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Bending Analysis of Orthotropic Plate using 5th Order Shear Deformation Theory

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Abstract

An extension of fifth order shear deformation theory which take into account transverse shear and normal strain deformation effect is presented for bending analysis of thick orthotropic square plate. The assumed displacement field accounts for non-linear variation of in-plane displacement as well as transverse displacement through the plate thickness. The closed form solution for analysis of simply supported orthotropic plates have been obtained using Navier solution. Numerical analysis is done using MATLAB software. The results of present theory are compared with the results obtained from classical plate theory, first order shear deformation theory, higher order shear deformation theory, trigonometric shear deformation theory and exact solutions. The results obtained from the present theory are found to be in good agreement with those obtained from higher order theories and exact solution.

Keywords: Orthotropic plate, fifth order shear deformation theory, Bending analysis, Navier solution.

1. Introduction

Plates are the basic structural elements commonly used in various engineering disciplines such as aerospace, civil, marine, mechanical, and electronics engineering. In shear flexible plates which may consist of isotropic, orthotropic, anisotropic, or laminated composite materials, the transverse shear and transverse normal deformation effects are more pronounced. Development of refined theories, which consider these effects in the static and dynamic study of structural elements, is an important task for identifying the correct structural behavior of structural elements composed of these materials.

The growing use of flexible shear materials has stimulated interest in the precise prediction of the structural behavior of thick plates. Moreover, thick beams and plates, either isotropic or anisotropic, are essentially two-and three-dimensional problems of the theory of elasticity. Reducing these problems to corresponding one-and two-dimensional approximate problems for their analysis is one of the main goals of the researchers. Further, shear deformation effects are more severe in thick plates than in thin plates when exposed to transverse loads. Such results are ignored in the classical plate theory [1, 2].

The classical plate theory based on the Navier-Kirchhoff hypothesis is well known to under-predict deflections and over-predict natural frequencies and buckling loads as it does not consider transverse shear deformation effects. Various

In view of this, Carrera [11] and Rohwer et. al. [12] suggested that theories with an expansion of the fifth order or higher order shear deformation theories are purposed for the study of homogeneous, isotropic, orthotropic, anisotropic, laminated composite and functionally graded plates subjected to mechanical and thermal loads. A review of these theories is given in detail in the literature [3, 4]. A brief review of theories developed in last few years is given below.

Klouche et. al. [5] purposed a single variable shear deformation theory to study the buckling analysis of thick isotropic plates. Sadoun et. al. [6] developed a quasi 3D trigonometric shear deformation theory for the dynamic analysis of thick orthotropic plate. Pandey and Pradyumna [7] presented a higher-order layer wise theory for analysis of functionally graded material (FGM) sandwich plates. Dehsaraji et. al. [8] performed bending analysis of a thick functionally graded piezoelectric rectangular plates using the higher-order shear and normal deformable plate theory. Boussoula et. al. [9] studied thermomechanical flexural analysis of functionally graded material sandwich plates by employing a n^{th} order shear deformation theory. Niyogi et al., [10] presented buckling analysis of laminated composites with the effect of orthotropic material.

Since of the isotropic nature of thermal loadings, assumption in analysis of plates that ignore transverse normal strain contribute to very unreliable results in study of thick and thin plates. The capture of linear thermal stresses requires at least a parabolic expansion of the transverse displacement. higher than the fifth order, including a term for transverse strain effect, is required for an appropriate explanation of the

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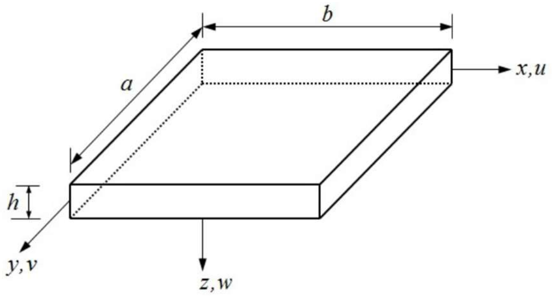

Figure 1: An orthotropic plate

thermal response. In this paper, a fifth order shear deformation theory [5th OSDT] purposed by Gajbhiye et al., [13,14,15,16,17] is applied for the flexure analysis of a square orthotropic plate to verify the accuracy of the 5th OSDT. The results of present theory are compared with classical plate theory, first order shear deformation theory, higher order shear deformation theory, trigonometric shear deformation theory and exact solutions.

