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On the Static Flexural Response of Shear Deformable IsotropicRectangular Nanobeams under the Parabolic Loading

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Abstract

It is well known that size-dependent effects, which are insignificant for macroscopic structures, become significant for small-scale structures like nanobeams. In addition, effects of the beam transverse shear deformation, which are insignificant for slender beams, become significant for shear deformable beams. Hence the need arises for a beam theory which is simple-to-use as well as which accounts for just-mentioned features of shear deformable nanobeams. Recently, the authors have developed a single variable new first-order shear deformation nonlocal beam theory (*NFSDNBT*, doi: 10.1007/s40430-019-2128-6). The *NFSDNBT* is applicable for the flexure of linear isotropic nanobeams undergoing small deformations. Displacement functions of the *NFSDNBT* give rise to the constant transverse shear strain through the beam thickness. Hence similar to the nonlocal Timoshenko beam theory (*NTBT*), the *NFSDNBT* also requires a shear correction factor. In the *NFSDNBT*, nonlocal differential stress-strain constitutive relations of Eringen are utilized, through which size-dependent effects have been taken into account. These relations relate not only the beam axial stress with the beam axial strain but also the beam transverse shear stress with the beam transverse shear strain. The governing equation of the *NFSDNBT* is obtained by utilizing beam gross equilibrium equations and it has a strong resemblance with the governing equation of the nonlocal Bernoulli-Euler beam theory (*NBEBT*). In this paper, the *NFSDNBT* is utilized for finding the static flexural response of shear deformable isotropic rectangular nanobeams under the action of parabolically distributed transverse loading. For the simply-supported, clamped-clamped and cantilever nanobeams, effects of variations in values of the nonlocal parameter of Eringen and beam thickness-to-length ratio on the maximum non-dimensional beam transverse displacement are presented. Obtained results are compared with corresponding results obtained by utilizing the *NTBT* and *NBEBT* so as to demonstrate the efficacy of the *NFSDNBT*. Along-the-length profiles of the non-dimensional beam transverse displacement for the just-mentioned cases of nanobeams, for various values of the nonlocal parameter and beam thickness-to-length ratio are also presented.

Keywords: Nanobeams, Eringen's Nonlocal Elasticity, Beam Transverse Shear Deformation, Single Variable Nonlocal Beam Theory, Static Flexure

1. Introduction

For small-scale structures like nanobeams and nanoplates, size-dependent / small-scale effects play a vital role in their respective deformation behaviour. However, these effects get attenuated / negligible for macroscopic structures. Classical continuum theories, as they do not take into account size-dependent effects, are not appropriate for analysing the deformation behaviour of small-scale structures (Eltaher et al. [1]). The pioneering work have been reported by Eringen (references [2, 3]) in the form of the nonlocal elasticity theory wherein, the importance of size-dependent effects with regard to small-scale structures has been brought to light. As the name suggests, the Eringen's nonlocal elasticity theory postulates that, the stress at a point in an elastic continuum depends on strains at the point under consideration as well as on strains at all other points in the continuum.

In addition, beam transverse shear deformation effects, which are negligible for slender beams, become considerable for thick / shear deformable beams. In order to incorporate effects of the transverse shear in the beam deformation, various displacement-based first-order shear deformation beam theories (*FSDT*s) and higher-order shear deformation beam theories (*HSDT*s) have been proposed in the literature (Ghugal and Shimpi [4]). The primary assumption with regard to *FSDT*s is that the straight line, which is normal to the undeformed beam neutral axis, remains straight but may or may not remain normal to the deformed beam neutral axis. As a result, *FSDT*s result in the constant transverse shear strain (and hence the constant transverse shear stress) through the beam thickness. Hence *FSDT*s require a shear correction factor while satisfying the constitutive relation relating the beam transverse shear stress with the beam transverse shear strain. Whereas, the primary assumption with regard to *HSDT*s is that the straight line, which is normal to the undeformed beam neutral axis, may or may not remain straight and may or may not remain

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normal to the deformed beam neutral axis. As a result, *HSDT*s result in the nonlinear variation of the transverse shear strain (and hence the nonlinear variation of the transverse shear stress) through the beam thickness. *HSDT*s do not require a shear correction factor while satisfying the constitutive relation relating the beam transverse shear stress with the beam transverse shear strain. As compared to *FSDT*s, *HSDT*s generally involve increased number of primary unknowns and require the specification of increased number of beam end conditions.

