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Dynamic Analysis of Laminated Composite One-fold Plates under Thermal Load

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Abstract

In this research, free vibration analysis of epoxy-based laminated composite folded plate structures for thermal loads have been considered using finite element method. Eight noded isoparametric element with five degrees of freedom per node have been considered in the study. Folded plate formulation using 6 X 6 transformation matrix is applied to transform the element mass and stiffness matrices to global system matrices. Yang-Norris-Stavsky (YNS) theory along with rotary inertia have been used in the present formulation. Lamina material properties at elevated temperature have been used in the study. Parametric studies have been performed for laminated composite one-fold plate structure for various thicknesses, crank angle and fibre angle under different temperatures. Results reveal that rising thermal load reduces the stiffness of the structure considerably. As the presence of fold increases stiffness of the plate structures significantly, it can withstand increased temperature. Proper choice of fibre angle and thickness increases the stiffness of the structures thus making it more capable of resisting higher thermal load.

Keywords: Folded plate; laminated composites; finite element; free vibration, thermal load

1. Introduction

Composite folded plate structures offer a wide range of engineering applications such as culverts, buildings, cooling towers, roofing , cladding structures etc. When the folds are applied sensibly on thin laminated composite plates, it could render them structurally stiffer and spanning broader. They have higher strength to weight ratio and higher load carrying capacity compared to flat plates. laminated composite folded plates with epoxy resin-based matrix are very cheap, easy to fabricate and the structure become even more lighter. These folded plate structures may quite often be exposed to hot weather causing significant impact on the dynamic response. Hence knowledge of dynamic properties such as natural frequencies and mode shapes are essential for these structures under rising temperature.

The research works on isotropic folded plate had started long back. It began with Goldberg and Leve [1] considering exact static analysis of folded plate structures. Irie et al. [2] reported a study on the free vibration analysis of isotropic cantilever folded-plates using Ritz method. Danial [3] and Danial et al. [4] presented the notion of spectral element method for dynamic analysis of isotropic folded plates. Liu and Huang [5] conducted free vibration analysis of cantilever folded-plates using a finite element-transfer matrix method. Bathe [6] and Zienkiewicz and Cheung [7] presented a method of flat shell analysis for use in folded plate structures directly. Later Guha Niyogi et al. [8] used nine-noded Mindlin-type plate elements using first-order transverse shear deformation along with rotary inertia to predict the free and forced vibration response of laminated composite folded plate structures. Pal et al.[9], extended this analysis for stiffened laminated composite and sandwich folded plate structures. Lee et al. [10] analyzed folded plates using third order plate theory.

Many works have been done on thermal analysis of laminated composite plate. Whitney and Ashton [11] employed Ritz method to analyze the effect of environment on the free vibration of symmetric laminates. Sai Ram and Sinha [12] conducted stress behavior and free vibration [13] analysis for cross-ply and angle-ply laminated plate, subjected to hygro-thermal environment. They adopted quadratic isoparametric plate bending finite element model. Static and dynamic analyses of the thick laminates, based on higher order shear deformation theory, and subjected to hygro-thermal environment, was carried out by Patel *et al.* [14]. The formulation accounts for the nonlinear variation of the in-plane and transverse displacements through the thickness. Zenkour [15] studied the influence of hygro-thermal environment on the angle-ply laminated plates by using a

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sinusoidal shear deformation theory. Das et al. [16] performed free vibration analysis of laminated composite folded plate subjected to hygrothermal load using FSDT.

In this research finite element free vibration behavior of two-layer laminated composite one-fold plates, subjected to thermal load, is being considered through parametric studies conducted by incorporating variation in fold angle along with overall thickness and layup sequence. Eight-noded isoparametric plate bending element using first-order transverse shear deformation theory and rotary inertia has been employed. A 6 x 6 transformation matrix is applied to transform the element mass and stiffness matrices to global system matrices. Drilling degree of freedom, θ_z, is appended with the usual five degrees-of-freedom (u,v,w,θ_x and θ_y, i.e., displacements along x, y, and z coordinates followed by rotations about x and y axes, respectively) per node to enact this transformation.

2. Theoretical Formulation

In the present finite element analysis of the folded plate structures, first order transverse shear deformation has been used including the effect of rotary inertia in element mass matrices. Eight-noded isoparametric plate finite elements with five degrees of freedom at each node has been used.