2. Formulation

2.1 The displacement field

A square plate having length of side as 'a' and total thickness 'h' is considered. The schematic view of plate is shown in Figure 1. The plate consists of linearly elastic orthotropic material. The z-direction downward is taken to be positive. In Cartesian coordinate system, the plate occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$. A transverse load q (x, y) is applied to the plate's upper surface.
According to the 5th OSDT, the displacement function in fifth order of the thickness coordinate is given as,

$$U(x,y,z) = u_0 - z\frac{\partial w}{\partial x} + f_1(z)\,\phi_x(x,y) + f_2(z)\,\psi_x(x,y)$$
$$V(x,y,z) = v_0 - z\frac{\partial w}{\partial y} + f_1(z)\,\phi_y(x,y) + f_2(z)\,\psi_y(x,y)$$
$$W(x,y,z) = w(x,y) + g_1(z)\,\phi_z(x,y) + g_2(z)\,\psi_z(x,y) \qquad (1)$$

where

$$f_1(z) = z\left(1 - \frac{4}{3}\frac{z^2}{h^2}\right), \quad f_2(z) = z\left(1 - \frac{16}{5}\frac{z^4}{h^4}\right)$$

$$\text{and} \quad g_1(z) = \left(1 - 4\frac{z^2}{h^2}\right), \quad g_2(z) = \left(1 - 16\frac{z^4}{h^4}\right)$$

Where *u, v, w,* are the displacements in the *x, y, z,* direction, respectively. ϕ_x , ψ_x, ϕ_y, ψ_y, ϕ_z, ψ_z are the shear and bending component in *x, y,* and *z* direction. u_0 , v_0 and w are the middle and transverse displacement in *x, y* and *z* direction respectively.

$$\varepsilon_x = \frac{\partial U}{\partial x}, \varepsilon_y = \frac{\partial V}{\partial y}, \varepsilon_z = \frac{\partial W}{\partial z},$$
$$\gamma_{xy} = \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}, \gamma_{zx} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x},$$
$$\gamma_{yz} = \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \qquad (2)$$

2.2 About the present theory

The kinematics of the present theory is based on nine independent field variables. The usage of theory could be highly effective in the bending, buckling, vibration and thermal analysis of nonhomogeneous, anisotropic, composite or sandwich thick plates. The effect of transverse normal strain is considered, and the theory presents the quasi-3D description of bending response. The displacement field of the theory enforces the realistic variation of the transverse shear stresses (parabolic and quartic) across the thickness of the plate.

2.3 Stress-strain Relationship

The stress-strain relations in the K^{th} layer of an orthotropic plate are given as:

$$\begin{Bmatrix} \sigma_x^k \\ \sigma_y^k \\ \sigma_z^k \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11}^k & \overline{Q}_{12}^k & \overline{Q}_{13}^k \\ \overline{Q}_{12}^k & \overline{Q}_{22}^k & \overline{Q}_{23}^k \\ \overline{Q}_{13}^k & \overline{Q}_{23}^k & \overline{Q}_{33}^k \end{bmatrix} \begin{Bmatrix} \varepsilon_x^k \\ \varepsilon_y^k \\ \varepsilon_z^k \end{Bmatrix} \text{ and} \qquad (3)$$

$$\begin{Bmatrix} \tau_{xy}^k \\ \tau_{zx}^k \\ \tau_{yz}^k \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{44}^k & 0 & 0 \\ 0 & \overline{Q}_{55}^k & 0 \\ 0 & 0 & \overline{Q}_{66}^k \end{bmatrix} \begin{Bmatrix} \gamma_{xy}^k \\ \gamma_{zx}^k \\ \gamma_{yz}^k \end{Bmatrix} \qquad (4)$$

Where $\sigma_x^k, \sigma_y^k, \sigma_z^k, \tau_{xy}^k, \tau_{zx}^k, \tau_{yz}^k$ are the normal and shear component, $\varepsilon_x^k, \varepsilon_y^k, \varepsilon_z^k, \gamma_{xy}^k, \gamma_{zx}^k, \gamma_{yz}^k$ are the normal and strain component. $[\overline{Q}_{11}^k, \overline{Q}_{12}^k, \overline{Q}_{13}^k, \overline{Q}_{22}^k, \overline{Q}_{23}^k, \overline{Q}_{33}^k, \overline{Q}_{44}^k, \overline{Q}_{55}^k, \overline{Q}_{66}^k]$ are the reduced stiffness matrix components.