From the above-mentioned discussion, it is evident that size-dependent effects as well as transverse shear deformation effects of at least first-order must be accounted for, so as to effectively predict the deformation behaviour of shear deformable nanobeams. Pakhare et al. [5] have recently proposed a single variable new first-order shear deformation nonlocal beam theory (*NFSDNBT*), by taking a cue from the work reported by Shimpi et al. [6], for the flexure of linear isotropic nanobeams undergoing small deformations. Unlike the nonlocal Timoshenko beam theory (*NTBT*) which involves two coupled governing equations with two primary unknowns (Wang et al. [7]), the *NFSDNBT* involves only one governing equation with one primary unknown. The governing equation of the *NFSDNBT*, which is obtained by utilizing beam gross equilibrium equations, also has a strong resemblance with the governing equation of the nonlocal Bernoulli-Euler beam theory (*NBEBT*, Peddieson et al. [8]). Hence the knowledge-base required to solve the governing equation of the *NBEBT* for the flexure of slender isotropic nanobeams can be used as it is, to solve the governing equation of the *NFSDNBT*.

In this paper, the *NFSDNBT* has been utilized for performing the static flexural response of shear deformable isotropic rectangular nanobeams under the action of parabolically distributed transverse loading. Effects of variations in values of the nonlocal parameter of Eringen and beam thickness-to-length ratio on the maximum non-dimensional beam transverse displacement, for the simply-supported, clamped-clamped and cantilever nanobeams are presented. To bring forward the efficacy of the *NFSDNBT*, obtained results are compared with corresponding results obtained by utilizing the *NTBT* and *NBEBT*. Along-the-length profiles of the non-dimensional beam transverse displacement for the just-mentioned cases of nanobeams, for various values of the nonlocal parameter and beam thickness-to-length ratio are also presented in this paper.

2. Theoretical Details of the *NFSDNBT*

The nanobeam geometry is as shown in Fig. 1.

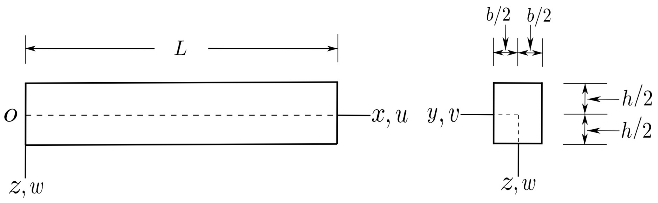

Fig. 1. The nanobeam geometry

Assumed displacement functions of the *NFSDNBT* (Pakhare et al. [5]) are as follows:

$$u(x,z) = -z\frac{dw_b}{dx} \tag{1}$$

$$w(x) = w_b + w_s \tag{2}$$

Where w_b and w_s are the bending component and shearing component respectively of the beam transverse displacement w, and are primary unknowns of the *NFSDNBT*. Through a rigorous procedure, it is possible to express w_s in terms of w_b, thereby reducing the total number of unknown functions of this theory from two to one. This relation between w_s in terms of w_b is as follows:

$$w_s = -\frac{2(1+\mu)I}{k_s A}\frac{d^2 w_b}{dx^2} \tag{3}$$

The governing equation of the *NFSDNBT*, obtained using the variationally inconsistent approach, is as follows:

$$EI\frac{d^4 w_b}{dx^4} + \beta\frac{d^2 q}{dx^2} - q = 0 \tag{4}$$

As the nanobeam is assumed to be under the action of parabolically distributed transverse loading in this paper, the distributed loading q is as follows:

$$q = q_o\left(\frac{x}{L}\right)^2 \tag{5}$$

Where q_o is the amplitude of the applied transverse loading.

The general solution of the governing equation (Eq. 4), after substituting Eq. 5 in it, is obtained which is as follows:

$$w_b = \frac{q_o x^6}{360\,EIL^2} - \frac{\beta q_o x^4}{12\,EIL^2} + C_1 x^3 + C_2 x^2 + C_3 x + C_4 \tag{6}$$

Where C_1, C_2, C_3 and C_4 in Eq. 6 are arbitrary integration constants, whose values can be found out by utilizing beam end conditions specific to the problem under consideration.