2.1. Displacement Equation

For Mindlin plate element, the displacement field for FSDT can be expressed as:

$$u(x,y,z) = u_0(x,y) + z\theta_y,$$
$$v(x,y,z) = v_0(x,y) - z\theta_x,$$
$$w = w_0$$
$$\emptyset_x = \theta_y + w_{,x},$$
$$\emptyset_y = -\theta_x + w_{,y} \quad (1)$$

Where *u, v* and *w* are displacements along the x, y and z-directions while θ_x and θ_y, are rotations about x- and y- axes. Here u_0, v_0 and w_0 are the mid-plane translations along x, y and z directions.

The constitutive equations, when subjected to uniform temperature is given by [13],

$$\{F\}=[D]\{\varepsilon\}-\{F^N\}, \quad (2)$$

where,

{F} = Mechanical force resultant

$= \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$,

$\{F^N\}$ = Thermal force resultant

$= \{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, Q_x^N, Q_y^N, 0, 0\}^T$,

$\{\varepsilon\} = \{\bar{\varepsilon}_x, \bar{\varepsilon}_y, \bar{\gamma}_{xy}, K_x, K_y, K_{xy}, \phi_x, \phi_y\}^T$

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix} \quad (3)$$

where, the thermal force and moment resultants are

$$\{N^N_x\ N^N_y\ N^N_{xy}\}^T = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left[Q'_{ij}\right]_k [e]_k dz$$

(*i, j* = 1, 2 and 6)

$$\{M^N_x\ M^N_y\ M^N_{xy}\}^T = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} z \left[Q'_{ij}\right]_k [e]_k dz$$

(*i, j* = 1, 2 and 6) (4)

where $\{e\}_k = \{e_x \ \ e_y \ \ e_{xy}\}^T$ = Thermal strain= [R] $\{\alpha_1, \alpha_2\}_k^T$ *(T-T₀)*

in which $[R] = \begin{bmatrix} cos^2\theta & sin^2\theta \\ sin^2\theta & cos^2\theta \\ \sin 2\theta & -sin\ 2\theta \end{bmatrix}$,

T and T_o is the instantaneous and reference temperature respectively.

The stiffness coefficient of the laminate are defined as

$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left[Q'_{ij}\right]_k \{1, z, z^2\}\ dz,$

for i, j =1,2,6,

and $(A_{ij}) = \alpha \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left[Q'_{ij}\right]_k dz,$

for i,j = 4,5 (5)

α is a shear correction factor, taken as 5/6, to take account for the non-uniform distribution of the transverse shear strain across the thickness of the laminate.

The non-linear strains have been expressed as [17],

2.2. Finite Element Formulation

Eight-noded isoparametric plate elements with five degrees of freedom per node have been implemented in the present computations. The stiffness matrix, the initial stress stiffness matrix, the mass matrix and the nodal load vectors of the element are derived by using the principle of minimum potential energy [13].

The element stiffness matrix is given by [18]

$$[K_e] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [B]^T [D][B] dx\ dy \quad (6)$$

Element s Geometric stiffness matrix[13] is given by $[K^r_{Ge}] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [G]^T\ \{S^r\}\ [G] dx\ dy$ (7)

The elemental mass matrix[13] is defined as $[M_e] = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} [N]^T [\rho]\ [N] dxdy$ (8)

The element load vector [13] due to thermal forces and moment is given by

$$\{P_e^N\} = \iint [B]^T \{F^N\} dxdy \quad (9)$$

The linear stiffness matrix, the geometric stiffness matrix, the mass matrix and the load vectors of the element are calculated by expressing the integral in local natural co-ordinates, ξ and η of the element and then performing Gaussian integration.

A transformation [T] is applied to transform local element linear stiffness, geometric stiffness and mass matrices into global stiffness and mass matrices [6] such that

$$\{u\} = [T]\{u'\} \quad (10)$$

Here $\{u\}$ and $\{u'\}$ denote local and global displacements, respectively. Thus, the local element stiffness and mass matrices (unprimed) are transformed to global (primed) co-ordinates using [16]

$$[K']_e = [T]^T[K]_e[T],$$

$$[M']_e = [T]^T[M]_e[T],$$

and, $[K'^{r}_{Ge}]_{\boldsymbol{e}} = [T]^T[K^r_{GE}]_e[T],$ (11)

Here $[T]^T = [T]^{-1}$ since $[T]$ is orthogonal. The natural frequencies of vibration of the plate are determined from the general equation of vibration,

$$|[K] + [K_{Ge}] - \omega_n{}^2[M]| = 0 \tag{12}$$

This generalized eigenvalue problem is solved by subspace iteration method.

3. Numerical Results

The theoretical formulation described above has been used to find vibration characteristics of laminated composite one-fold plate structure. Study of mesh convergence and validation with published literature have been furnished elsewhere [16]. A FORTRAN program has been developed for this purpose and parametric studies are carried out for variations in overall thickness, crank angle, fibre angles and external temperatures.