2.4 Governing Equation.

The principle of virtual work is applied to derive the governing equations using the stress-strain relationship given in Eq. (3) and (4) for $\delta u_0, \delta v_0, \delta w, \delta\phi_x, \delta\psi_x, \delta\phi_y, \delta\psi_y, \delta\phi_z, \delta\psi_z$.

$$\int_{z=-h/2}^{z=+h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a} \begin{bmatrix} \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z\, \delta\varepsilon_z \\ +\tau_{xy}\delta\gamma_{xy} + \tau_{zx}\delta\gamma_{zx} + \tau_{yz}\delta\gamma_{yz} \end{bmatrix} dx\,dy\,dz - \int_{y=0}^{y=a}\int_{x=0}^{x=a} q(x,y)\,\delta w\,dx\,dy = 0 \qquad (5)$$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{44}\frac{\partial^2 u_0}{\partial y^2} - A_{12}\frac{\partial^2 v_0}{\partial x \partial y} - A_{44}\frac{\partial^2 v_0}{\partial x \partial y} + \left(B_{11}\frac{\partial^3 w}{\partial x^3} + \left(B_{12} + 2B_{44}\right)\frac{\partial^3 w}{\partial x \partial y^2}\right) - I_{11}\frac{\partial^2 \phi_x}{\partial x^2} - I_{44}\frac{\partial^2 \phi_x}{\partial y^2}$$
$$-J_{11}\frac{\partial^2 \psi_x}{\partial x^2} - J_{44}\frac{\partial^2 \psi_x}{\partial y^2} - \left(I_{12} + I_{44}\right)\frac{\partial^2 \phi_y}{\partial x \partial y}$$
$$-\left(J_{12} + J_{44}\right)\frac{\partial^2 \psi_y}{\partial x \partial y} + \frac{8}{h^2}B_{13}\frac{\partial \phi_z}{\partial x} + \frac{64}{h^4}E_{13}\frac{\partial \psi_z}{\partial x} = 0 \qquad (6)$$

$$\begin{aligned}
&-\left(A_{12}+A_{44}\right)\frac{\partial^2 u_0}{\partial x\partial y}-A_{44}\frac{\partial^2 v_0}{\partial x^2}-A_{22}\frac{\partial^2 v_0}{\partial y^2}+B_{22}\frac{\partial^3 w}{\partial y^3}+\\
&\left(B_{12}+2B_{44}\right)\frac{\partial^3 w}{\partial x^2\partial y}-\left(I_{12}+I_{44}\right)\frac{\partial^2 \phi_x}{\partial x\partial y}-\left(J_{12}+J_{44}\right)\frac{\partial^2 \psi_x}{\partial x\partial y}\\
&-I_{22}\frac{\partial^2 \phi_y}{\partial y^2}-I_{44}\frac{\partial^2 \phi_y}{\partial x^2}-J_{22}\frac{\partial^2 \psi_y}{\partial y^2}-J_{44}\frac{\partial^2 \psi_y}{\partial x^2}\\
&+\frac{8}{h^2}B_{23}\frac{\partial \phi_z}{\partial y}+\frac{64}{h^4}E_{23}\frac{\partial \psi_z}{\partial y}=0
\end{aligned}\tag{7}$$

$$\begin{aligned}
&-B_{11}\frac{\partial^3 u_0}{\partial x^3}-\left(B_{12}+2B_{44}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}-B_{22}\frac{\partial^3 v_0}{\partial y^3}\\
&-\left(B_{12}+2B_{44}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+D_{11}\frac{\partial^4 w}{\partial x^4}+\left(2D_{12}+4D_{44}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}\\
&+D_{22}\frac{\partial^4 w}{\partial y^4}-L_{11}\frac{\partial^3 \phi_x}{\partial x^3}-\left(L_{12}+2L_{44}\right)\frac{\partial^3 \phi_x}{\partial x\partial y^2}-M_{11}\frac{\partial^3 \psi_x}{\partial x^3}\\
&-\left(M_{12}+2M_{44}\right)\frac{\partial^3 \psi_x}{\partial x\partial y^2}-L_{22}\frac{\partial^3 \phi_y}{\partial y^3}+\left(L_{12}+2L_{44}\right)\frac{\partial^3 \phi_y}{\partial x^2\partial y}\\
&-M_{22}\frac{\partial^3 \psi_y}{\partial y^3}-\left(M_{12}+2M_{44}\right)\frac{\partial^3 \psi_y}{\partial x^2\partial y}+D_{13}\frac{8}{h^2}\frac{\partial^2 \phi_z}{\partial x^2}\\
&+D_{23}\frac{8}{h^2}\frac{\partial^2 \phi_z}{\partial y^2}+F_{13}\frac{64}{h^4}\frac{\partial^2 \psi_z}{\partial x^2}+F_{23}\frac{64}{h^4}\frac{\partial^2 \psi_z}{\partial y^2}-q=0
\end{aligned}\tag{8}$$