Based on the physical understanding of the nanobeam deformation, Pakhare et al. [5] have specified physically meaningful beam end conditions of the *NFSDNBT*. These end conditions with regard to the beam end $x = 0$, and can be defined at the beam end $x = L$ based on a similar logic, for an illustrative purpose are as follows:

- When the beam end $x = 0$ is simply-supported:

$$\left[w_b - \frac{2(1+\mu)I}{k_s A}\frac{d^2 w_b}{dx^2}\right]_{x=0} = 0 \tag{7}$$

$$\left[EI\frac{d^2 w_b}{dx^2} + \beta q\right]_{x=0} = 0 \tag{8}$$

- When the beam end $x = 0$ is clamped:

$$\left[w_b - \frac{2(1+\mu)I}{k_s A}\frac{d^2 w_b}{dx^2}\right]_{x=0} = 0 \tag{9}$$

$$\left[\frac{dw_b}{dx}\right]_{x=0} = 0 \tag{10}$$

- When the beam end $x = 0$ is free:

$$\left[E\,I\,\frac{d^2 w_b}{d x^2} + \beta\, q\right]_{x\,=\,0} = 0 \qquad (11)$$

$$\left[E\,I\,\frac{d^3 w_b}{d x^3} + \beta\,\frac{d q}{d x}\right]_{x\,=\,0} = 0 \qquad (12)$$

Appropriate beam end conditions are chosen from Eqs. 7 through 12 for illustrative examples considered in this paper.

3. Illustrative Examples, Numerical Results and Discussion

Three cases of the nanobeam configuration *i. e.*, the simply-supported (*SS-SS*), clamped-clamped (*C-C*) and cantilever (*C-F*) nanobeams (illustrative examples 1 through 3 respectively) for the static flexure have been considered. The beam end conditions of the *NFSDNBT*, at beam ends $x = 0$ and $x = L$ for an evaluation of arbitrary integration constants of Eq. 6, for each illustrative example are as follows:

- The illustrative example 1 (the *SS-SS* nanobeam):

$$\left[w_b - \frac{2\,(1+\mu)\,I}{k_s\,A}\,\frac{d^2 w_b}{d x^2}\right]_{x\,=\,0,\,L} = 0 \qquad (13)$$

$$\left[E\,I\,\frac{d^2 w_b}{d x^2} + \beta\, q\right]_{x\,=\,0,\,L} = 0 \qquad (14)$$

- The illustrative example 2 (the *C-C* nanobeam):

$$\left[w_b - \frac{2\,(1+\mu)\,I}{k_s\,A}\,\frac{d^2 w_b}{d x^2}\right]_{x\,=\,0,\,L} = 0 \qquad (15)$$

$$\left[\frac{d w_b}{d x}\right]_{x\,=\,0,\,L} = 0 \qquad (16)$$

- The illustrative example 3 (the *C-F* nanobeam):

$$\left[w_b - \frac{2\,(1+\mu)\,I}{k_s\,A}\,\frac{d^2 w_b}{d x^2}\right]_{x\,=\,0} = 0 \qquad (17)$$

$$\left[\frac{d w_b}{d x}\right]_{x\,=\,0} = 0 \qquad (18)$$

$$\left[E\,I\,\frac{d^2 w_b}{d x^2} + \beta\, q\right]_{x\,=\,0} = 0 \qquad (19)$$

$$\left[E\,I\,\frac{d^3 w_b}{d x^3} + \beta\,\frac{d q}{d x}\right]_{x\,=\,0} = 0 \qquad (20)$$

Results of the maximum non-dimensional beam transverse displacement (w^*_{max}) obtained using the *NFSDNBT* and corresponding results of the *NTBT* (Wang et al. [7]) and of the *NBEBT* (Peddieson et al. [8]) for illustrative examples 1 through 3 for various values of the nonlocal parameter of Eringen (β) and beam thickness-to-length ratio ($h\,/\,L$) are presented in Tables 1 through 3 respectively. The w^*_{max} by using the *NTBT* and *NBEBT* is calculated by present authors by utilizing the work reported by Wang et al. [7] and Peddieson et al. [8] respectively.

