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Development of Moment Coefficients For Two Adjacent Side Supported Slab By Using Yield Line Theory

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Abstract

The slab is a key and generally used structural component. The shape, arrangement, and stiffness of the supporting beams all have a significant role in the slab's performance.IS 456:2000 coefficients are only applicable to rectangular slabs with four non-yielding side supports. But due to architectural requirements, structural engineers are now confronted with a variety of complications, such as slabs supported on two adjacent sides. In the past analytical model was suggested for slab supported over two adjacent edges carrying uniformly distributed area load but design aids were not prepared. The analytical model presented herein to analyze two adjacent edges supported slab carrying uniformly distributed area load on its top edge using yield line analysis. It is also validated with well-established literature. Design aids are also prepared and validated with FE nonlinear analysis in software ATENA and the results are in favour.

Keywords: Moment Coefficients, Yield Line Theory, Energy Principles, FEM Analysis

1. Introduction

Slab is the most important structural element in any building. It is used almost in every building to enclose the space along with some other structural elements such as walls, columns etc. In comparison to other structural elements, a slab is highly indeterminate in nature due to the integrity of the internal stress resultants and it offers several load paths to the applied loading. Due to the integrity of internal force-resultants, the structural behaviour of the slabs is highly susceptible to the type, layout of the supporting system and stiffness of the supporting structural element. Change in any parameters of the supporting systems causes a significant change in the moment-field induced in the slab under applied loading. Removal of even a single support will lead to a considerable change in the behaviour of the slab. In addition, even small changes in the depth of the supporting beams/system will cause a significant change in the moment-field in the slab panel.

The coefficients for moment, which are suggested by Indian codes (IS 456:2000) are only applicable for a rectangular slab with four side non-yielding support having different aspect ratio and edge-conditions [1]. But in routine design practice, numbers of case are encountered by the designers such as opening in slab, three side supported slab, two side supported slab system. For which the moment coefficient suggested by IS 456:2000 cannot be applicable [1]. Design of this type of problem can be done by FEM based software or developing own solution (which is time consuming and also leads to uneconomical design) or can follow the published literature [3], [4], [6], [7], [9], [12], [13].Various mathematical expressions are available in published literature, which can be used for the precise estimation of collapse load and bending moments induced in a reinforced concrete slab with different edge conditions and various shapes [2], [5], [7], [11], [14]. All these outcomes are either based upon lower or upper-bound theorems of limit analysis or the elastic theory of plates, but these are mathematically too difficult to get a solution in routine design practices; and also, it is a time consuming process. Solution to predict the most precise collapse load of a reinforced concrete (RC) slab must satisfy the equilibrium, the mechanism conditions and the yield criteria [6].

These requirements are normally too hard to achieve directly, therefore upper- or lower-bound theorems are oftenly used for find the solutions. Both theorems satisfy the equilibrium condition, but in the upper-bound approach yield criterion is ignored, although lower-bound technique does not require any condition to satisfy for the collapse mechanism at the ultimate state. The upper bound method can predict a collapse mechanism, which gives the unsafe yield criterion of the slab under applied of loading, and anticipate results on highly unsafe side if the analyst fails to predict the accurate yield line pattern of the slab at ultimate

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load. The current state-of-art available for the design of reinforced concrete slabs does not satisfactorily provide the solution for two adjacent side supported slab [7]. This literature gives you simple and optimised design calculations.

2. Objective of the Study

Keeping in mind the limitation and constraints of various theories and design guidelines available in the various published literature, this project attempts to present an analytical model that can be used to find the moment-field induced in a slab-system supported over the two adjacent sides. It has been developed using the principle of limit analysis with following objectives:-

- To develop moment coefficient for two adjacent side supported slab.
- To perform FEM analysis.
- Validate the results of proposed design aids and FEM analysis.

3. Proposed Methodology

If a RC slab continuously loaded to a failure, it develops a strip of flexural cracks at its tensile face along the lines of maximum curvature. This strip of cracks across which the tensile steel in the slab has been yielded is represented by a single line at the center of strip and all plastic rotation of the slab segments are assumed to be along this single line. This failure line is called as yield line [6]. After the complete formation of a yield line pattern, the slab system starts behaving like a rigid-perfectly plastic material which absorbs no energy during further deformation. Hence at this stage, the total external energy must disappear for equilibrium of slab when an arbitrary value of a virtual displacement given to the mechanism. This condition can satisfy only, if the total external work done by the external forces is equal to the total internal work done by the moment induced in the system. This will fulfill the equilibrium of the assumed collapse mechanism of the slab at ultimate state. This condition gives the collapse load on the higher side or at the most equal to the true value depending upon the shape of the assumed yield line pattern.