$$\begin{aligned}
&-I_{11}\frac{\partial^2 u_0}{\partial x^2}-I_{44}\frac{\partial^2 u_0}{\partial y^2}-\left(I_{12}+I_{44}\right)\frac{\partial^2 v_0}{\partial x\partial y}+L_{11}\frac{\partial^3 w}{\partial x^3}\\
&+\left(L_{12}+2L_{44}\right)\frac{\partial^3 w}{\partial x\partial y^2}-N_{11}\frac{\partial^2 \phi_x}{\partial x^2}-N_{44}\frac{\partial^2 \phi_x}{\partial y^2}\\
&+C_{55}\phi_x-P_{11}\frac{\partial^2 \psi_x}{\partial x^2}-P_{44}\frac{\partial^2 \psi_x}{\partial y^2}+H_{55}\psi_x\\
&-\left(N_{12}+N_{44}\right)\frac{\partial^2 \phi_y}{\partial x\partial y}-\left(P_{12}+P_{44}\right)\frac{\partial^2 \psi_y}{\partial x\partial y}\\
&+\left(\frac{8}{h^2}L_{13}+C_{55}\right)\frac{\partial \phi_z}{\partial x}+\left(\frac{64}{h^4}S_{13}+H_{55}\right)\frac{\partial \psi_z}{\partial x}=0
\end{aligned}\tag{9}$$

$$\begin{aligned}
&-J_{11}\frac{\partial^2 u_0}{\partial x^2}-J_{44}\frac{\partial^2 u_0}{\partial y^2}-\left(J_{12}+J_{44}\right)\frac{\partial^2 v_0}{\partial x\partial y}+M_{11}\frac{\partial^3 w}{\partial x^3}\\
&+\left(M_{12}+2M_{44}\right)\frac{\partial^3 w}{\partial x\partial y^2}-P_{11}\frac{\partial^2 \phi_x}{\partial x^2}-P_{44}\frac{\partial^2 \phi_x}{\partial y^2}\\
&+H_{55}\phi_x-R_{11}\frac{\partial^2 \psi_x}{\partial x^2}-R_{44}\frac{\partial^2 \psi_x}{\partial y^2}+G_{55}\psi_x\\
&-\left(P_{12}+P_{44}\right)\frac{\partial^2 \phi_y}{\partial x\partial y}-\left(R_{12}+R_{44}\right)\frac{\partial^2 \psi_y}{\partial x\partial y}\\
&+\left(\frac{8}{h^2}M_{13}+H_{55}\right)\frac{\partial \phi_z}{\partial x}+\left(\frac{64}{h^4}T_{13}+G_{55}\right)\frac{\partial \psi_z}{\partial x}=0
\end{aligned}\tag{10}$$

$$\begin{aligned}
&-\left(I_{12}+I_{44}\right)\frac{\partial^2 u_0}{\partial x\partial y}-I_{22}\frac{\partial^2 v_0}{\partial y^2}-I_{44}\frac{\partial^2 v_0}{\partial x^2}+L_{22}\frac{\partial^3 w}{\partial y^3}\\
&+\left(L_{12}+2L_{44}\right)\frac{\partial^3 w}{\partial x^2\partial y}-\left(N_{12}+N_{44}\right)\frac{\partial^2 \phi_x}{\partial x\partial y}-\left(P_{12}+P_{44}\right)\frac{\partial^2 \psi_x}{\partial x\partial y}\\
&-N_{22}\frac{\partial^2 \phi_y}{\partial y^2}-N_{44}\frac{\partial^2 \phi_y}{\partial x^2}+C_{66}\phi_y-P_{22}\frac{\partial^2 \psi_y}{\partial y^2}-N_{44}\frac{\partial^2 \psi_y}{\partial x^2}\\
&+H_{66}\psi_y+\left(\frac{8}{h^2}L_{23}+C_{66}\right)\frac{\partial \phi_z}{\partial y}+\left(\frac{64}{h^4}S_{23}+H_{66}\right)\frac{\partial \psi_z}{\partial y}=0
\end{aligned}\tag{11}$$

