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A study of impact induced stress due low velocity impact on laminated composite skewed hypar shell roof

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Abstract

The doubly ruled hypar shell, being one of the most feasible shell roof configurations, enjoys industrial preference for covering large column free areas. This class of shells is unique as the only curvature here is the cross curvature and these do not admit easy closed form solution particularly when the boundary conditions are complicated. Laminated composite, an innate choice to different industrial sectors for its huge specific strength and stiffness, good weathering resistance, is now being extensively employed in civil engineering. However the low transverse shear strength of the composite shell impelled the researchers to study the response of the same under the action of impact loads. In the present study, a finite element code is functioned to investigate the impact induced stress history generated in simply supported laminated composite hypar shell for different impact velocities of a spherical solid striker. Contact behavior is described by modified Hertzian contact law where as time dependent equations are solved using Newmark's time integration algorithm in present analysis. A practicing engineer often has to select a particular shell option consisting stacking sequence among a number of possibilities. The present paper discusses the behaviour of composite hypars under low velocity impact and different stresses induced due to the same from engineering standpoint to propose a selection guide line among the available options and to proposes values of practical parameters like the equivalent static loads and dynamic magnification factors for designing such shells through static simplification of the problems.

Keywords: Composite material, Impact responses, Hypar shell, Stress analysis, Design guideline

1. Introduction

Composites are engineered materials and are chosen in several industrial sectors for their high specific stiffness, strength, low specific weight and weathering resistance.

In spite of these advantages impact-induced damage, due to low transverse shear capacity of laminated composites, has become a great concern. Civil engineering shell roofs, might often be exposed to such impact due to wind born debris or snowfall.

The classical contact law between isotropic elastic solids derived by Hertz [1] was found to be inadequate for composite materials. Yang and Sun [2] proposed a power law framed on static indentation tests. The modified version of the same was further proposed by Tan and Sun [3].

Impact response of simply supported initially stressed plate was reported by Sun and Chen [4] using modified contact law [3].Toh et al. [5] studied impact analysis of an orthotropic laminated cylindrical shell under low-velocity impact.

Shim et al.[6] reported the elastic response of glass/epoxy laminated composite ogival shell subjected to low velocity impact using bi-harmonic polynomial solution. A numerical solution was proposed by Chun and Lam [7] for analysing a laminated composite plate under low-velocity impact. In all these research related impact behaviour of shell roof and its stress histories remained untouched.

A look through the literature reveals the fact that impact response of civil engineering shell structures has not received due attention. A parallel review reveals that the hypar shell was studied recently by Sahoo and Chakravorty[8] for free vibration issues only. The single report on impact response of composite hypar shell was due to Das Neogi et al [9] where in, impact problem was studied.

Hence, this paper aims to carry out a study on stress histories under low velocity impact for fully clamped boundary condition.

2. Mathematical formulation

The surface considered in the present study is a thin, shallow, doubly curved, anticlastic skewed hyparshell (Fig.-1)

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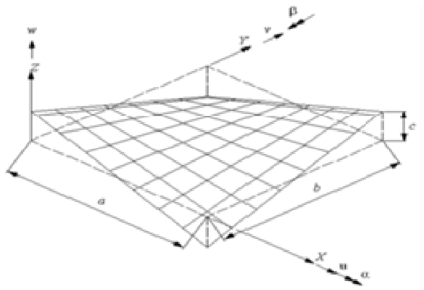


Surface equation: $z = \frac{4c}{ab}\left(x - \frac{a}{2}\right)\left(y - \frac{b}{2}\right)$

Fig. 1. Surface of a skewed hypar shell and degrees of freedom

of laminated composite and linearly elastic material having cross curvature $1/R_{xy}$, with uniform thickness h. A shell is characterized as shallow if any infinitesimal line element of its middle surface is approximated by the length of its projection on the XY plane. This implies that

$$\left(\frac{\partial z}{\partial x}\right)^2 << 1, \qquad \left(\frac{\partial z}{\partial y}\right)^2 << 1, \qquad \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) << 1$$

(1)Similarly, the lateral boundary of a shallow shell is also estimated by its projection on the XY plane with accordance to its boundary conditions.