It is assumed that, the load vs. deformation response of a RC slab at near the collapse, is rigid-perfectly plastic, which requires a ductile slab section which is appropriately supported and provided with evenly distributed reinforcing steel at the tensile face of the slab. In addition to this requirement, the analytical model has been developed with following constraints [12].

- The slab carrying an out-of-plane uniformly distributed area load over its entire top face.
- Reinforcement is orthotropic in nature and provided at the tensile face of the slab and reinforcing steel bars placement is parallel to the edges of the slab.
- Corner levers effect has ignored because this will cause a small reduction in collapse load and also the yield line theory is conservative.
- Arching or compressive membrane action (CMA) in reinforced concrete slabs section has been ignored and if considered, this will only increase the yielding strength of the slab system.
- Slab openings, punching and shear effect have not been considered in the analysis.
- Kinking effect of the reinforcing bars across the crack / yield line and biaxial stress conditions in the concrete compression zone of slab has been ignored because these do not affect the yield criterion of reinforced concrete slabs.

These assumptions must be satisfied to use the proposed analytical model for the analysis of slab-systems.

3.1 Solution Using Energy Principle

The work equations are calculated by using the equilibrium of the total energies i.e.the internal work done by yield lines are equated with the work done by the external load of the orthogonal rectangular slab where a small virtual displacement is given to the slab [6], [13]. The work equation is given by:

$$\int\int W_{ult}\delta(x,y)dx.dy \; = \sum(m_{ult}\,x.\theta_x.y_o \; + m_{ult}y,\theta_y.x_o$$

Here, W_{ult} is the ultimate load per unit area carried by the slab at yield criterion, δ(x,y) is the applied virtual displacement in the direction of the loading at the area of element having dimensions $d_x \times d_y$, m_{ultx} and m_{ulty} are the ultimate yield moments developed per unit within the x and y directions, θ_x and θ_y are the virtual rotation components of the slab segments in the x and y directions and x_0& y_0 are the projected lengths of the yield lines in two perpendicular x and y directions of slab.

3.2 Analytical Derivation

There is several possible yield line patterns associated with different edge conditions of the slab. But in this case only one yield line pattern (shown in fig. 1) is possible.

In the yield line pattern shown below the point e is given unit vertical virtual displacement. The external and internal work done can be computed as follows.

Let, L_X = Short Span

L_y = Long Span

W_u = Ultimate load per unit area.

M_{u1} = Short span moment

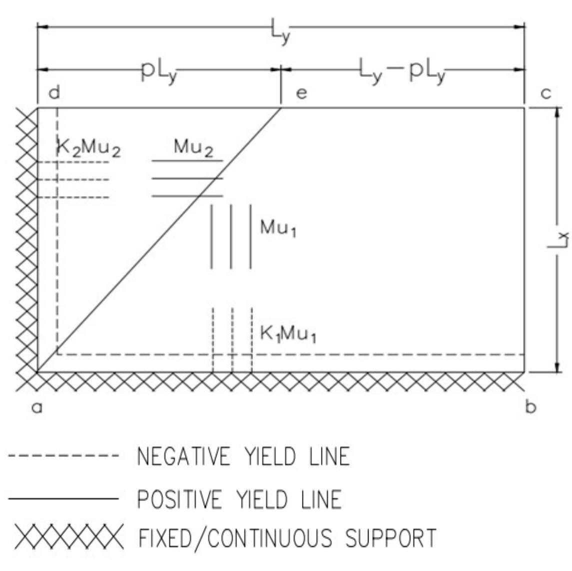

Fig. 1: -Proposed yield line pattern for two adjacent side supported slab.

M_{u2} = Long span moment
K_1 = Ratio of negative moment to positive moment along short span
K_2 = Ratio of negative moment to positive moment along longer span.
μ = orthotropy $= \left(\frac{M_{u2}}{M_{u1}}\right)$
W_e = Total external work done by load
W_i= Total internal work done by load
Therefore, external work done (W_e)
$W_e = W_{e(aed)} + W_{e(abce)}$

$$=W_u\left\{\left(\frac{pL_yL_x}{2}\right)\left(\frac{1}{3}\right)\right\} + W_u\left\{\left(\frac{pL_yL_x}{2}\right)\left(\frac{1}{3}\right) + (L_y - pL_y)L_x\left(\frac{1}{2}\right)\right\}$$