$$\begin{aligned}
&-\left(J_{12}+J_{44}\right)\frac{\partial^2 u_0}{\partial x\partial y}-J_{22}\frac{\partial^2 v_0}{\partial y^2}-J_{44}\frac{\partial^2 v_0}{\partial x^2}+M_{22}\frac{\partial^3 w}{\partial y^3}\\
&+\left(M_{12}+2M_{44}\right)\frac{\partial^3 w}{\partial x^2\partial y}-\left(P_{12}+P_{44}\right)\frac{\partial^2 \phi_x}{\partial x\partial y}\\
&-\left(R_{12}+R_{44}\right)\frac{\partial^2 \psi_x}{\partial x\partial y}-P_{22}\frac{\partial^2 \phi_y}{\partial y^2}-P_{44}\frac{\partial^2 \phi_y}{\partial x^2}+H_{66}\phi_y\\
&-R_{22}\frac{\partial^2 \psi_y}{\partial y^2}-R_{44}\frac{\partial^2 \psi_y}{\partial x^2}+G_{66}\psi_y+\left(\frac{8}{h^2}M_{23}+H_{66}\right)\frac{\partial \phi_z}{\partial y}\\
&+\left(\frac{64}{h^4}T_{23}+G_{66}\right)\frac{\partial \psi_z}{\partial y}=0
\end{aligned}\tag{12}$$

$$\begin{aligned}
&-\frac{8}{h^2}B_{13}\frac{\partial u_0}{\partial x}-\frac{8}{h^2}B_{23}\frac{\partial v_0}{\partial y}+\frac{8}{h^2}D_{13}\frac{\partial^2 w}{\partial x^2}+\frac{8}{h^2}D_{23}\frac{\partial^2 w}{\partial y^2}\\
&-\left(\frac{8}{h^2}L_{13}+C_{55}\right)\frac{\partial \phi_x}{\partial x}-\left(\frac{8}{h^2}M_{13}+H_{55}\right)\frac{\partial \psi_x}{\partial x}\\
&-\left(\frac{8}{h^2}L_{23}+C_{66}\right)\frac{\partial \phi_y}{\partial y}-\left(\frac{8}{h^2}M_{23}+H_{66}\right)\frac{\partial \psi_y}{\partial y}-C_{55}\frac{\partial^2 \phi_z}{\partial x^2}\\
&-C_{66}\frac{\partial^2 \phi_z}{\partial y^2}+D_{33}\frac{64}{h^4}\phi_z-H_{55}\frac{\partial^2 \psi_z}{\partial x^2}-H_{66}\frac{\partial^2 \psi_z}{\partial y^2}\\
&+F_{33}\frac{512}{h^6}\psi_z=0
\end{aligned}\tag{13}$$

$$\begin{aligned}
&-\frac{64}{h^4}E_{13}\frac{\partial u_0}{\partial x}-\frac{64}{h^4}E_{23}\frac{\partial v_0}{\partial y}+\frac{64}{h^4}F_{13}\frac{\partial^2 w}{\partial x^2}+\frac{64}{h^4}F_{23}\frac{\partial^2 w}{\partial y^2}\\
&-\left(\frac{64}{h^4}S_{13}+H_{55}\right)\frac{\partial \phi_x}{\partial x}-\left(\frac{64}{h^4}T_{13}+G_{55}\right)\frac{\partial \psi_x}{\partial x}\\
&-\left(\frac{64}{h^4}S_{23}+H_{66}\right)\frac{\partial \phi_y}{\partial y}-\left(\frac{64}{h^4}T_{23}+G_{66}\right)\frac{\partial \psi_y}{\partial y}\\
&-H_{55}\frac{\partial^2 \phi_z}{\partial x^2}-H_{66}\frac{\partial^2 \phi_z}{\partial y^2}+F_{33}\frac{512}{h^6}\phi_z-G_{55}\frac{\partial^2 \psi_z}{\partial x^2}\\
&-G_{66}\frac{\partial^2 \psi_z}{\partial y^2}+O_{33}\frac{4096}{h^8}\psi_z=0
\end{aligned}\tag{14}$$