Cross curvature is approximately represented as

$$\frac{1}{R_{xy}} = \frac{\partial^2 z}{\partial x \partial y} \qquad (2)$$

An eight-noded curved quadratic isoparametric finite element is used for analysis with five degrees of freedom at each node as shown in Fig. 1. The generalised displacement vector of an element is expressed in terms of the shape functions and nodal degrees of freedom as

$$[u] = [N]\{d_e\} \qquad (3)$$

The element stiffness and mass matrices are derived by using the minimum energy principle. The element stiffness matrix is

$$[K] = \iint [B]^T [D][B] dxdy \qquad (4)$$

Incorporating both the translatory and rotatory inertia terms, the generalised inertia matrix takes the following form

$$[M] = \iint [N]^T [\rho][N] dxdy \qquad (5)$$

The dynamic equilibrium equation of the target shell for low velocity impact is given by the following equation:

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{F\} \qquad (6)$$

where [M] and [K] are global mass and elastic stiffness matrices, respectively.{δ} is the global displacement vector. For the impact problem {F}, the force vector is given as

{F} = {0 0 0 ….FC……0 0 0} T (7)

Here FC is the contact force given by the indentation law and the equation of motion of the rigid impactor is given as

$$m_i \ddot{\omega}_i + F_C = 0 \qquad (8)$$

where mi and $\ddot{\omega}_i$ are the mass and acceleration of the impactor respectively.

Assessment of the contact force depends on a contact law relating the former with indentation. The contact force model following Sun and Chen [4] has been incorporated in the present finite element formulation with appropriate modification for friction generated due to oblique impact. If k is the contact stiffness and $\alpha_m$is the maximum local indentation, the contact force $F_c$during loading is given by

$F_c = k\alpha_i^{1.5} 0 < \alpha_i \le \alpha_m (9)$

The indentation parameter α_i at any ith iteration depends on the difference of the displacements of the impactor and the target structure at any instant of time, and the contact force as well i.e. the values of α_i keeps changing with time on account of time-varying displacements of both the rigid impactor and the target structure.

Considering displacements along any arbitrary global directions for oblique impact, the indentation αi at any ith iteration is given as

αi=wi(t)cosθi- ws(x_c,y_c,t_c) (10)

Where wi and ws are displacement of impactor and target shell along any arbitrary direction (θ_i) at the point of contact (x_c ,y_c) and at any time instant (t_c), respectively. In present analysis θ_i = 0^0

Subsequently, the displacement of the impactor gradually decreases, but the target point displacement keeps on changing and finally increases to a maximum and there comes a time when these two displacements become equal. This leads to zero value of indentation. Eventually the contact force becomes zero when the impactor loses the contact with the target. This process of attaining the maximum contact force till the declining of the same to zero is fundamentally referred to as unloading. Provided that the mass of the impactor is not very small,a second impact may occur upon the rebound of the target structure leading to an identical phenomenon of contact deformation and attainment of the maximum. This is known as reloading. If Fm is the maximum contact force at the onset of unloading and αm is the maximum indentation during loading, the contact force Fc for unloading and reloading are expressed as [4].

Unloading phase: $$F_c = F_m \left[\frac{\alpha_i - \alpha_0}{\alpha_m - \alpha_0}\right]^{2.5} \qquad (11)$$

Reloading phase: $$F_c = F_m \left[\frac{\alpha_i - \alpha_0}{\alpha_m - \alpha_0}\right]^{1.5} \qquad (12)$$

The solution for the equations of motion given by Equations (6) and (7) is solved using Newmark constant-acceleration time integration algorithm in the present analysis. Equation (1) may be expressed in iteration form at each time step.

