

On Nonlinear First Ply Failure Study of Laminated Composite Thin Skewed Hypar Shell Roofs

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Paper ID - 010079

Abstract

The skewed hypar shells have gained immense popularity in practical civil engineering fields due their highly aesthetic look. Light weight laminated composite material has increased its wide acceptance day by day in practical structures by virtue of superior material properties like high strength, high stiffness to weight ratio, prolonged fatigue life etc. Different types of bending as well as free and forced vibration studies of laminated composite skewed hypar shells were done by earlier researchers. In some literatures, the first ply failure was also investigated using geometric linear strains only. So, the present paper aims to study the first ply failure behaviour of laminated composite skewed hypar shell roofs considering geometric nonlinearity. An eight noded isoparametric curved finite element having five degrees of freedom at each node is used here for finite element method. To establish the correctness of the present approach different benchmark problems are solved in this paper. Various types of lamina stacking sequences are taken up by the authors' problems for first ply failure analysis of symmetric and antisymmetric skewed hypar thin shells with practical boundary condition. Along with the maximum stress and maximum strain failure criteria the authors use interactive failure criteria like Hoffman, Tsai-Hill and Tsai-Wu criteria as well as failure mode based criteria like Hashin and Puck criteria for the present study. The results obtained from this numerical investigation are analysed and post processed from different engineering standpoints to extract meaningful conclusions regarding first ply failure behaviour of the composite skewed hypar thin shells and to arrive at important practical guidelines which are expected to be beneficial for practicing civil engineers.

Keywords: Skewed Hypar Shell, Thin Shell, First Ply Failure, Geometric Nonlinearity

1. Introduction

Among different types of shell structures, thin skewed hypar shells are mostly preferred as the roofing units due to its high aesthetic appearance. This geometry may be used to cover large column free open spaces such as auditoriums, car parking lots, stadiums, shopping malls, airports etc.

Laminated composite structure is distinguished by its high strength/stiffness to weight ratio and are used in various weight sensitive branches of engineering. High strength to weight ratio, high stiffness to weight ratio, high fatigue strength, capacity of being assembled fast, less susceptibility to thermal expansion and low decay due to weathering action and moisture makes the laminated composite a profitable material to the practicing structural engineers. A laminated composite may fail due to its static overloading under service condition. The failure of laminated composite is different from isotropic material due to anisotropy of composites. The failure of laminates does not take place suddenly. The most heavily stressed ply in the laminate fails first (first ply failure) and then the material stiffness will be redistributed so that remaining laminate begins to carry the imposed load. The first ply failure may

initiate within the laminate and remain undetected and unattended. Such latent damages ultimately lead to sudden catastrophic collapse of the shell under service. Hence the load at which the failure initiates (first ply failure) in the laminate is very important to be known to a practicing engineer.

Saha et al. [1] studied the effects of several parameters like offset distance, twist angle, mid plane delamination and non-dimensional speed on first ply failure load of shallow conical shell. Authors obtained numerical results using finite element method with the implementation of various failure criteria such as maximum stress, Tsai-Hill, Tsai-Wu and Hoffman. The first ply and progressive failure of hypar shells and first ply failure of composite spherical shells using geometric linear finite element formulation were studied by Ghosh and Chakravorty [2, 3]. The first ply failure of laminated composite conoidal shells were studied by Bakshi and Chakravorty using geometrically linear strains [4, 5]. Joshi [6] presented the failure analysis of laminated composite plate, of graphite/epoxy, using higher order shear deformation theory. Various failure theories