3.1. Case Study 1:

Effect of variation in fibre angle, thicknesses and temperature on clamped one- fold plate with crank angle β= 90° and 135°

A one-fold laminated composite folded plate, as shown in Fig. 1, has been selected for this study. Length of the two sides are taken as 0.75m each. The plate is 1.5m long. The elastic properties for the graphite-epoxy surface material taken are: E_1 = 130.0 x 10^9 N/m^2, E_2 = 9.5 x 10^9 N/m^2, G_{12} = G_{13} = 6.0 x 10^9 N/m^2, G_{23} = 0.5G_{12}, and ρ = 1600 Kg/ m^3. Along AB and BC four elements and along CD eight elements have been used in the analysis. Clamped boundary conditions have been applied to all edges. The laminate comprises of two layers of graphite-epoxy laminae with layup θ/-θ. Fibre angle θ has been varied as 30, 45, 60, 75 and 90. Investigations are done for three different thicknesses, 10mm, 20mm and 40mm. The elastic properties of the material have been altered with change in temperature. Variation in elastic properties of the graphite-epoxy composite, listed in reference 13, are reproduced in Table 1. Table 2 and Table 3 show fundamental frequency (Hz) for one-fold plate with β = 90° and 135° respectively with different fibre angle and thicknesses.

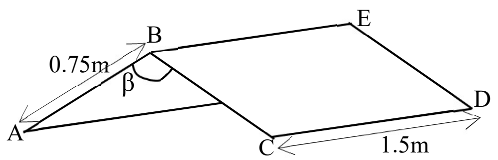

Fig. 1 Geometry of one-fold folded plate

Table 1. Properties of Graphite-Epoxy composite at different temperatures (ν_{12} = 0.3, α_1 = - 0.3 x 10^{-6}/K and α_2 = 28.1 x 10^{-6}/K)

Elastic constants (GPa)	Temperature (K)					
	300K	325K	350K	375K	400K	425K
E_1	130.0	130.0	130.0	130.0	130.0	130.0
E_2	9.5	8.5	8.0	7.5	7.0	6.75
G_{12}=G_{13}	6.0	6.0	5.5	5.0	4.75	4.5

Table 2. Fundamental Frequency (Hz) of one-fold plate (β = 90°) with fibre angle θ/-θ

Temp (K) θ	h(mm)	300	325	350	375	400	425
30/-30	40	204.72	196.50	184.50	152.25	24.49	-
	20	105.30	95.57	78.55	-	-	-
	10	53.55	35.04	-	-	-	-
45/-45	40	230.37	226.44	219.78	204.60	170.26	51.57
	20	118.61	113.04	101.16	-	-	-
	10	60.26	48.92	28.42	-	-	-
60/-60	40	272.54	271.36	267.12	258.38	243.23	207.14
	20	140.64	137.79	132.37	95.38	-	-
	10	71.31	64.18	55.24	-	-	-
75/-75	40	344.00	342.53	337.49	331.88	328.21	324.06
	20	179.54	177.16	174.12	171.12	165.75	130.81
	10	91.09	86.35	81.63	68.67	-	-
90/-90	40	429.07	427.75	423.65	419.18	416.45	413.63
	20	228.17	226.33	224.20	222.19	220.71	219.04
	10	116.20	112.81	106.27	89.42	60.22	

Table 3. Fundamental Frequency (Hz) of one-fold plate (β = 135°) with fibre angle θ/-θ

Temp (K) θ	h(mm)	300	325	350	375	400	425
30/-30	40	205.19	196.62	189.35	182.20	176.10	171.22
	20	105.36	95.84	87.02	78.13	78.13	60.58
	10	53.55	36.30	-	-	-	-
45/-45	40	230.74	223.94	217.56	211.31	206.30	202.37
	20	118.64	111.45	104.86	98.55	93.11	87.90
	10	60.24	48.42	34.55	12.49	-	-
60/-60	40	272.88	267.83	261.43	255.02	250.44	246.70
	20	140.64	135.57	130.26	125.21	121.31	117.71
	10	71.28	63.46	55.26	46.72	36.50	-
75/-75	40	344.45	341.15	333.91	326.31	321.43	316.99
	20	179.54	176.24	171.48	166.75	163.47	160.35
	10	91.05	86.10	80.39	69.02	40.58	-
90/-90	40	429.92	428.61	424.49	420.01	417.27	414.43
	20	228.21	226.40	224.28	222.27	220.80	219.14
	10	116.16	112.81	106.33	89.63	60.85	-

Fig. 2 plots variation in fundamental frequency (Hz) for different temperature for one-fold plate of fibre angle 45°/-45°. Fig. 3 shows the variation in frequency with rising temperature for different fibre angle orientation. Fig. 4

depicts the first three mode shapes of 20mm thick one-fold plate with crank angle 135° for fibre angle 45°/-45° and temperature 300K, 350K and 400K.