$$\left[\bar{K}\right]\{\Delta\}_{t+\Delta t}^{i+1} = \frac{\Delta t^2}{4}\{F\}_{t+\Delta t}^{i} + [M]\{b\}_i \qquad (13)$$

Where

$$\left[\bar{K}\right] = \frac{\Delta t^2}{4}\left[K\right] + \left[M\right] \quad (14)$$

$$\{b\}_t = \{\Delta\}_t + \Delta t\left\{\dot{\Delta}\right\}_t + \frac{\Delta t^2}{4}\left\{\ddot{\Delta}\right\}_t \quad (15)$$

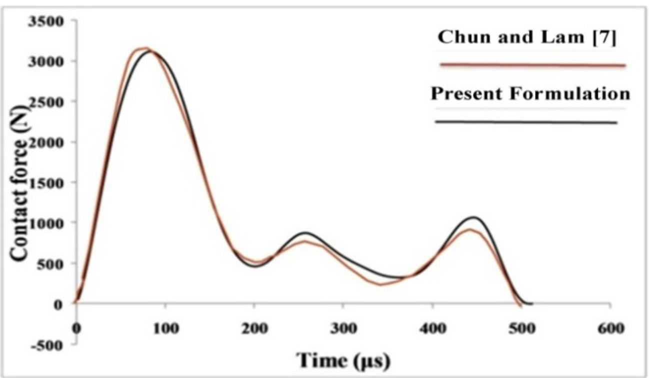
The same solution scheme is also utilized for solving the equation of motion of the impactor, i.e. Equation (7). It is to be noted that a modified contact force obtained from the previous iteration is used to solve the current responseThe iteration procedure is continued until the equilibrium criterion is met.

3. Numerical example

Problems are solved in this paper to validate the present finite element code and to numerically explore the different behavioral aspects of composite skewed hypar shell roof under low-velocity impact. Firstly the present formulation is applied to solve natural frequencies of graphite-epoxy twisted plates which are structurally similar to skewed hypar shells.
This problem is expected to validate both the stiffness and mass matrix formulation of present finite element code comparing with the published one [10]. Results of existing literature [7] regarding the impact response of composite plate considering clamped boundary condition, is taken up as the second benchmark to validate the correct incorporation of clamed support in the present code.
The details of the benchmark problems are furnished along with Table-1, Fig-2.

Table 1 Non dimensional natural frequencies (ϖ) for three layer graphite epoxy twisted plates $[\theta/-\theta/\theta]$

Angle of twist	θ (deg)	0^0	15^0	30^0
$\phi = 15^\circ$	Qatu and Lessia[10]	1.0035	0.9296	0.7465
	Present formulation	0.9990	0.9257	0.7445
$\phi = 30^\circ$	Qatu and Lessia[10]	0.9566	0.8914	0.7205
	Present formulation	0.9490	0.8842	0.7181



E_{11}=142.73GPa, E_{22}=13.79GPa, G_{12}=4.64GPa, ν_{12} =0.3, ρ =1.61x 10^3 kg/m^3, a=b=0.14m, h=3.81x10^{-3}m, ply orientation = [$0^0/90^0/0^0$/] Velocity of impactor =22.6m/s

Fig. 2. Contact force history of clamped plate

Besides the aforementioned problems, responses of skewed hypar shells being impacted at the central point are also studied for eight different shell options combining two boundary conditions and four laminations. Six impact velocities with four different angle of impact are considered. The details of the authors' own problems are furnished below.

i)	Boundary conditions	Clamped (CL)
ii)	Laminations	$+45^\circ/-45^\circ/+45^\circ$ (Angle ply symmetric or AP/SY) $0^\circ/90^\circ/0^\circ$ (Cross ply symmetric or CP/SY)
iii)	Velocityof impact (m/s)	1, 3, 5, 10
iv)	Details of shell geometry	a = 1.0m, b =1.0m, t=0.02m, c=0.2m
V)	Material details	E_{11}=120Gpa, E_{22}= 7.9 GPa, G_{12}= G_{23}= G_{13}= 5.5GPa ν_{12} = 0.30, ρ = 1.58x10^{-5}N-sec^2/cm^4