$$\begin{pmatrix} A_{ij}, I_{ij}, J_{ij}, \\ N_{ij}, P_{ij}, R_{ij} \end{pmatrix} = \int_{-h/2}^{+h/2} \bar{Q}_{ij} \begin{pmatrix} 1, f_1(z), f_2(z), f_1^2(z), \\ f_1(z) f_2(z), f_2^2(z) \end{pmatrix} dz, \ (i, j = 1, 2, 4)$$

$$\begin{pmatrix} B_{ij}, D_{ij}, \\ L_{ij}, M_{ij} \end{pmatrix} = \int_{-h/2}^{+h/2} \bar{Q}_{ij} \begin{pmatrix} z, z^2, z f_1(z), \\ z f_2(z) \end{pmatrix} dz, \ (i, j = 1, 2, 3, 4)$$

$$\begin{pmatrix} E_{ij}, F_{ij}, \\ S_{ij}, T_{ij} \end{pmatrix} = \int_{-h/2}^{+h/2} \bar{Q}_{ij} \begin{pmatrix} z^3, z^4, z^3 f_1(z), \\ z^3 f_2(z) \end{pmatrix} dz, \ (i, j = 1, 2, 3)$$

$$\begin{pmatrix} C_{ij}, G_{ij}, \\ H_{ij} \end{pmatrix} = \int_{-h/2}^{+h/2} \bar{Q}_{ij} \begin{pmatrix} g_1^2(z), g_2^2(z), \\ g_1(z) g_2(z) \end{pmatrix} dz, \ (i, j = 5, 6)$$

$$\left(O_{ij}\right) = \int_{-h/2}^{+h/2} \bar{Q}_{ij} \left(z^6\right) dz, \quad (i, j = 3)$$

$$f_1(z) = \left(z - \frac{4}{3}\frac{z^3}{h^2}\right); \quad f_2(z) = \left(z - \frac{16}{5}\frac{z^5}{h^4}\right);$$

$$f_1'(z) = g_1(z) = \left(1 - 4\frac{z^2}{h^2}\right);$$

$$f_2'(z) = g_2(z) = \left(1 - 16\frac{z^4}{h^4}\right) \tag{15}$$

2.5 Navier's solution

According to Navier's solution procedure, the following solution form for unknown variables is assumed to satisfy precisely the simply supported boundary conditions.

$$\left(u_0, \phi_x, \phi_y\right) = \left(u_{0mn}, \phi_{xmn}, \phi_{ymn}\right) \sin \alpha x \ \cos \beta y$$

$$\left(v_0, \psi_x, \psi_y\right) = \left(v_{0mn}, \psi_{xmn}, \psi_{ymn}\right) \cos \alpha x \ \sin \beta y$$

$$\left(w, \phi_z, \psi_z\right) = \left(w_{mn}, \phi_{zmn}, \psi_{zmn}\right) \sin \alpha x \ \sin \beta y \tag{16}$$

Where $u_{0mn}, v_{0mn}, w_{mn}, \phi_{xmn}, \psi_{xmn}, \phi_{ymn}, \psi_{ymn}, \phi_{zmn}, \psi_{zmn}$ are unknown coefficient.

$$q(x, y) = q_{mn} \ \sin \ \alpha x \ \sin \ \beta y$$

where, q_{mn} is Fourier coefficient of load. For sinusoidal load $q_{mn} = q_0 = 1.0$.

The expression in matrix from as,

$$\left[K\right]_{9X9} \ \left\{\Delta\right\}_{9X1} = \left\{F\right\}_{9X1}$$

where, $[K]$ is the stiffness matrix, $\{F\}$ is the force vector and $\{\Delta\}$ is the vector of unknowns.