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such as maximum stress, maximum strain, Tsai-Wu etc. are used with both symmetric and antisymmetric lamination scheme under simply supported condition. The results suggest that failure load increases with increase in fiber orientation angle. Uniyal et al. [7] presented the multi scale modelling and failure analysis of composites using finite element software ANSYS. Authors used various failure theories such as maximum stress, maximum strains, Tsai-Wu and first ply failure load values are obtained for different lamination schemes under uniaxial and biaxial loading conditions. The effect of fiber orientation on failure load of a laminated curved panel subjected to uniaxial compression was studied by Adali and Cagdas [8]. They defined failure load as minimum of first ply failure and buckling load. The results suggest that for a thick cylindrical panel failure is mostly due to first ply failure while for thin cylindrical panels, buckling mode of failure dominates. Pal and Bhattacharya [9] carried out the progressive failure analysis of laminated composite plate subjected to transverse loading using first order shear deformation theory. The results suggest that ultimate failure load increases with the increase in angle of fiber orientation for both symmetric and antisymmetric cross ply laminated plate consisting of different number of layers. Akhras and Li [10] observed that load at which total laminate fails is far higher than the first ply failure load. However, for the design reliability an engineer should be well known about first ply failure because if initiation of damage remains undetected then it may lead to severe damage and sudden collapse of structure. A progressive intralaminar failure methodology was proposed by Falzon and Apruzzese [11] proposed to simulate damage growth of laminated composite materials and some structural applications of this failure model on composite plates were implemented later by the same authors [12].

The close review of literature reveals that the first ply failure of composite plates was done by a number of researchers but the same study is very less on shell geometries. Very few of researchers focussed on failure behaviour of laminated composite conoidal and spherical shells. But these are limited to geometric linear finite element formulations. So there is a gap on first ply failure of laminated composite skewed hyar shell roofs using geometric nonlinearity. The present paper aims to fulfil this lacuna.

2. Mathematical Formulations

In this paper a geometrically nonlinear finite element code is developed to study the first ply failure of laminated composite thin skewed hyar shells. A schematic diagram of laminated composite skewed hyar shell of uniform thickness h and twist radius of curvature R_{xy} is shown in Fig. 1. Keeping the total thickness constant, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the X axis of the coordinate system. The surface equation and the twist radius of curvature of this shell are expressed as:

$$z = (4c/ab)(x - a/2)(y - b/2) \quad (1)$$

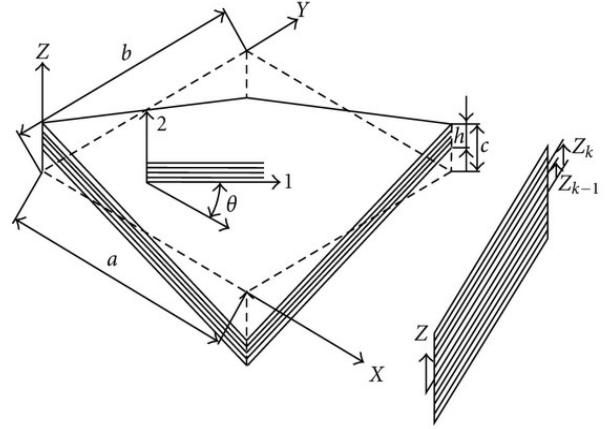


Fig.1. Surface of a skewed hyar shell

$$1/R_{xy} = \partial^2 z / \partial x \partial y = 4c/ab \quad (2)$$

The governing equations related to laminate constitutive relations and the systematic development of the geometric linear stiffness matrix of the shell have been reported in other publication of the present authors [13]. The mid-surface strain vector $\{\epsilon\}$ of the hyar shell is expressed in Eq. (3) following von-Kármán assumptions and Sanders' nonlinear strain-displacement relations.

$$\{\epsilon\} = \{\epsilon\}^{\text{Linear}} + \{\epsilon\}^{\text{Nonlinear}} \quad (3)$$