4. Results and discussions

Table-1 and Fig-2 furnish the results of the benchmark problems including the published ones and those obtained by the present approach. The fundamental frequencies of the composite twisted plate as obtained here closely match with those reported by Qatu and Lessia[10] and hence the correct incorporation of the stiffness and mass matrices of composite twisted plate, which are geometrically analogous to skewed hypar shells, in the present code, is established. The fact that contact force history of clamped composite plate reported by Chun and Lam [7] closely match with the present values establish that the present formulation accurately takes into account the clamped support condition.
To study the impact response of simply supported (SS) cross ply (CP) shell Fig. 2. and Table-2 are studied. All the resultsof contact force and displacement that are presented in either graphical or tabular form are arrived at after the study of time step convergence. The finite element mesh adopted is also based on force and displacement convergence criteria. When low velocity normal impact response of simply supported angle ply shell is studied being struck by the spherical impactor centrally, it is observed that the contact force shows a sort of parabolic variation with a single peak. After a given time span which is 100µs or less the contact force converges to a null value. It is interesting to note that higher the impactor velocity higher is the contact force as expected, but the force dies down to a null value earlier. This behaviour may be attributed to the fact that the higher the velocity more rapid is the elastic rebound of the impactor followed by detachment which causes contact force to decay out. It is also very interesting to observe that the time instant corresponding to peak contact force and that for peak displacement do not match. This is because the resultant displacement at any time instant is a cumulative effect

Table 2. Maximum contact force, maximum dynamic displacement, bending stress, torsional stress, normal stress and shear stress for different impact velocities

Boundary condition	Ply orient -ation	Impact Velocity (m/s)	Maximum Contact force (N)	Maximum deflection (m)	Maximum bending stress and torsional stress (N/m^2)			Maximum normal stress and shear stress (N/m^2)		
					Mx	M_y	M_{xy}	N_X	N_y	N_{xy}
Simply supported	0^0/ 90^0	1	310.18	0.000169	50.71	50.71	1.48	431.6	202.8	236.9
		3	1169.58	0.000552	55.99	55.99	6.04	1625.90	1610.20	8605.2
		5	2169.13	0.000965	99.50	99.50	15.4	3064.40	3064.40	16378
		10	5025.04	0.002031	166.6	166.6	26.7	5011.30	5011.40	30192
	$+45^0$/-45^0	1	309.28	0.000168	51.6	51.6	2.63	501.62	501.88	3019.2
		3	1165.87	0.000523	55.99	55.99	8.26	1625.90	1625.9	9188.1
		5	2162.33	0.00090	99.5	99.5	15.4	3065.4	3065.4	16278
		10	5010.83	0.001875	167.2	167.2	27.0	5018.8	5016.2	30154