From Table 2 and 3 it is observed that, with increase in thickness, stiffness increases and hence fundamental frequency also increases. More stiff the plate is, more thermal load it can withstand. Though the fundamental frequency for crank angle 90° and 135° are nearly same when there is no thermal load present, with increase in temperature, 135° folded plate shows much rigid behaviour than 90° crank angle. It can withstand more thermal load than 90° as seen in Fig. 2. For stiffer structure fundamental frequency decreases linearly whereas for weaker structure, the frequency reduces exponentially at higher temperature. For 10mm and 20mm thick folded plate with β = 90° numerical instability is observed after 350K as seen from Fig. 2. The stiffness reduced to zero and hence fundamental frequency could not be found out at 375K.

From Fig. 3 it is observed that 90°/-90° fibre angle gives maximum stiffness and hence maximum fundamental frequency. Here fibres at 90° acts as reinforcements perpendicular to the supports, hence providing maximum stiffness. 30°/-30° provides least stiffness. In Fig. 4, all mode shapes show bending behaviour and the mode shapes are similar even if the temperature changes.

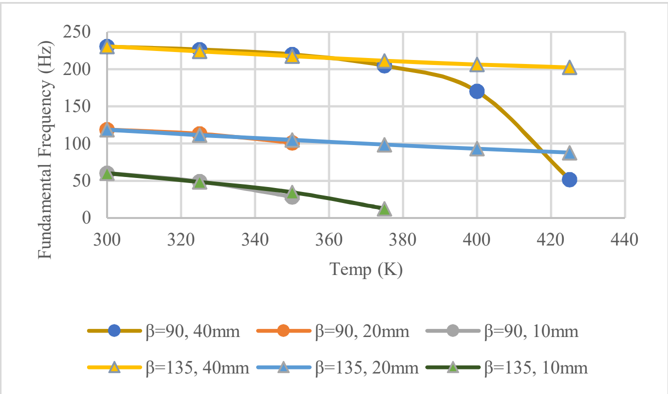

Fig. 2. Variation of fundamental frequency for crank angle 90° and 135° with fibre angle 45°/-45°

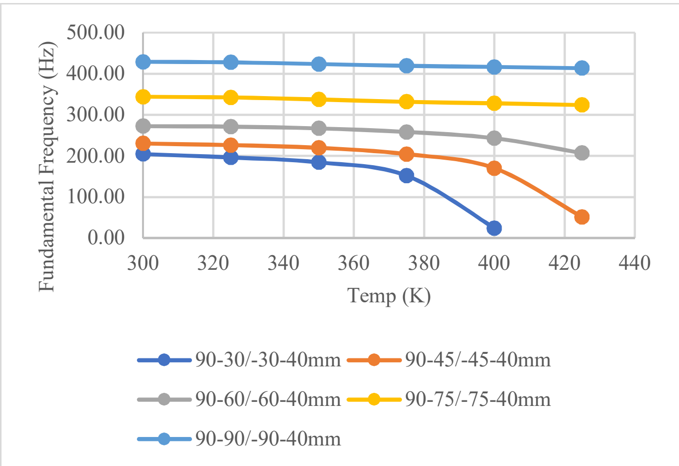

Fig. 3. Variation of fundamental frequency for different fibre angle with crank angle 90° and 40mm thickness

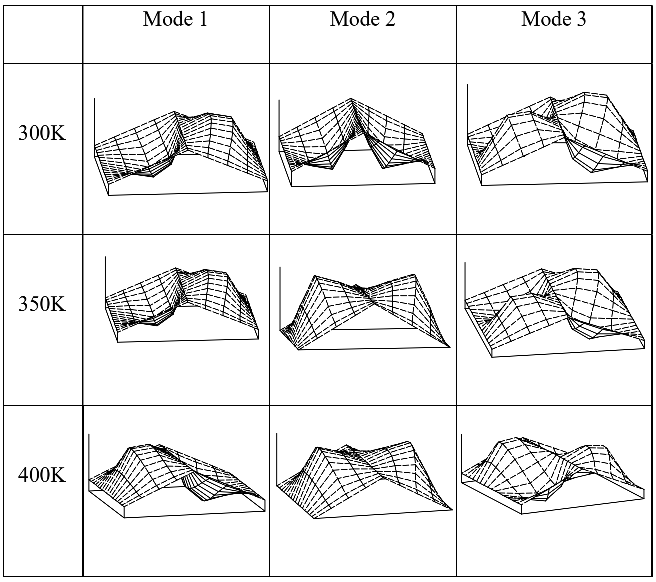

Fig. 4. First three mode shapes of 20mm thick one-fold plate with crank angle 135° for fibre angle 45°/-45°