The governing nonlinear static equilibrium equations are solved by Newton – Raphson iteration method and the tangent $[K]_T$ and secant $[K]_S$ stiffness matrices are expressed as:

$$[K]_T = [K]_L + [K]_{LT} + [K]_F \quad (4)$$

$$[K]_S = \oint_A [B]^T [E] [B] dA + 0.5 \oint_A [B]^T [E] [B'] dA + \oint_A [B']^T [E] [B] dA + 0.5 \oint_A [B']^T [E] [B'] dA \quad (5)$$

$$\text{where, } [K]_L = \oint_A [B]^T [E] [B] dA \quad [K]_F = \oint_A [G']^T \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} [G'] dA$$

$$[K]_{LT} = \oint_A [B]^T [E] [B'] dA + \oint_A [B']^T [E] [B] dA + \oint_A [B']^T [E] [B'] dA$$

The developed lamina stresses and strains are used in well accepted failure theories like maximum stress, maximum strain, Tsai-Hill, Tsai-Wu, Hoffman, Hashin and Puck failure criterion to evaluate the first ply failure load of laminated composite thin skewed hyar shell roofs. The expressions of the failure theories are adopted from authors another paper [13].

3. Numerical Examples

In order to validate the correct incorporation of the present geometrically nonlinear static bending formulation the present authors solve a first benchmark problem. Fig. 2 represents the comparative studies between the central displacement values from present code with the displacement values obtained from earlier literature by

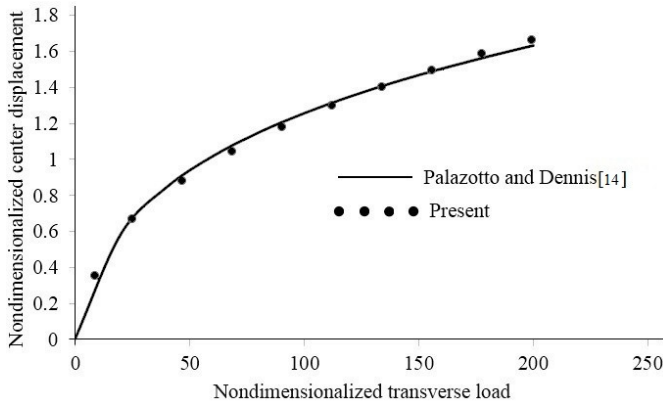


Fig. 2. Nonlinear deflection of isotropic plate

Table-1. Comparison of first ply failure loads in Newton for a $(0^\circ/90^\circ)_s$ plate

Failure criteria	Length/plate thickness	Experimental failure load (Kam et al. [15])	First ply failure loads (Kam et al. [15])	First ply failure loads (present formulation)
Maximum stress	105.26	157.34	108.26 ^L	112.15 ^L
Maximum strain			147.61 ^{NL}	139.94 ^{NL}
Hoffman			122.86 ^L	127.56 ^L
			185.31 ^{NL}	194.58 ^{NL}
Tsai-Wu			106.45 ^L	103.36 ^L
			143.15 ^{NL}	137.12 ^{NL}
Tsai-Hill			112.77 ^L	110.46 ^L
			157.78 ^{NL}	150.71 ^{NL}
			107.06 ^L	104.40 ^L
			144.42 ^{NL}	151.22 ^{NL}

Note: “L” and “NL” indicate the linear and nonlinear failure loads respectively

Note: Length=100mm, load details = central point load

Palazotto and Dennis [14] for an isotropic simply supported plate. For the isotropic plate problem, the transverse load and central plate displacements are nondimensionalized as $\bar{q} = (q_0 a^4) / (E_1 h^4)$ and $\bar{w} = w / h$ respectively where w is the transverse displacement and q_0 is intensity of the uniformly distributed surface pressure. Here, a and E_{11} represent the length and modulus of elasticity along fiber direction of square plate respectively. For validating the first ply failure formulation the authors compare the nonlinear first ply failure load values of a partially clamped laminated composite square plate evaluated using the present finite element formulation with the results published by Kam et al [15]. The results are furnished in Table 1.