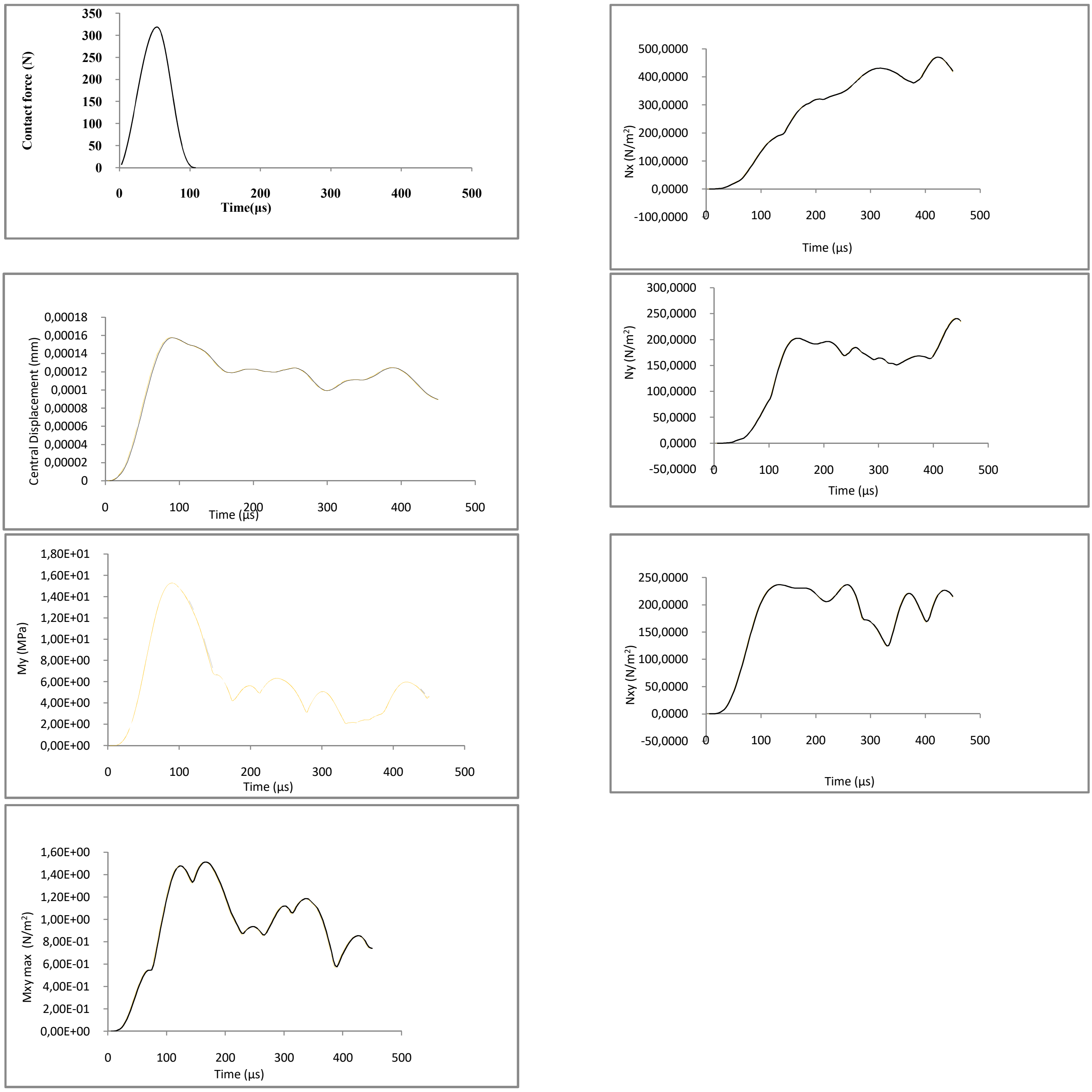


Fig. 3. Impact response of clamped symmetric cross ply (SY/CP) composite hypar shells for impact velocity 1m/s