Along-the-length profiles of the non-dimensional beam transverse displacement (w^*) obtained using the *NFSDNBT* for various values of the β and $h\,/\,L$ for illustrative examples 1 through 3 are presented in Figs. 1 through 3 respectively.

Table-1. The maximum non-dimensional beam transverse displacement ($w^*_{max} = (100\ w_{max}\ E\ I)\ /\ (q_o\ L^4)$) for the illustrative example 1 (the *SS-SS* nanobeam) with $\mu = 0.3$.

Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
		w^*_{max} for $\beta = 0$		
NFSDNBT [\$]	0.388384	0.390736	0.398094	0.410378
NTBT [\$] [7]	0.388384	0.390736	0.398094	0.410378
NBEBT [8]	0.388384	0.388384	0.388384	0.388384
		w^*_{max} for $\beta = 1$		
NFSDNBT [\$]	0.426195	0.428408	0.435330	0.446885
NTBT [\$] [7]	0.426202	0.428570	0.435975	0.448336
NBEBT [8]	0.426195	0.426195	0.426195	0.426195
		w^*_{max} for $\beta = 2$		
NFSDNBT [\$]	0.464236	0.466308	0.472787	0.483602
NTBT [\$] [7]	0.464249	0.466630	0.474074	0.486496
NBEBT [8]	0.464236	0.464236	0.464236	0.464236
		w^*_{max} for $\beta = 3$		
NFSDNBT [\$]	0.502463	0.504391	0.510422	0.520490
NTBT [\$] [7]	0.502482	0.504873	0.512348	0.524820
NBEBT [8]	0.502463	0.502463	0.502463	0.502463

[\$] A shear correction factor (k_s) with a value 5 / 6 is used.

Table-2. The maximum non-dimensional beam transverse displacement ($w^*_{max} = (100\ w_{max}\ E\ I)\ /\ (q_o\ L^4)$) for the illustrative example 2 (the *C-C* nanobeam) with $\mu = 0.3$.

Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
		w^*_{max} for $\beta = 0$		
NFSDNBT [\$]	0.075036	0.077433	0.084950	0.097547
NTBT [\$] [7]	0.075036	0.077433	0.084950	0.097547
NBEBT [8]	0.075036	0.075036	0.075036	0.075036
		w^*_{max} for $\beta = 1$		
NFSDNBT [\$]	0.069906	0.072157	0.079215	0.091052
NTBT [\$] [7]	0.069913	0.072333	0.079926	0.092663
NBEBT [8]	0.069906	0.069906	0.069906	0.069906
		w^*_{max} for $\beta = 2$		
NFSDNBT [\$]	0.064790	0.066894	0.073499	0.084584
NTBT [\$] [7]	0.064804	0.067249	0.074929	0.087824
NBEBT [8]	0.064790	0.064790	0.064790	0.064790
		w^*_{max} for $\beta = 3$		
NFSDNBT [\$]	0.059689	0.061650	0.067805	0.078147
NTBT [\$] [7]	0.059711	0.062186	0.069964	0.083039

NBEBT [8]	0.059689	0.059689	0.059689	0.059689

$ A shear correction factor (k_s) with a value 5 / 6 is used.

Table-3. The maximum non-dimensional beam transverse displacement ($w^*_{max} = (100\ w_{max}\ E\ I) / (q_o\ L^4)$) for the illustrative example 3 (the *C-F* nanobeam) with $\mu = 0.3$.

	w^*_{max} for $\beta = 0$			
Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
NFSDNBT $	7.22287	7.23847	7.28722	7.36847
NTBT $ [7]	7.22287	7.23847	7.28722	7.36847
NBEBT [8]	7.22287	7.22287	7.22287	7.22287
	w^*_{max} for $\beta = 1$			
Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
NFSDNBT $	7.13956	7.15579	7.20649	7.29099
NTBT $ [7]	7.13954	7.15514	7.20389	7.28514
NBEBT [8]	7.13956	7.13956	7.13956	7.13956
	w^*_{max} for $\beta = 2$			
Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
NFSDNBT $	7.05626	7.07311	7.12576	7.21351
NTBT $ [7]	7.05621	7.07181	7.12056	7.20181
NBEBT [8]	7.05626	7.05626	7.05626	7.05626
	w^*_{max} for $\beta = 3$			
Theory	h / L = 0.01	h / L = 0.05	h / L = 0.10	h / L = 0.15
NFSDNBT $	6.97295	6.99042	7.04502	7.13602
NTBT $ [7]	6.97287	6.98847	7.03722	7.11847
NBEBT [8]	6.97295	6.97295	6.97295	6.97295

$ A shear correction factor (k_s) with a value 5 / 6 is used.