3. Numerical Results.

For numerical study, flexural analysis of an orthotropic plate is done for different side to thickness ratios ($a/h = 4, 10, 100$) and aspect ratio ($a/b = 1$). The results of the present theory are compared with exact solution, classical plate theory (CPT), first order shear deformation theory (FSDT), higher order shear deformation theory (HSDT), and trigonometric shear deformation theory (TSDT). Material properties from Ghugal and Sayyad [18] are used to carry out numerical analysis. The properties of orthotropic plate are given below:

$$E_1 = 25E_2,$$
$$G_{13} = G_{12} = 0.5E_2, \ G_{23} = 0.2E_2 \tag{17}$$
$$\mu_{12} = 0.25$$

The results obtained for in-plane displacements, transverse displacement, in-plane normal and shear stresses, transverse normal stress, and transverse shear stresses are presented in the following non-dimensional forms commonly used in the literature.

$$\bar{u}\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{uE_2}{qhS^3}; \ \bar{w}\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{w100E_2}{qhS^4};$$

$$(\bar{\sigma}_x, \bar{\sigma}_y)\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{(\sigma_x, \sigma_y)}{qS^2};$$

$$(\bar{\tau}_{xy})\left(0, 0, \frac{z}{h}\right) = \frac{(\tau_{xy})}{qS^2};$$

$$(\bar{\tau}_{zx})\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{(\tau_{zx})}{qS};$$

$$(\bar{\tau}_{yz})\left(\frac{a}{2}, 0, \frac{z}{h}\right) = \frac{(\tau_{zx}, \tau_{yz})}{qS};$$

The results of other theories for comparison purpose are taken from Shinde et. al.[19].

4. Discussion

Table 1 shows different response quantities obtained through bending analysis of thick orthotropic square plate subjected to sinusoidal load. Non-dimensional in-plane displacement ($\bar{u}$) obtained through present theory exactly matches with the exact and higher order reform theory solutions. Similarly, other response quantities like in-plane normal displacement ($\bar{w}$), in-plane normal stresses $(\bar{\sigma}_x, \bar{\sigma}_y)$ and in-plane shear stress $(\bar{\tau}_{xy})$, and non-dimensional transverse shear stresses ($\bar{\tau}_{zx}, \bar{\tau}_{yz}$) are comparable with exact and higher order reform theory solutions. Figure 2-4 shows distribution of in-plane displacement ($\bar{u}$), in plane normal stress ($\bar{\sigma}_x$), in plane normal stress ($\bar{\sigma}_y$) and transverse shear stress ($\bar{\tau}_{zx}$), respectively across the thickness obtained through present theory for aspect ratio of 10.

Table1: Simply supported thick orthotropic plate with under transverse sinusoidally distributed load.

S	Theory	Model	$\overline{u}$	$\overline{w}$	$\overline{\sigma}_x$	$\overline{\sigma}_y$	$\overline{\tau}_{xy}$	$\overline{\tau}_{zx}$	$\overline{\tau}_{yz}$
4	Kirchhoff	CPT	0.0068	0.431	0.5387	0.0267	0.0213	-	-
	Mindlin	FSDT	0.0060	1.6616	0.4784	0.0579	0.0358	0.2692	0.0491
	Reddy	HSDT	0.0092	1.6206	0.7379	0.0640	0.0289	0.3903	0.0714
	Sayyad & Ghugal	TSDT	0.0096	1.4973	0.7725	0.0383	0.0306	0.3427	0.1371
	Pagano	Elasticity	0.0093	1.5978	0.7276	0.0727	-	0.3620	0.0738
	Present	5th OSDT	0.0093	1.7107	0.7319	0.0733	0.0338	0.3671	0.0694
10	Kirchhoff	CPT	0.0068	0.431	0.5387	0.0267	0.0213	-	-
	Mindlin	FSDT	0.0066	0.6383	0.5385	0.0339	0.0246	0.2877	0.0306
	Reddy	HSDT	0.0071	0.6371	0.5700	0.0347	0.0225	0.3903	0.0714
	Sayyad & Ghugal	TSDT	0.0072	0.6141	0.5770	0.0286	0.0228	0.3506	0.1402
	Pagano	Elasticity	0.0071	0.634	0.5680	0.0360	-	0.422	0.0460
	Present	5th OSDT	0.0072	0.6746	0.5989	0.0325	0.0133	0.4347	0.0339
100	Kirchhoff	CPT	0.0068	0.431	0.5387	0.0267	0.0213	-	-
	Mindlin	FSDT	0.0068	0.4333	0.5385	0.0267	0.0213	0.2931	0.0252
	Reddy	HSDT	0.0068	0.4333	0.5390	0.0268	0.0213	0.4397	0.0378
	Pagano	Elasticity	0.0068	0.4330	0.5385	0.026	-	0.4390	0.0377
	Present	5th OSDT	0.0072	0.4619	0.5674	0.0228	0.0091	0.4523	0.0250