$$= \frac{W_uL_xL_y}{6}(3-p)$$

$$= \frac{W_uL_x^2\, r}{6}(3-p)$$

Here, $\left(r = \frac{L_y}{L_x}\right)$
$W_i = W_{i(aed)} + W_{i(abce)}$

$$=M_{u2}L_x\left(\frac{1}{pL_y}\right) + K_2M_{u2}L_x\left(\frac{1}{pL_y}\right) + M_{u1}\,pL_y\left(\frac{1}{L_x}\right) + K_1M_{u1}L_y\left(\frac{1}{L_x}\right)$$

$$= \frac{M_{u2}(1+K_2)}{p\,r} + \left(M_{u1}r(p+K_1)\right)$$

$$= M_{u1}r\left(\frac{\mu(1+K_2)}{p\,r^2} + (p+K_1)\right)$$

$$\mu = \left(\frac{M_{u2}}{M_{u1}}\right)$$

The work equation $W_e = W_i$ gives,

$$\frac{W_uL_x^2\, r}{6}(3-p) = M_{u1}r\left(\frac{\mu(1+K_2)}{p\,r^2} + (p+K_1)\right)$$

Therefore,

$$W_u = \left(\frac{6M_{u1}}{L_x^2}\right)\left[\frac{\mu(1+K_2)}{p\,r^2} + (p+K_1)\right]\left(\frac{1}{3-p}\right) \qquad \text{...... (1)}$$

For critical load Wu, $\frac{\delta W_u}{\delta p} = 0$, i.e.

$$\left\{\left[-\frac{\beta(1+K_2)}{p^2} + 1\right]\left(\frac{1}{3-p}\right) + \left[\frac{\beta(1+K_2)}{p} + (p+K_1)\right]\left(\frac{1}{(3-p)^2}\right)\right\} = 0$$

$$\left(\beta = \frac{\mu}{r^2}\right)$$

$$p^2 + \frac{2\beta(1+k_2)}{(3+K_2)}p - 3\frac{\beta(1+k_2)}{(3+K_2)} = 0$$

Put,

$$\left[\frac{(1+K_{2)}}{(3+K_1)}\right] = s$$

Therefore,

$$p^2 + 2\beta sp - 3\beta s = 0$$

Where,

$$p = \beta s\left(\sqrt{1-\frac{3}{\beta s}} - 1\right) \qquad \text{......(2)}$$

Equation (1) and (2) gives the required moment coefficients.

4. DESIGN AID

Design aids were prepared using an analytical model along with orthotropic coefficients and continuity factors (given in **Erreur ! Source du renvoi introuvable.**) for different aspect ratios. These design aids can be used by the structural engineers for designing the two adjacent edges supported slab carrying uniformly distributed are load along with line load (UDL) at free edges.
Therefore, moment field in x-direction can be calculated by using equation (3) and (4),

$$m_x^+ = \alpha_x^+ \, w\, l_x^2 \qquad (3)$$

$$m_x^- = \alpha_x^- \, w\, l_x^2 \qquad (4)$$

Moment field in y-direction can be calculated by using equation (5) and (6),

$$m_y^+ = \alpha_y^+ \, w\, l_x^2 \qquad (5)$$

$$m_y^- = \alpha_y^+ \, w\, l_x^2 \qquad (6)$$

Where,
α_x^+ is the coefficient for a span positive moment and α_x^- is for support negative moment along the short span (l_x). α_y^+ is the span moment coefficient and α_y^- is the support negative moment along a long span (l_y). The design aids prepared are given in **Erreur ! Source du renvoi introuvable.**to **Erreur ! Source du renvoi introuvable.** for different cases.
w = uniform area load carried by the slab.
l_x = short span

5. VALIDATION

This research has been carried out to prepare design aids and to validate the proposed analytical model with the available literature. Design aids are validated with FE nonlinear analysis carried out in ATENA 3D software.

5.1 Validation with available literature

The proposed analytical model has been validated with available literature(Park, R. & Gamble, 2000).

To validate the proposed analytical model for calculating the moment field induced intwo adjacent edges fixed supported RC rectangular slab, a comparison is drawn in Table for two adjacent edges fixed slab subjected to uniformly distributed area load of 10 kN/m^2 having short span 2m with aspect ratio 1.3, i_2 or k_1= 11.68, i_1 or k_2 = 6.44, μ = 1.74 (μ = 0.575 for park and gamble).