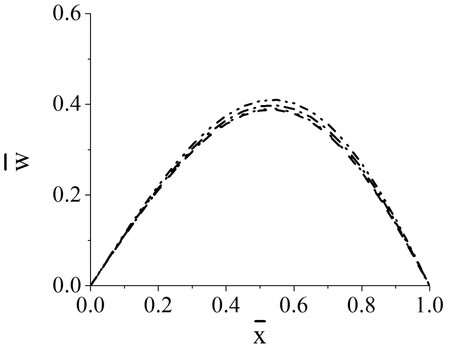

Fig. 2 (a). For $\beta = 0$

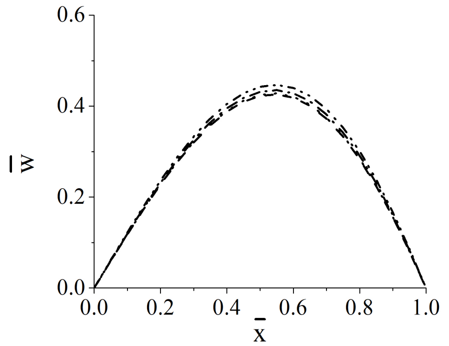

Fig. 2 (b). For $\beta = 1$

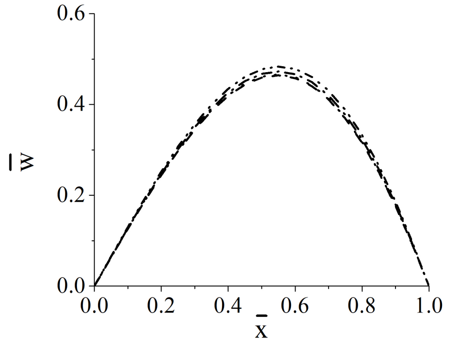

Fig. 2 (c). For $\beta = 2$

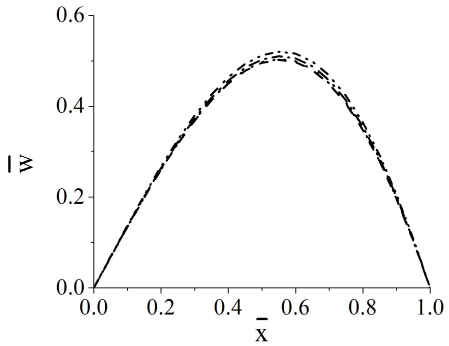

Fig. 2 (d). For $\beta = 3$

Fig. 2 (a, b, c, d). For the illustrative example 1 (the *SS-SS* nanobeam), the non-dimensional beam transverse displacement ($w^* = (100\ w\ E\ I) / (q_o\ L^4)$) versus the non-dimensional beam axial location ($x^* = x / L$) for various values of the nonlocal parameter of Eringen (β) and beam thickness-to-length ratio (h / L), - - - for $h / L = 0.01$, . . . for $h / L = 0.05$, - . - for $h / L = 0.10$ and - . . - for $h / L = 0.15$.