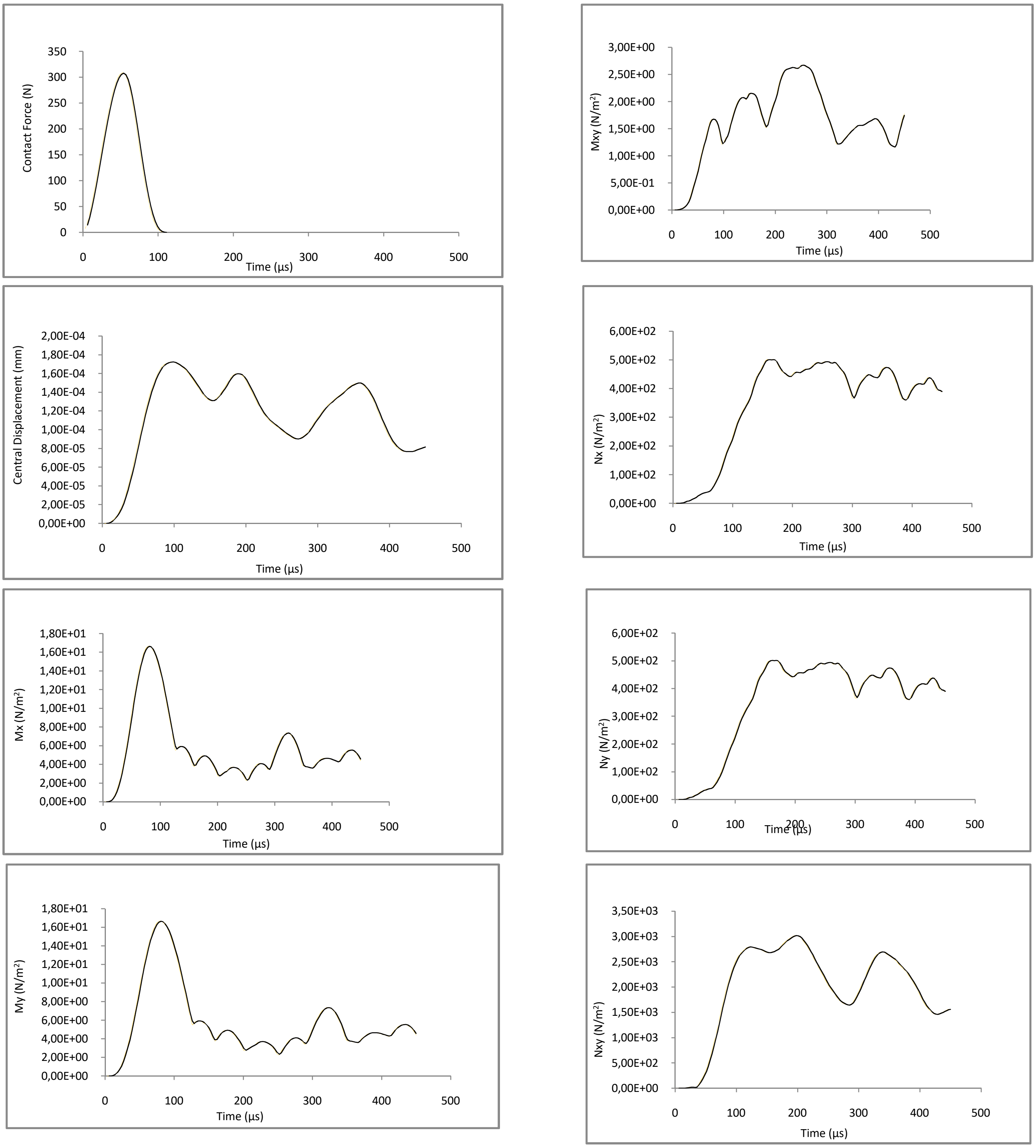


Fig 4. Impact response of clamped symmetric angle ply (SY/AP) composite hypar shells for impact velocity 1m/s.

of the instantaneous contact force value and the inertia effect of the previous instant. The figure showing the transient displacement reflects the fact that vibration continues even after the force dies down with successively occurring peak though the peak values are less in magnitude than the highest peak which occurs a bit after the instant of maximum contact force but before the full decay of it.

The behavior of the impact response of simply supported (SS) cross ply (CP) shell may be studied through Fig.10 to 16 and Table-2. The nature of contact force and dynamic displacement for this class (SY/ CP) of shell is more or less similar to what is discussed before for SY/AP shell. One interesting difference is that for SY/ CP shell the peak dynamic displacement does not only show a phase lag with respect to the peak contact force but by time displacement value reaches the peak the contact force value dies down totally. This shows that the after-effect of impact are some times more severe than the shell response during the impact and study of displacement variation even after the contact force decays to a null value is absolutely necessary. However, after passage of some more time the subsequent local maxima which are obtained do not touch the peak.

