both approaches have a curvature & 6th order of -0.1 at 0.5 units and 0 at 0,1. Curvature & 6th order error is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. Sharpness & 7th order is -0.5 at 0 units and grows exponentially to 0.5 at 1 unit and zero at 0.5 units for both approaches. Sharpness error is 2×10^{-5} at 0 unit and drops exponentially to 1×10^{-3} at 1 unit. For both methods, the impendent function is one at 0 and 1 units. The impendent error is -1.5×10^{-5} at 0 units and decreases exponentially to -1×10^{-5} at 1 unit. The novel and computational solutions closely correspond, and the maximum displacement, slope, curvature, and sharpness values measured are 0.001, 0.04, -0.1 , 0.5, and 1 unit, respectively.

5. Conclusions

A novel method is employed to manage a range of technical challenges and continuing support bridge challenges.

Consequently, the unique technique's results are contrasted with the precise solution. The novel approach is simple to utilize, even for multi-media tasks. The answer is supplied as rapidly converging progressions with easily computable components using the present unique method. In the fourth order non-homogeneous Problem, for a beam length varying from $[-1 \leq \xi \leq 1]$ from case 1 to case 3.

- ❖ The maximum displacement, slope, curvature, sharpness, and impendent values observed are 0.8, 5, -30, 200, and 1000 units. For beam, length varies from $[-1 \leq \xi \leq -\frac{1}{2}]$.
- ❖ A beam length differs from $[-\frac{1}{2} \leq \xi \leq \frac{1}{2}]$. The maximum displacement, slope, curvature, sharpness, and impendent values measured are 50, 200, 1×10^4 , and 4×10^4 units, respectively.
- ❖ The maximum displacement, slope, curvature, sharpness, and impendent values measured are 800, 5, -30, 200, and 1000 units, respectively for beam length varies from $[\frac{1}{2} \leq \xi \leq 1]$.

In the fourth order non-homogeneous Problem, for a beam length varying from $[0 \leq \xi \leq 1]$ from case 4 to case 6.

- ❖ The maximum displacement, slope, curvature, sharpness, and impendent values measured are 0.01, 0.04, -0.1, 0.5, and 1 unit, respectively.
- ❖ The maximum displacement, slope, curvature, sharpness, and impendent values observed are 800, 200, -800, 2×10^4 , and 10×10^5 units, respectively.
- ❖ The maximum displacement, slope, curvature, sharpness, and impendent values measured are 0.01, 0.04, -0.1, 0.5, and 1 unit, respectively.

Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

References

1. Ganesh, E., Ramana, P. V., & Shrimali, M. K. (2022). Solved structural dynamic mathematical models via a novel technique approach. *Materials Today: Proceedings*.
2. Anamika Agnihotri, P.V. Ramana (2021), Modified Adomian Decomposition Method for Uni-Directional Fracture Problems, *Sadhana*, unique addition, 1-9.
3. B K Raghu Prasad, P.V. Ramana (2012), "Modified Adomian decomposition method for fracture of laminated uni-directional composites," *Sadhana Vol. 37, Part-1*, pp.33-57.
4. Ayush Meena, P.V. Ramana (2021), Mathematical Model for Recycled PolyEthylene Terephthalate Material Mechanical Strengths, *Materials Today: Proceedings*, 38, Part 5.
5. Arigela Surendranath, P.V. Ramana (2021), Mathematical Approach on Recycled Material Strength Performance Via Statistical Mode, *Materials Today: Proceedings*, 38, Part 5.
6. Ayush Meena, P.V. Ramana (2021), High-Rise Structural Stalling and Drift Effect Owe to Lateral Loading, *Materials Today: Proceedings*, 38, Part 5.
7. P.V. Ramana (2021), Statistical concert of solid effect on fiber concrete, *Materials Today: Proceedings*, 38, Part 5.
8. Ganesh, E., Ramana, P. V., & Shrimali, M. K. (2022). Unsolved structural dynamic mathematical models via Novel technique approach. *Materials Today: Proceedings*.
9. Vivek Singh, P.V. Ramana (2013), "The Magnitude of Linear Problems," "International Conference on Emerging Trends in Engineering & Applied Sciences (ICETEAS)," published in *International Journal of Advanced Engineering & Computing Technologies*, ISSN: 2249-4928.
10. Ganesh, E., Shrimali, M. K., & Ramana, P. V. (2021). Estimation of Seismic Fragility in Structural Systems. *i-Manager's Journal on Structural Engineering*, 10(2), 1.
11. Arigela Surendranath, P.V. Ramana (2021), Interpretation of bi-material interface through the mechanical and microstructural possessions *Materials Today: Proceedings*, 17, Part 3.
12. Anamika Agnihotri, P.V. Ramana (2021), Mathematical Model for Recycled PolyEthylene Terephthalate Material Mechanical Strengths, *Materials Today: Proceedings*, 38, Part 5.
13. Anamika Agnihotri, P.V. Ramana (2021), GGBS: fly-Ash Evaluation and Mechanical properties within high strength concrete, *Materials Today: Proceedings*, 38, Part 5.
14. Ayush Meena, P.V. Ramana (2021), Assessment of endurance and microstructural properties effect on polypropylene concrete, *Materials Today: Proceedings*, 38, Part 5.
15. P.V. Ramana (2020), Functioning of bi-material interface intended for polypropylene fiber concrete, *Materials Today: Proceedings*, 14, Part 2.
16. Anamika Agnihotri, P. V. Ramana "Strength and Durability Analysis of GGBS & Recycled Materials," *International Conference on Advances in Civil and Structural Engineering (ICACSE-2020)*, May 28-30, 2020.
17. Ganesh, E., Ramana, P. V., & Shrimali, M. K. (2022). Inelastic materials and mathematical variables for obstacle bridge problem evaluation. *Materials Today: Proceedings*.
18. A. Wazwaz (1998), "A comparison between Adomian decomposition method and Taylor series method in the series solution", *applied mathematics computations*, vol. 97, pp. 37-44.
19. A. Wazwaz (1999), "A reliable modification of Adomian decomposition method", *applied mathematical computations*, vol. 102, pp. 77-86.