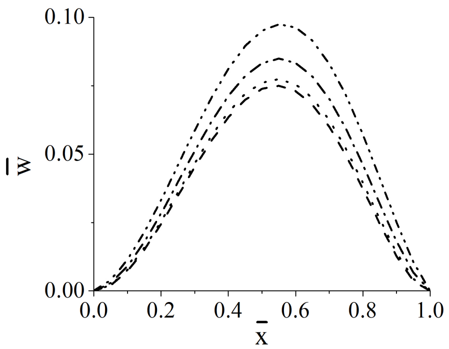

Fig. 3 (a). For $\beta = 0$

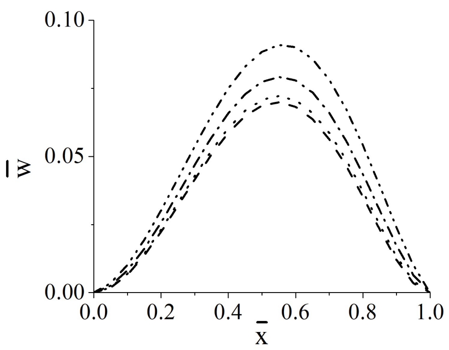

Fig. 3 (b). For $\beta = 1$

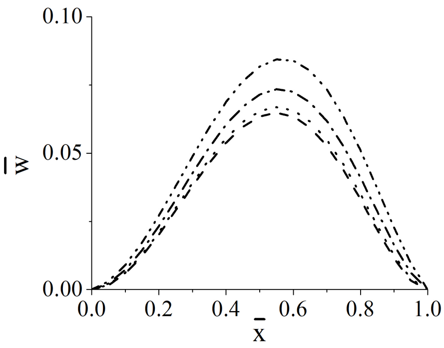

Fig. 3 (c). For *β = 2*

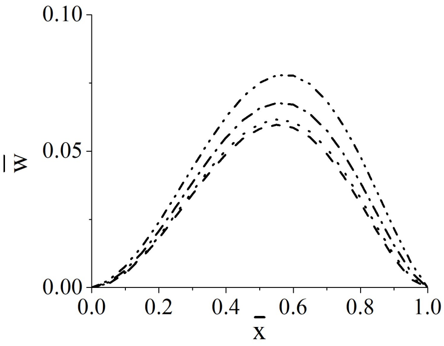

Fig. 3 (d). For *β = 3*

Fig. 3 (a, b, c, d). For the illustrative example 2 (the *C-C* nanobeam), the non-dimensional beam transverse displacement ($w^* = (100\ w\ E\ I) / (q_o\ L^4)$) versus the non-dimensional beam axial location ($x^* = x / L$) for various values of the nonlocal parameter of Eringen (*β*) and beam thickness-to-length ratio (*h / L*), - - - for *h / L* = 0.01, . . . for *h / L* = 0.05, - . - for *h / L* = 0.10 and - . . - for *h / L* = 0.15.