3.2. Case Study 2:

Effect of variation in fibre angle, thicknesses and temperature on clamped flat plate with crank angle β= 180°

In this study, similar type of plate as discussed in case 1, with β= 180° i.e., a flat plate of size 1.5m x 1.5m, subjected to rising temperatures, is analyzed and shown in Table 4. Comparing Table 4 with Table 2 and Table 3, it is noticed that the fundamental frequency for flat plate is far less than that of folded plate.
It is obvious as a fold in a structure incorporates rigidity in it. In this case also, 90°/-90° fibre angle produces maximum stiffness. 30°/-30° and 60°/-60° being symmetrical for flat

Table 4. Fundamental Frequency (Hz) of flat plate (β = 135°) with fibre angle θ/-θ

θ	h(mm) \ Temp (K)	300	325	350	375	400	425
30/-30	40	121.39	116.69	112.26	107.97	104.45	101.39
	20	61.61	55.51	49.53	43.52	37.83	31.18
	10	31.03	18.43	-	-	-	-
45/-45	40	116.27	111.35	106.99	102.80	99.28	96.25
	20	58.97	52.56	46.39	40.19	34.25	27.30
	10	29.70	16.32	-	-	-	-
60/-60	40	121.39	116.69	112.26	107.97	104.45	101.39
	20	61.61	55.51	49.53	43.52	37.83	31.18
	10	31.03	18.43	-	-	-	-
75/-75	40	141.76	137.93	133.57	129.22	125.86	122.84
	20	72.28	67.22	61.95	56.54	51.17	32.63
	10	36.41	25.71	-	-	-	-
90/-90	40	170.20	167.76	165.01	162.35	160.32	158.35
	20	87.49	83.71	79.96	75.39	62.89	45.55
	10	44.13	30.70	-	-	-	-

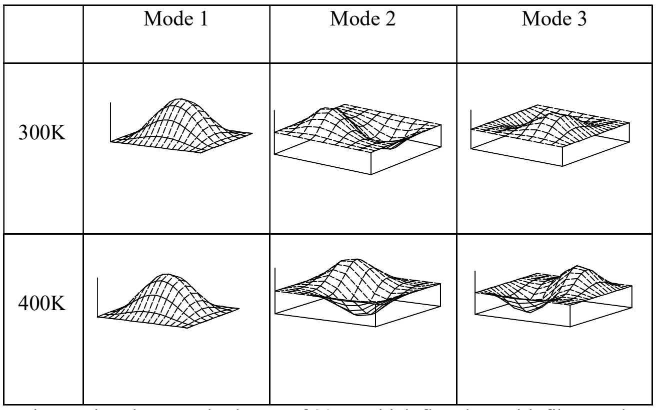

Fig. 5. First three mode shapes of 20mm thick flat plate with fibre angle $45^o/-45^o$

plate, produces same fundamental frequency. $45^o/-45^o$ develops least stiffness for this case. Instability problem arises on all 10mm thick plates after 325K due to geometric nonlinearity. Fig. 5 shows first three mode shapes for 20mm flat plate with fibre angle $45^o/-45^o$ and temperature 300K and 400K. It is seen that the mode shape remains unchanged with rise in temperature.

4. Conclusion

This research is concerned with the free vibration analysis of laminated composite one-fold folded plate structure. Finite element technique has been used for the numerical simulation of the problem. The first order shear deformation theory (FSDT) has been employed so that moderately thick folded plates could be analyzed. A new set of parametric studies are presented with variations in crank angles, wall thickness and fiber stacking sequence under rising temperature. The FSDT is found to work well for folded plates where the five regular degrees of freedom is appended with a sixth drilling degree of freedom. It is seen that presence of fold or ridge makes the structure stiffer. The ridge lines of folded plates acted as spine thus increasing its stiffness. Change of crank angle does not change the fundamental frequency noticeably. But at higher temperature it plays an important role in the dynamics of folded plate structure. Actually, the in-plane pre-stress effect due to thermal load changes the stiffness of the structure creating possible instability due to geometric nonlinearity.

Disclosures

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