Table 1 Comparison of non-dimensional in-plane displacement $\overline{u}$, transverse displacement $\overline{w}$, in-plane normal stresses $(\overline{\sigma}_x, \overline{\sigma}_y)$ and in-plane shear stress $(\overline{\tau}_{xy})$, non-dimensional transverse shear stresses $\overline{\tau}_{zx}$, $\overline{\tau}_{yz}$ in simply supported square orthotropic plate (*a*=*b*) subjected to sinusoidally distributed load.

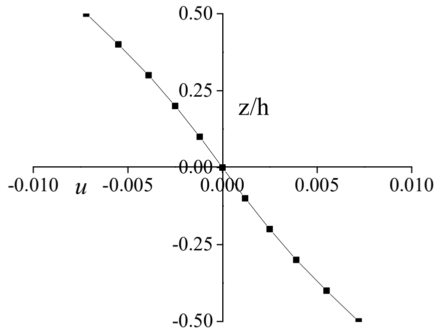

Figure 2: Variation of in-plane displacement ($\overline{u}$) across thickness of simply supported square orthotropic plate subjected to sinusoidal distribution load for aspect ratio of 10

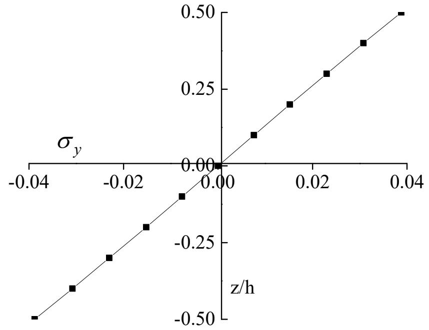

Figure 4: Variation of in plane normal stress ($\overline{\sigma}_y$) across thickness of simply supported square orthotropic plate subjected to sinusoidal distribution load for aspect ratio of 10

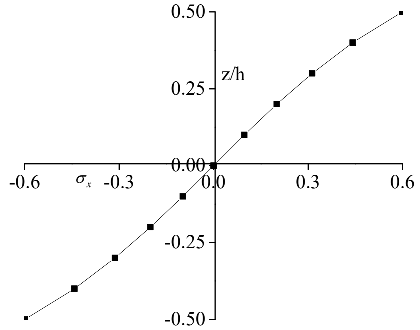

Figure 3: Variation of in plane normal stress ($\overline{\sigma}_x$) across thickness of simply supported square orthotropic plate subjected to sinusoidal distribution load for aspect ratio of 10

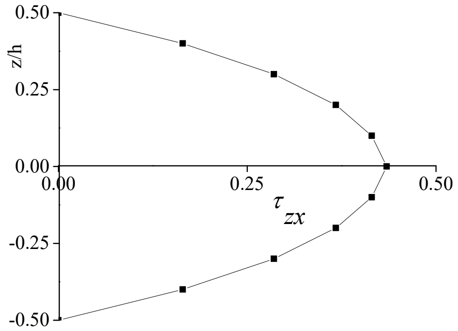

Figure 5: Variation of transverse shear stress ($\overline{\tau}_{zx}$) across thickness of simply supported square orthotropic plate subjected to sinusoidal distribution load for aspect ratio of 10

5. Conclusion

In this paper, the fifth order shear deformation theory [12] is applied for the static flexural analysis of square orthotropic plates subjected to sinusoidal load. A comparison of various response quantities like non-dimensional in-plane displacement $\bar{u}$, transverse displacement $\bar{w}$, in-plane normal stresses $(\bar{\sigma}_x, \bar{\sigma}_y)$, in-plane shear stress $(\bar{\tau}_{xy})$, and non-dimensional transverse shear stresses $(\bar{\tau}_{zx}, \bar{\tau}_{yz})$ obtained through present theory, exact solution, classical plate theory and higher order deformation theories is done. It is observed that the results of displacement and stresses predicted by the present theory are in good agreement with the exact results. Further, the proposed theory can produce reasonably accurate transverse shear stresses using constitutive relations.

Disclosures

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