Apart from these benchmark problems the authors solve a number of problems to determine the nonlinear first ply failure load values of laminated composite skewed hyar thin shells of a practical boundary condition where two opposite edges are clamped and other two edges are free. For this boundary condition, two and three layered anti-symmetric (AS) and symmetric (SY), cross (CP) and angle ply (AP) laminations are considered in the present study for developing ASCP ($0^\circ/90^\circ$), SYCP ($0^\circ/90^\circ/0^\circ$), ASAP ($45^\circ/-45^\circ$) and SYAP ($45^\circ/-45^\circ/45^\circ$) laminates. The material properties of graphite – epoxy composites are taken from Kam et al [15] paper. The plies are numbered from top to bottom of the laminate. The first ply failure load values (FL) which are nondimensionalized as $\bar{FL} = (FL / E_{22}) (a/h)^4$ and the other related failure information are furnished in Table 2

using different well-established failure criteria like maximum stress, maximum strain, Hoffman, Tsai-Hill, Tsai-Wu, Hashin and Puck failure criteria. The minimum failure load values obtained from these criteria are considered as the first ply failure loads (marked by italics in Table 2) on which engineering factor of safety may be imposed.

4. Results and Discussions

The nondimensional static geometrically nonlinear central displacement values of isotropic plates obtained from the present formulation are compared with the displacement values published by Palazotto and Dennis [14] in Fig. 2. The close agreement of this result establishes the correct incorporation of geometrically nonlinear bending formulation in the present finite element code. The values of linear and nonlinear first ply failure loads of laminated composite plate determined from present code are also compared with the results reported earlier by Kam et al. [15] in Table 1. The correctness of the present approach for determination of first ply failure loads considering geometrically linear and nonlinear strains is strongly established by very good agreement of these failure results with the results published in the literature.

The uniformly distributed non-dimensional first ply failure load values of skewed hyar shells with the present boundary condition are reported in the Table 2 considering both geometrically linear and nonlinear strains. The failure modes and failed plies along with the corresponding failure criteria are also shown in this table. The least failure load values obtained through different failure criteria for each cases are considered as the first ply failure loads for that shell options and the failure criteria which govern the minimum first ply failure load values are treated as the governing failure criteria. The Puck failure criterion yields the minimum values of the first ply failure loads for most of the present cases except ASAP shells. For this exceptional case, the maximum strain failure criterion gives the lowest value of failure load but this value is very close (within 5.1%) to the value obtained through the Puck failure criterion. So, the present authors suggest that the Puck failure criteria is the governing failure criteria of all the cases considered here.

It is interestingly noticed from the results furnished in Table 2 that the cross ply shells perform better than the angle ply shell options in terms of first ply failure loads. The failure load values of cross ply shells may increase up to 1.3 times of the load values obtained from angle ply shells for this edge condition. The close look at the results reported in this table also reflect that, the anti-symmetric cross ply skewed hyar shells give the maximum first ply failure load values among all the cases taken up here. So it can safely be concluded that among all the cases considered here, the anti-symmetric cross ply ($0^\circ/90^\circ$) are the best skewed hyar shell choice to the practicing design engineers.

Apart from the failure load values, the failed plies and the failure modes / tendencies are also indicated in this tables for different stacking sequences. This information may help to the practicing engineers for fabricating new laminated composite materials for skewed hyar shells.

Table-2. Nondimensional first ply failure pressures \overline{FL} for skewed hypar shells