Table 1 Proposed Moment Coefficients for slab supported over two adjacent edges.

r	Mu$_1$	Mu$_1$-	Mu$_2$	Mu$_2$-
1	0.0188	0.1482	0.0188	0.1482
1.1	0.0184	0.1645	0.0225	0.1616
1.2	0.0177	0.1799	0.0259	0.1742
1.3	0.0167	0.1945	0.0289	0.1862
1.4	0.0152	0.2084	0.0315	0.1975
1.5	0.0137	0.2219	0.0336	0.2076
1.6	0.0123	0.2345	0.0352	0.2170
1.7	0.0107	0.2467	0.0364	0.2250
1.8	0.0090	0.2582	0.0372	0.2324
1.9	0.0077	0.2701	0.0380	0.2354
2	0.0065	0.2784	0.0383	0.2453

Table 2 Comparison of moment field for two adjacent edges fixed supported slab.

Moment-field	**Park and Gamble (2000) (kN-m/m)**	**Proposed analytical model (kN-m/m)**
Positive moment parallel to the long edge	0.66	0.66
Negative Moment parallel to the long edge	7.77	7.77
Positive moment parallel to the short edge	1.15	1.15
Negative moment parallel to the short edge	7.45	7.45

Table 3 Reinforcement details for slab supported over two adjacent fixed edges.

Slab ID.	**SLAB DIMENSION**			**REINFORCEMENT DETAILS**			
	lx(m)	**ly(m)**	**Thickness(mm)**	**Along l_x^+**	**Along l_x^-**	**Along l_y^+**	**Along l_y^-**
AF1	2	2.5	160	8Φ @ 273 mm	8Φ @273 mm	8Φ @280 mm	8Φ @280 mm
AF2	2.1	3.6	160	8Φ @296 mm	8Φ @255 mm	8Φ @294 mm	8Φ @258 mm
AF3	1.8	3.5	150	8Φ @288 mm	8Φ @288 mm	8Φ @297 mm	8Φ @297 mm

5.2 Validation using FEM based software ATENA

Numerical simulation is done to validate the proposed design aids; in FEM-based software ATENA.Three numbers of slabs for each case has been considered for the validation. The slab considered for the validation has been designed using the proposed design aids in chats above for different cases.
In this research work, M20 concrete was used for microelement having an elastic modulus of 22360.67 and Poisson's ratio 0.15. For steel bars, Fe 415 Grade was used having an elastic modulus of 200000 MPa with Poisson's ratio of 0.3.Sensitivity analysis has been carried out to find the optimum mesh size in terms of the accuracy and the time consumed to analyse the slab. Square size mesh of 0.1m used to carry out the analysis.The loading rate has been kept $1/10^{th}$ of the design load. Arc length parameter used for peak load estimation. The detail of the slabs used for validation is given in Table – 3.

6. RESULT AND DISCUSSION

Analytical modelling of a slab supported over two adjacent edges supported slab using yield line analysis has been carried out and the equations from (1) and (2) are proposed to find moment field induced in the slab. Proposed equations were validated with the available literature. Results of validation for different cases are given in Table . The results, while validating with available literature are found satisfactory as the values of the moment field are matched exactly. Design aids were prepared using equations (1) and (2) along with orthotropic coefficient and continuity factors (given in Annexure – A). Slabs designed by proposed design aids were analysed in ATENA 3D software. Incremental load analysis is done, in which slab is loaded up to complete collapse is achieved. The total load carried by the slab at the collapse is then calculated by adding all values of load at nodes. The total load carried by the slab is then compared with the design load coming after satisfying equilibrium. The comparison is drawn between design load and ultimate load carried by the slab, obtained from nonlinear analysis output are given in Table 4.

The crack pattern obtained (shown in fig.) is much comparable with the assumed yield line pattern in the case of two adjacent edges fixed.

Table 4 Comparison between design load and ultimate load carried by the slab.

Case	**Slab Id**	**Design load (kN)**	**Ultimate load carried by the slab (kN)**
	AF1	97	106.28
Case 1	AF2	100.5	114.22
	AF3	88.2	99.04

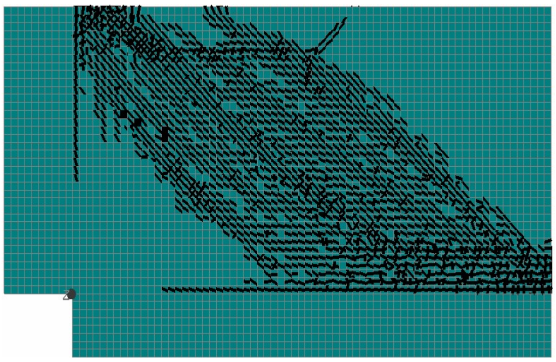

Figure 2 Crack pattern at ultimate load on the top surface of a slab supported over two adjacent edges supported slab.