While studying the stresses generated under the impact load for clamped supported cross ply (SY/CP) and angle ply (SY/AP) maximum stress generated in the shell increases with the increment of impact velocity. For brevity of space the time history curve of impact response are only represented for one impact velocity (1m/s). Maximum values of contact force, displacement and other stresses are reported in Table 2. From Fig. 2 and 3 it is discernible that maximum bending stresses ($M_{x,}$ $M_{y)}$ and torsional stress (M_{xy}) occurs with a phase lag with maximum contact force but the normal stresses (N_x,N_y) and shear stress (N_{xy}) occurs with huge phase lag in compare to the previous. While scrutinizing the results presented in Table 2 in is clearly concluded that the normal stresses are the governing load for design as it normally occurs in case of thin shell. While the shear stress is also very high which also very common for composite structure.

Comparing the performance of cross ply (CP) and angle ply (AP) shell option from the data of Table 2. it could be concluded that cross ply shell is acting more stiffer than angle ply shell as the higher contact force confirmemore stiffer elastic rebound. However, the stresses in the angle ply shell is bit higher in many cases. This suggests cross ply (CP) shell as preferred option.

Conclusion

The following conclusions may be derived from the present study.

1. The close agreement of the results obtained by the present method with those available in the published literature establishes the correctness of the approach used here.
2. Under the influence of normal low velocity impact the contact force shows a parabolic combined loading and unloading curve with a single peak for the practical class of shells considered here. Higher magnitude of impact velocity results in higher value of the peak contact force dynamic displacement and stresses. However, due to a sharp elastic rebound the total duration of contact force is less for higher velocity of impactor.
3. The time instants at which the maximum contact force and the maximum dynamic displacement and the stresses occur show a phase difference and interestingly in some cases the maximum displacement and hence stresses may occur even after the contact force dies down totally .Thus it is concluded that the study should be continued only after when the major peaks of the dynamic displacement die down and not after the full decay of the contact force only.
4. From the study of the results it could be concluded that simply supported cross ply (SS/CP) is a better option over simply supported angle ply (SS/AP)

Disclosures

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References

[1] H.Hertz, On the contact of elastic solids. *Journal fur die reine und angewandteMathematik*, 92 (1881) 156-171.

[2] S.H. Yang, C.T.Sun, Indentation law for composite laminates, *Compos. Mat.: Testing and Desig., ASTME STP* 787 (1985) 425-446.

[3] T.M.Tan, C.T. Sun, Use of statical indentation laws in the impact analysis of laminated composite plate, *J. Appl. Mech.* 52 (1983) 6-12.

[4] C.T. Sun, J.K. Chen, On the impact of initially stressed laminates, *J. Compos. Mat.* 19 (1985) 490-503.

[5] S.L. Toh, S.W. Gong, V.P.W.Shim, Transient stress generated by low velocity impact on orthotropic laminated cylindrical shell, *Compos. Struct.* 31(3) (1995) 213-228.

[6] V.P.W. Shim, S.L. Toh, S.W.Gong, The elastic impact response of glass/epoxy laminated ogival shells, *Int. J. Impt. Engg.* 18(6) (1996) 633-655.

[7] L. U. Chun , K. Y. Lam, Dynamic response of fully-clamped laminated composite plates subjected to low-velocity impact of a mass, *Int. J. Solids Struc.*, 35 (11) (1998) 963–979.

[15] S. Sahoo, D.Chakravorty, Finite element vibration characteristics of composite hypar shallow shells with various edge supports, *J. Vib. Control* 11(10) (2005) 1291-1309.

[16] S.DasNeogi, A. Karmakar, D. Chakravorty, Impact response of simply supported skewed hypar shell roofs by finite element, *J ReiforcedPlactics Compos.*30(21) (2011) ,1795-1805.

[19] Qatu, M. S. and Leissa, A. W., "Natural frequencies for cantilevered doubly-curved laminated composite shallow shells", Computers and Structures. 17(3), 227-256, (1991)