Lamination	Failure theory	\overline{FL}	Failed ply	Failure mode / failure tendency
ASCP	Maximum stress	3782.43 ^L	2	Matrix cracking
		3773.24 ^{NL}	2	Matrix cracking
	Maximum strain	3712.97 ^L	2	Matrix cracking
		3656.79 ^{NL}	2	Matrix cracking
	Hoffman	3411.64 ^L	2	Matrix cracking
		3368.74 ^{NL}	1	Fiber breakage
	Tsai-Hill	3544.43 ^L	2	Matrix cracking
		3518.90 ^{NL}	2	Matrix cracking
	Tsai-Wu	3414.71 ^L	2	Matrix cracking
		3384.07 ^{NL}	2	Matrix cracking
	Hashin	3540.35 ^L	2	Matrix cracking
		3516.85 ^{NL}	2	Matrix cracking
	Puck	1813.07 ^L	2	Matrix crushing mode C
		1676.20 ^{NL}	2	Matrix crushing mode C
SYCP	Maximum stress	2855.98 ^L	2	Matrix cracking
		2861.08 ^{NL}	2	Matrix cracking
	Maximum strain	2802.86 ^L	2	Matrix cracking
		2804.90 ^{NL}	2	Matrix cracking
	Hoffman	2705.82 ^L	2	Matrix cracking
		2700.72 ^{NL}	2	Matrix cracking
	Tsai-Hill	2766.09 ^L	2	Matrix cracking
		2767.11 ^{NL}	2	Matrix cracking
	Tsai-Wu	2707.87 ^L	2	Matrix cracking
		2702.76 ^{NL}	2	Matrix cracking
	Hashin	2760.98 ^L	2	Matrix cracking
		2764.04 ^{NL}	2	Matrix cracking
	Puck	1493.36 ^L	2	Matrix crushing mode C
		1407.56 ^{NL}	2	Matrix crushing mode C
ASAP	Maximum stress	1331.97 ^L	1	Matrix cracking
		1340.14 ^{NL}	1	Matrix cracking
	Maximum strain	1255.36 ^L	1	Matrix cracking
		1261.49 ^{NL}	1	Matrix cracking
	Hoffman	1317.67 ^L	1	Matrix cracking
		1324.82 ^{NL}	1	Matrix cracking

SYAP	Tsai-Hill	1345.25 ^L	1	cracking
		1352.40 ^{NL}	1	Matrix cracking
	Tsai-Wu	1301.33 ^L	1	Matrix cracking
		1307.46 ^{NL}	1	Matrix cracking
	Hashin	1321.76 ^L	1	Matrix cracking
		1328.91 ^{NL}	1	Matrix cracking
	Puck	1319.71 ^L	1	Matrix cracking mode A
		1326.86 ^{NL}	1	Matrix cracking mode A
	Maximum stress	1591.42 ^L	1	Matrix cracking
		1589.38 ^{NL}	1	Matrix cracking
	Maximum strain	1494.38 ^L	1	Matrix cracking
		1502.55 ^{NL}	1	Matrix cracking
	Hoffman	1575.08 ^L	1	Matrix cracking
		1572.01 ^{NL}	1	Matrix cracking
	Tsai-Hill	1609.81 ^L	1	Matrix cracking
		1601.63 ^{NL}	1	Matrix cracking
	Tsai-Wu	1553.63 ^L	1	Matrix cracking
		1553.63 ^{NL}	1	Matrix cracking
	Hashin	1579.16 ^L	1	Matrix cracking
		1577.12 ^{NL}	1	Matrix cracking
	Puck	1324.82 ^L	2	Matrix crushing mode C
		1330.95 ^{NL}	2	Matrix crushing mode C

Note: "L" and "NL" indicate the linear and nonlinear failure pressures respectively

Note: $a/b = 1$, $a/h = 100$, $c/a = 0.2$

5. Concluding Remarks

The following conclusions are evident from the present study.

- The present finite element code can efficiently predict the geometrically nonlinear first ply failure loads of laminated composite thin skewed hypar shell roofs as it is evident from the solutions of the benchmark problems.
- The present authors suggest that the Puck failure criteria is the governing failure criteria of all the cases considered here.
- Among all the cases considered here, the anti-symmetric cross ply ($0^\circ/90^\circ$) are the best skewed hypar shell choice to the practicing design engineers.

Disclosures

Free Access to this article is sponsored by SARL ALPHA CRISTO INDUSTRIAL.

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