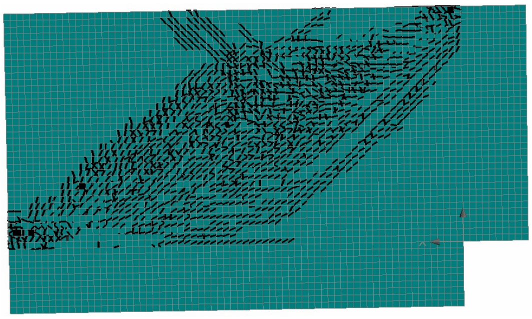

Figure 3 Crack pattern at ultimate load on the bottom surface of a slab supported over two adjacent edges supported slab.

7. CONCLUSIONS

The analytical model proposed by the authors in this research is to analyse the slab supported over two adjacent edges carrying uniformly distributed area load over the top surface and line load on its free edges. This analytical model is also validated with the available literature(Park, R. & Gamble, 2000) and results are found in favour.

Design aids are proposed in this research to predict the moment field induced in the slab supported over two adjacent edges. Design aids are also validated by using FE nonlinear analysis and the results are in favour. The ultimate load-carrying capacity of slab obtained after FE nonlinear analysis is about 9- 14% more than the design load after satisfying the equilibrium of work done.

Disclosures

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ANNEXURE A
A.1 ORTHOTROPIC COEFFICIENTS AND CONTINUITY FACTORS

Orthotropic coefficients (μ) and continuity factors (k_1 and k_2) are important to calculate the design aids for slab supported over two adjacent edges. These were developed by using the linear finite element analysis (FEA). For the development of these coefficients and factors total 11 numbers of slabs were analysed in STAAD Pro.

FE linear analysis was done to obtain the elastic moment induced in the slab. For FE linear analysis quadratic mesh (4- node element) was used. For obtaining the optimum mesh size sensitivity analysis was done for various mesh sizes (0.2 x 0.2m, 0.1 x 0.1m and 0.05 x 0.05m). From this sensitivity analysis, it is found that 0.1 x 0.1m mesh size gives less percentage variation and also took less time for complete analysis in STAAD Pro. The maximum bending moment induced in the slab was obtained from the post-processing. Bending moment contour graphs along long and short spans are shown inFigure A - 2 and Figure A - 2. To obtain the orthotropic coefficients and continuity factors, maximum positive bending moment at span and the maximum negative bending moment at support in both direction of the slab was used. The orthotropic coefficients and continuity factors obtained after FE linear analysis are given in Table A - 1.

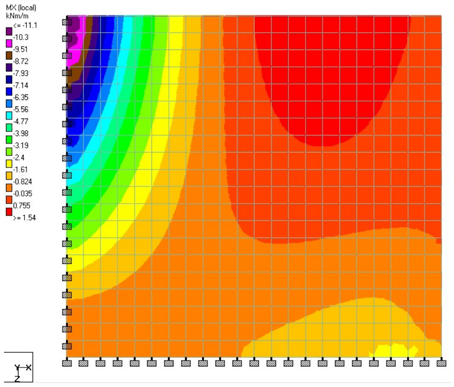

Figure A - 1 Bending moment contour graph along long span moment for slab supported over two adjacent edges.

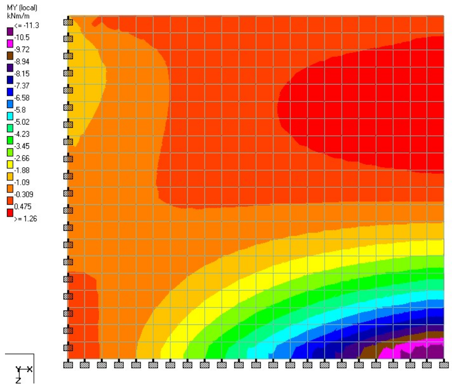

Figure A - 2 Bending moment contour graph along long span moment for slab supported over two adjacent edges.

Table A - 1Orthotropic coefficients (μ) and continuity factors (k_1 and k_2) for slab supported over two adjacent fixed edges.

r	μ	k_1	k_2
1	1.00	7.90	7.90
1.1	1.23	8.96	7.18
1.2	1.46	10.17	6.73
1.3	1.74	11.68	6.44
1.4	2.07	13.70	6.27
1.5	2.45	16.19	6.19
1.6	2.87	19.12	6.17
1.7	3.41	23.15	6.18
1.8	4.14	28.72	6.25
1.9	4.95	35.15	6.19
2	5.92	43.04	6.41