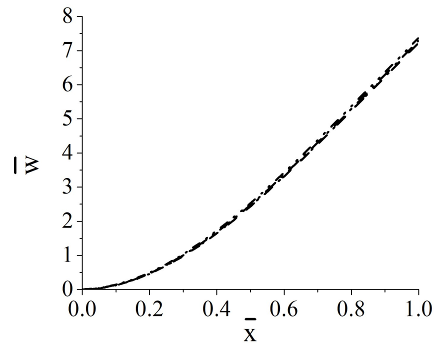

Fig. 4 (a). For *β = 0*

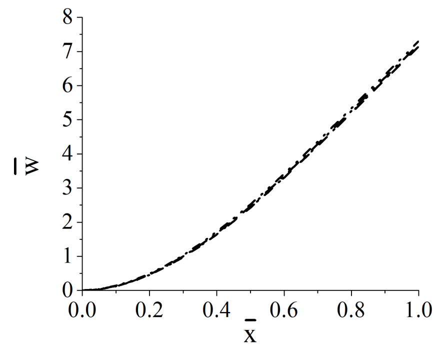

Fig. 4 (b). For *β = 1*

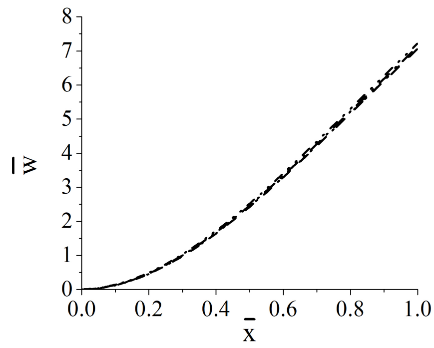

Fig. 4 (c). For *β = 2*

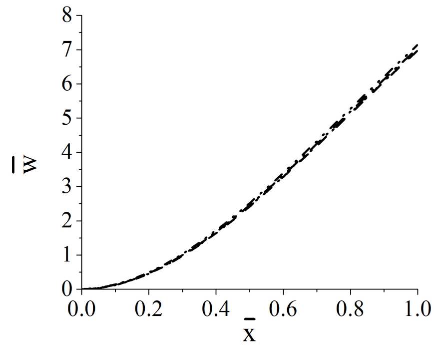

Fig. 4 (d). For *β = 3*

Fig. 4 (a, b, c, d). For the illustrative example 3 (the *C-F* nanobeam), the non-dimensional beam transverse displacement ($w^* = (100\ w\ E\ I) / (q_o\ L^4)$) versus the non-dimensional beam axial location ($x^* = x / L$) for various values of the nonlocal parameter of Eringen (*β*) and beam thickness-to-length ratio (*h / L*), - - - for *h / L* = 0.01, . . . for *h / L* = 0.05, - . - for *h / L* = 0.10 and - . . - for *h / L* = 0.15.

From Tables 1 through 3, following observations can be made with regard to the maximum non-dimensional beam transverse displacement (w^*_{max}) for the *SS-SS*, *C-C* and *C-F* nanobeams (illustrative examples 1 through 3 respectively):

1. The w^*_{max} obtained using the *NFSDNBT* have a quantitative agreement with corresponding results obtained using the *NTBT* for various values of the nonlocal parameter of Eringen (*β* = 0, 1, 2 and 3) and beam thickness-to-length ratio (*h / L* = 0.01, 0.05, 0.10 and 0.15) with nanobeams under the action of parabolically distributed transverse loading. Only in the case of *β* = 3 and *h / L* = 0.15 for the *C-C* nanobeam, an appreciable quantitative disagreement is observed in values of the w^*_{max} obtained using the *NFSDNBT* and *NTBT*.
   However, it should be noted that the *NTBT* involves two coupled governing equations with two primary unknowns as opposed to the *NFSDNBT* which has only one governing equation with single primary unknown.
2. As the *NBEBT* does not take into account beam transverse shear deformation effects, the w^*_{max}

obtained using the *NBEBT* remains significantly underestimated for higher values of h / L as compared to corresponding values obtained using the *NFSDNBT* and *NTBT*.

From Figs. 2 through 4, following observations can be made with regard to along-the-length variations of the non-dimensional beam transverse displacement (w^*) for the *SS-SS*, *C-C* and *C-F* nanobeams (illustrative examples 1 through 3 respectively):

1. For the *SS-SS* nanobeam, the w^* increases with an increase in the value of the nonlocal parameter of Eringen (β) as well as with an increase in the value of the beam thickness-to-length ratio (h / L).
2. For the *C-C* and *C-F* nanobeams, the w^* decreases with an increase in the value of β for a given value of h / L. Whereas, the w^* increases with an increase in the value of h / L for a given value of β.

From just-mentioned observations with regard to the w^* for nanobeams under the action of parabolically distributed transverse loading, Pakhare et al. [5] (where nanobeams are assumed under the action of uniformly distributed transverse loading) and Pakhare et al. [9] (where nanobeams are assumed under the action of sinusoidally distributed transverse loading), it is notable that variation trends of w^* for nanobeams is dependent on beam end conditions as well as on the type of distributed transverse loading applied on it.

4. Concluding Remarks

In this paper, the newly-developed single variable new first-order shear deformation nonlocal beam theory (*NFSDNBT*) is utilized for performing the flexure of shear deformable isotropic rectangular nanobeams under the action of parabolically distributed transverse loading. Effects of the nonlocal parameter of Eringen and beam thickness-to-length ratio on profiles of the non-dimensional beam transverse displacement and maximum non-dimensional beam transverse displacement for shear deformable isotropic rectangular nanobeams with various beam fixity conditions are presented. Obtained results are compared with corresponding results of the nonlocal Timoshenko beam theory and of the nonlocal Bernoulli-Euler beam theory to demonstrate the effectiveness of the *NFSDNBT*. These results and comparisons prove that the flexural analysis of shear deformable isotropic rectangular nanobeams carried out using an easy-to-use *NFSDNBT* is reliable for a range of nonlocal parameter, beam thickness-to-length ratio and beam fixity conditions.

Disclosures

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